MECHANISM DESIGN AND DYNAMIC FORMULATION FOR CART/SEESAW/PENDULUM SYSTEM

J. Lin, Julian Chang, S.-W. Wang, and C.J. Huang

***Abstract*—Underactuated mechanism have recently attracted a lot of attention because they represent a rich class of control system from a control stand point and because they offer robot price reduction due to robot link mass reduction and lower a number of robot actuators. In contrast to the system with full controls, a super articulated mechanical system (SAMS) is a controlled mechanical system in which the dimension of the configuration space exceeds the dimension of the control input space. The main objectives of the research are to develop a new SAMS model, which is called cart-seesaw-pendulum system. The system consists of two mobile carts, which are coupled via rack and pinion mechanics to two parallel tracks mounted on pneumatic rodless cylinders. The cylinder was double-acting. One cart carries an inverted pendulum, which is free to fall. The other cart serves as a counterbalance. Forces are applied to each cart. All the elements are mounted on the seesaw platform. The seesaw is jointed and frees to rotate in unison about the pivot point. The goal is to control the seesaw angle and the pendulum-carrying cart position while keeping the pendulum upright. The dynamic formulation is the first proposed for control purposes of such new model. Numerous simple real-time experiments were performed to verify the function of this new model. Consequently, the proposed software/hardware platform can be also profitable for the standardization of laboratory equipment, as well as for the development of entertainment tools.**

I. INTRODUCTION

Underactuated mechanisms have recently attracted significant attention owing to their ability not only to represent a rich class of control system from a control stand point, but to reduce the cost of robots by decreasing the link mass and the number of robot actuators [1]. Contrary to the system with full controls, a super articulated mechanical system (SAMS) is a controlled underactuated mechanical system in which the dimension of the configuration space exceeds the dimension of the control input space. For example, inverted pendulum on a cart, ball and beam problem, mass sliding on a cart and robots with joint elasticity, underactuated bipedal robot, nonholonomic mobile robot, etc, all is SAMS [2].

A super-articulated mechanical system presents challenges that are not found in a system with full controls, in which the dimensions of the configuration space equals the dimension of the control input space. Literature [3-6] on motion planning and control of nonholonomic systems has indicated the difficