A Novel Hybrid Optimization Procedure for Distribution Capacitor Placement with Varying Load Condition

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ABSTRACT

A combination of fuzzy logic, genetic algorithm and dynamic programming has been applied to optimize the problem of capacitor placement on distribution feeders. The cost function consists of three terms: the total cost of energy loss, the total cost of capacitors, including the purchase and installation costs, and one constant term related to total cost of produced power in peak load condition. A multi-objective and non-differentiable optimization problem is formulated. The proposed method of this article uses fuzzy reasoning for siting of capacitors, optimized by genetic algorithm, which finds the optimum weighting factors of fuzzy membership functions. Dynamic programming is used for sizing of the capacitors. Varying load condition of the distribution system has been considered in the optimization problem. The proposed method has been implemented in a software package and its effectiveness has been verified on a 34-bus radial distribution feeder, as a practical case study. A comparison has been made between the proposed method of this paper and similar methods in other research works, which shows the effectiveness of the proposed method of this paper for solving optimum capacitor planning problem.

***Keywords*-- Capacitor Placement, Fuzzy Logic, Genetic Algorithm, Dynamic Programming, Radial Distribution Feeder.**

1. INTRODUCTION

Shunt capacitors are installed on radial distribution feeders for many reasons, such as power flow control, improving system stability, power factor correction, voltage profile management, and losses minimization. Optimal site and size of capacitors to be installed on the buses of a radial distribution system must be determined during the planning of these systems.

In the literature, many approaches have been proposed to solve the capacitor planning problem. For example, [1] proposed a nondominated sorting genetic algorithm for the capacitor placement. In [2] optimal capacitor placement was solved using a hybrid method drawn upon the tabu search (TS), extended with features taken from other combinatorial approaches such as genetic algorithm (GA) and simulated annealing (SA). Reference [3] proposed a method based on a heuristic technique for reactive loss reduction in a distribution network. A mathematical formulation for the optimal reactive power planning taking into account the static security constraints and the uncertainty in load values was presented in [4]. Reference [5] presented a solution methodology based on a SA technique, then implemented the solution methodology in a software package and tested it on a distribution system with 69 buses. In Reference [6], GA was implemented to obtain the optimal selection of capacitors, but the objective function only considered the capacitor cost and power losses without involving operational constraints.

The capacitor planning problem is formulated as a multiple objective problem. The formulation proposed in this paper considers two distinct objectives related to (1) total cost of energy loss and (2) total cost of capacitors including the purchase and installation costs and one constant term related to total cost of produced power in peak load condition. It also considers (1) load flow restrictions (2) security and operational constraints like loading of feeders and voltage profile (3) maximum reactive compensation (4) varying load condition as practical constraints of the problem. A combination of fuzzy logic (FL)–dynamic programming (DP) approach along with GA solves the multiple objective problem with its constraints.

The rest of this article is organized as follows: Section II describes a novel formulation of the capacitor planning problem. Section III explains the fuzzy logic usage for capacitor siting. A solution algorithm based on the combination FL-DP-GA method for the multi-objective problems is developed in section IV. Section V demonstrates the effectiveness of the solution algorithm on two distribution case studies. Conclusions are finally made in section VI.

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2. MATHEMATICAL MODEL OF THE PROBLEM

The capacitor planning problem for radial distribution feeders under varying load condition can be formulated as follows:

$$Minimize \sum_{k=1}^{NLL} [(K_E \times (\sum_{i=0}^{N-1} P_{loss(i,i+1)}^{T_k}) \times T_k) + (\sum_{i=1}^{N_C} (C^{Q_i(T_k)}{}_{inst} + C^{Q_i(T_k)}{}_{purc})) + \Lambda^{T_k}]$$

(1)

Such that for each load duration:

$$P_{gi} - P_{di} - V_i \sum_{j=1}^{N} V_j Y_{ij} \cos (\delta_i - \delta_j - \theta_{ij}) = 0$$

(2)

$$Q_{gi} - Q_{di} - V_i \sum_{j=1}^{N} V_j Y_{ij} \sin (\delta_i - \delta_j - \theta_{ij}) = 0$$

(3)

$V_i^{\min} \le V_i \le V_i^{\max}$ i=1… N

(4) $P_{ij}^{\min} \le P_{ij} \le P_{ij}^{\max}$ i=1… N

(5)

$Q_C^{Total} \le Q_L^{Total}$

(6)

Where:

K_E, Cost per energy loss, \$/kWh/year

NLL, Total number of load level in a year

N, Total Number of buses in radial distribution network

$P_{loss(i,i+1)}^{T_k}$, Active power loss of (i, i+1) branch for the duration of T_k

T_k, duration of the kth load level

N_C, Total number of capacitors

$C^{Q_i(T_k)}{}_{inst}$, the cost of installation of a capacitor bank of Q (Var) on bus i for the duration of T_k

$C^{Q_i(T_k)}{}_{purc}$, the cost of purchasing of a capacitor bank of Q (Var) for bus i for the duration of T_k

he constraints, equations (3) and (4) point to the well-known load flow restrictions while security and operational constraints such that voltage profile and loading of feeders have been formulated in inequality of (5) and (6).

As a general rule, for reactive-power compensation, the maximum capacitor size should not exceed the connected reactive load. This results in a limited number of available capacitor sizes for installing on the radial distribution network. This concept has been formulated by equation (7) in the set of constraints of introduced objective function.

3. CAPACITOR SITING USING FUZZY LOGIC

Conventional crisp sets assign membership values of either 0 or 1, i.e. an object is either a member of a set or is not a member. Fuzzy sets generalize the values of set membership to any value in the interval [0, 1]. A fuzzy set can be defined mathematically by assigning a value to each possible element of membership in the set. The application of fuzzy set theory in power system engineering has been investigated by many researchers, for example [7] and [8]. Reference [9] uses approximate reasoning to determine

Λ^{T_k}, cost of produced power in peak load for the duration of T_k

P_{gi}, Q_{gi}, active and reactive power generations at bus i

P_{di}, Q_{di}, active and reactive power load at bus i

V's, δ's, system bus voltages magnitudes and phase angles

Y_{ij}, θ_{ij}, bus admittance matrix elements

Q_C^{Total}, Total connected Var by capacitor banks

Q_L^{Total}, Total Var of connected loads.

In the constant load condition, the above objective function can be reduced to the following objective function with the same constraints:

$$Minimize \{(K_P \times (\sum_{i=0}^{N-1} P_{loss(i,i+1)}^{k}) + (\sum_{i=1}^{N_C} (C^{Q_i}{}_{inst} + C^{Q_i}{}_{purc}))\}$$

(7)

The objective function introduced by equation 1, consists of three terms. The first term, $(K_E \times \sum_{k=1}^{nll} ((\sum_{i=0}^{n-1} P_{loss(i,i+1)}^{T_k}) \times T_k)$, denotes the cost of energy loss obtained by summing up the energy losses at each load considering varying load condition, the second term, $(\sum_{i=1}^{n_c} (C^{Q_i}{}_{inst} + C^{Q_i}{}_{purc}))$, includes the total cost of capacitors, that is the purchase and installation costs and finally the last term, Λ, which is a constant term, is cost of produced power in peak load condition. Building of power generation plants is according to the maximum demand of clients in a network. It is obvious that by reducing the power loss, the system operator can use the available generation capacity more effectively. In this way, a constant term has been added to the objective function for considering the cost of produced power in peak load.

Regarding t suitable candidate nodes in a distribution system for capacitor placement by utilizing fuzzy membership functions.

This paper proposes a combination of fuzzy reasoning method for finding the suitable bus for installing capacitor banks (siting), combined with a GA, which optimizes the weighting factors of the membership functions.

The membership function of equation (8) has been assigned to bus voltages and the membership function in equation (9) to line losses.

$$\mu_v(i) = e^{-w_v \cdot \left[\frac{V(i)-1}{V_{\max}-V_{\min}}\right]^2} \qquad (8)$$

$$\mu_p(i) = e^{\frac{-w_p . L(i)}{P_{T,Loss}}} \qquad (9)$$

Where:

$\mu_v(i)$: exponential membership function of voltage for bus i

W_v: weighting factor of voltage membership function

V (i): voltage of bus i

$V_{\max}$: maximum permitted voltage

$V_{\min}$: minimum permitted voltage

$\mu_p(i)$ Exponential membership function of real loss of branch i

W_p : weighting factor of real loss membership function

L(i) : real loss for line between bus i and bus i+1

$P_{T,loss}$: total power loss.

The intersection inference method is used to obtain fuzzy decision membership function, thus

$$\mu_s(i) = \min\{\mu_P(i) , \mu_V(i)\} \qquad (10)$$

The solution algorithm is explained in section IV, where the proposed method has been firstly explained for constant load condition and then it is applied to varying load condition as a practical distribution system.

4. PROPOSED HYBRID OPTIMIZATION METHOD (FL-DP-GA)

According to the principle of optimality that was introduced by Bellman and Dreyfus [10], a policy is optimal if, at a stated stage, whatever the preceding decisions may have been, the decisions still to be taken constitute an optimal policy when the result of the previous decisions is included. Using the policy of DP, this paper finds the optimal size of capacitors on each bus in radial distribution networks.

The solution algorithm for optimal distribution capacitor planning considering constant load condition, using a combined FL-DP approach is summarized as follows.

Step1. Input network and constant load data.

Step2. Determine membership functions by setting w_v and w_p.

Step3. Run power flow equations, find each bus voltage and each line loss.

Step4. Identify the bus with the lowest membership function μ_s as the candidate node for installing the capacitor bank.

Step5. Find the optimum size of capacitor in the candidate bus (determined in step4) according to the objective function and DP approach.

Step6. Check the stop criterion. If all bus voltages and line currents are in the range, go to the next step. Otherwise, go to the step 3.

Step7. Find the objective function of the whole distribution network with installed capacitors for the w_v and w_p parameters.

By running steps 1 through 7 for a constant load, the objective value (optimal site and size of capacitors) can be found for specific weighting factors (w_v and w_p) of voltage and power loss exponential membership functions.

However, the shape of membership functions (or the values of w_v and w_p) have a direct effect on the objective value. So finding the best values of w_v and w_p for having the minimum amount of objective function is of great interest. GA can be used for this purpose, i.e. finding the optimal membership functions. The main idea of GA is that “the best member of a population has the highest probability for survival and reproduction” [11]. In the literature, GA is reported to be capable of finding a global optimum for mathematical problems having a multiplicity of local optima and hard non-convexities. GA has also proved powerful in the optimization process in various power engineering applications, e.g. [12], [13]. Hence, GA is utilized to find optimal membership functions.

For this application, the decision variables of GA (chromosomes) are two variables, namely w_v and $w_{p,}$. An initial population is introduced and then the well known operators for genetic algorithm, namely, crossover and mutation are used. In this step, the original population grows through the addition of new members, which are obtained from the crossover and mutation steps. This enlarged population is ranked with a fitness function defined as follows:

$$Fitness(w_i) = \begin{cases} Obj\,Val(w_i) & If\ w_i\ meets\ all\ constra\text{int}\,s \\ B & If\ w_i\ does\ not\ meet\ all\ constrains \end{cases} \qquad (11)$$

w_i: A sample chromosome

B: A large number

Obj Val (w_i): Objective value for chromosome w_i

The proposed hybrid optimization method for optimum distribution capacitor planning is depicted in Figure 1.

As it is clear from this figure, the objective function of distribution capacitor planning problem is minimized in two steps ,namely, by FL-DP approach in placing capacitor banks and by GA method in finding w_v and w_p parameters. Thus, fuzzy logic is utilized for siting of capacitors, which is optimized by GA, and DP is used for sizing of capacitors. This feature is one of the unique powerful aspects of the proposed method for radial distribution network planning which leads to very promising results.

As a matter of fact, the load that an individual customer or a group of customers present to the distribution system constantly changes. Although it is possible to use switchable capacitors to obtain the optimum size of capacitors of relevant load duration to radial distribution feeder, but this approach is not an industrial solution for distribution radial feeders. Generally, distribution system operators are interested in one scenario of capacitors for a relatively long period, say one year. In order to consider the changing load in optimum distribution capacitor planning, the following steps are proposed.

Step1. The load curve is digitized to small time intervals, so that the power on each interval can be modelled as a constant load.

Step2. Capacitor siting is done by using fuzzy logic method, explained as follows.

By performing the load flow program during each load duration,, the membership function of bus voltages and sectional losses can be calculated. In this view, there are sets of bus voltage and sectional loss membership functions.

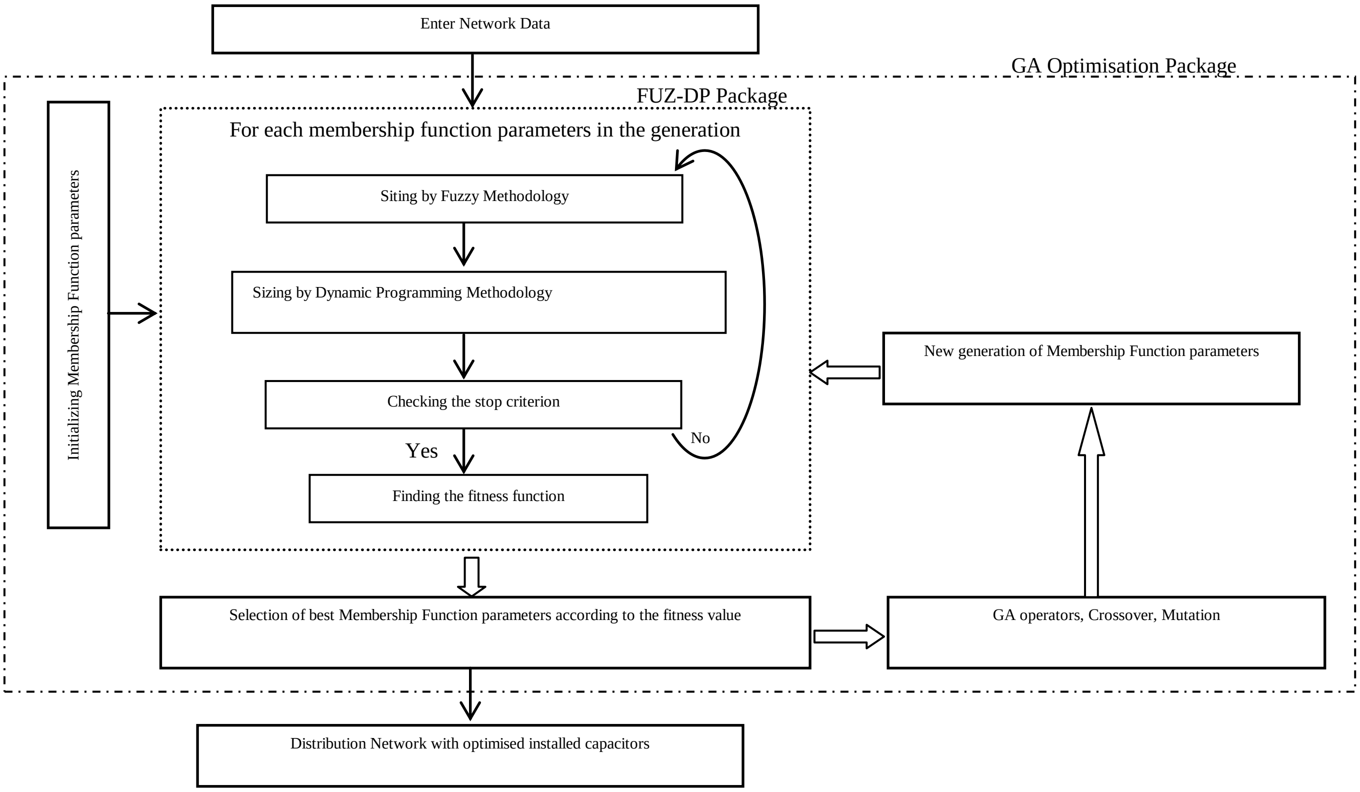

Fig. 1. Proposed Hybrid Optimization Method

The method reaches to a unique set of bus voltage and also loss membership functions by using the fuzzy inference concept. That is, by applying intersection operator to different sets of bus voltage membership functions and also sectional loss membership functions, only one set of voltage membership function as well as sectional loss membership function will be considered as the output of fuzzy inference. From this point forward, the algorithm keeps the same procedure as the one introduced for the constant load condition.

5. CASE STUDY

A 34-bus system with varying load condition

A radial distribution network with 34 load points is used to simulate the proposed hybrid optimization methodology for capacitor planning with varying load condition. The data of this test system has been taken from reference [14]. The system is shown in Figure 2.

The system voltage is 11 kV. Before compensation, the cost is US$37212, which is based on the previously defined cost function. The active and reactive losses are 221.5 kW and 65.04 kVAr, respectively, and the voltage limits in per unit are 0.9417 and 1.0.

Considering total connected reactive load of 2873.5 kVar of this system, 19 capacitor bank combinations can be used.

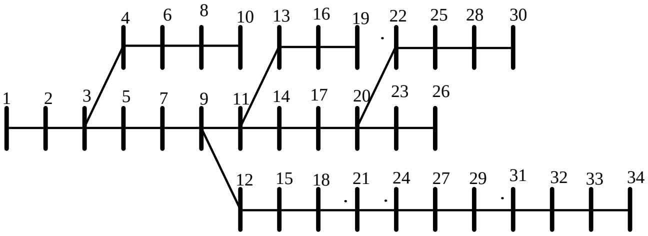

Fig. 2. A 34-bus test feeder

For applying the hybrid optimization method to varying load condition the loads of 34 bus system have been modeled as 3 load levels, namely, 0.5 p.u for a period of 1000 hours, 0.8 p.u. for a period of 6760 hours, and finally 1 p.u. for a period of 1000 hours in a year.

The system performance in each load level has been collected in Table I. Applying the hybrid optimization methodology for each load level leads to results as collected in Table II.

TABLE I
34-BUS RADIAL DISTRIBUTION FEEDER PERFORMANCE BEFORE OPTIMUM DISTRIBUTION CAPACITOR PLANNING FOR EACH LOAD LEVEL

Load Level (S,p.u.)	Real loss(kW)	Reactive loss(kVar)	Min. V(p.u.)	Max. V (p.u.)	Lossed Energy (MWh)
1	221.72	65.11	0.94169	1	221.72
0.8	139.16	40.881	0.95385	1	943.53
0.5	52.855	15.535	0.9716	1	52.855

TABLE II
34-BUS RADIAL DISTRIBUTION FEEDER PERFORMANCE AFTER OPTIMUM DISTRIBUTION CAPACITOR PLANNING FOR EACH LOAD LEVEL

Load Level(S,p.u).	Real loss(kW)	Min. V (p.u.)	Max. V (p.u.)	Optimum Site and Size of Capacitors (Bus Name-kVar)				
1	160.5	0.95068	1	9-900	24-450	30-450	32-300	34-450
0.8	101.185	0.96087	1	9-900	27-450	30-300	34-450	
0.5	39.276	0.97563	1	9-600	30-150	34-450		

It is worth noting that in the case of S =1 p.u., the proposed methodology leads to a radial compensated feeder with minimum voltage of 0.95068p.u. and total cost of US$27593, which is much better than proposed scenario of capacitors in reference [14] that leads to a minimum value of 0.95017 for bus voltages with a total cost of US$28373.

It is observed that the hybrid optimization technique proposed by this paper gives the best cost and loss reductions with a promising voltage profile among all other methodologies proposed by references [14], [15], [16] and [17]. Also, the results of the loss reduction of the proposed method are better than those of the heuristic method of [16], the analytical method of [18], and that of the fuzzy expert system presented in [9], with the same advantage of compromising between the voltage and losses importance.

Table III shows the optimum weighting factors of membership functions found by GA. The results of optimum capacitor planning by this method for considering varying load condition have been collected in Table IV.

TABLE III
EXPONENTIAL MEMBERSHIP FUNCTION WEIGHTING FACTORS FOR EACH LOAD LEVEL FOUND BY GA

S(p.u.) Weighting	1 Pu	0.8 Pu	0.5 Pu
W_v	16.411	6.6602	16.762
W_p	7.5614	2.4611	2.7304

TABLE IV
OPTIMUM CAPACITOR PLANNING USING FUZZY PRODUCT METHOD

Real loss before optimum capacitor planning (MWh)	Real loss after optimum capacitor planning (MWh)	Optimum Site and Size of Capacitors (Bus Name-kVar)			
1218.105	891.1324	9-900	27-600	30-150	34-450

By installing the optimal scenario of capacitors in 34-bus radial distribution feeder the results of Table V are obtained. The last column of Table V gives the percentage of reduction in energy losses in comparison with the network without capacitors with the same load level.

TABLE V
OPTIMUM CAPACITOR PLANNING BY FUZZY PRODUCT METHOD APPLIED TO EACH LOAD LEVEL INDEPENDENTLY

Load Level (S,p.u).	Real Loss (kW)	Reactive Loss (kVar)	Min. V (p.u.)	Max. V (p.u.)	% of Reduction in energy losses
1	162.97	47.725	0.94892	1	26.5
0.8	101.19	29.574	0.96087	1	27.5
0.5	44.118	12.794	0.97832	1	16.53

Figure 3 shows how the GA converges in finding optimum values of membership function weighting factors. As shown in the figure, the weighting factors have converged to 6.646 and 17.428 for W_P and W_V, respectively.

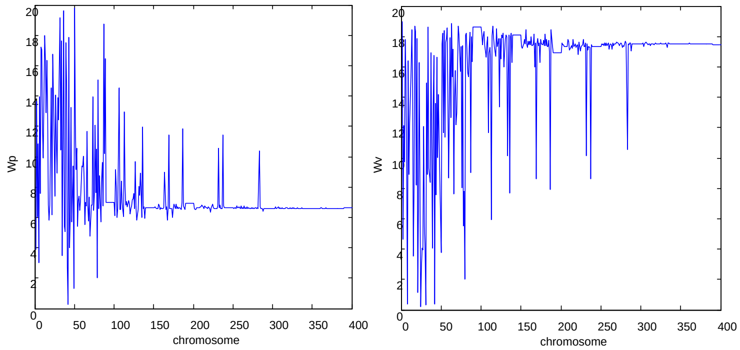

Fig. 3. Convergence of weighting Factors to optimum values for an effective load of 0.788 p.u.

6. CONCLUSIONS

This article presents a new hybrid optimization method for optimum capacitor planning problem. The proposed method uses a combination of fuzzy logic, dynamic programming and genetic algorithm for optimum siting and sizing of the capacitors. Fuzzy logic, whose membership functions are optimized by genetic algorithm, is used for siting of the capacitors. Dynamic programming is used for sizing the capacitors.

The method developed herein is tested on a 34-bus distribution system with varying loads and the results have been compared with similar research works. The comparison shows the effectiveness of the proposed method taking into account both costs and operation of the distribution network.

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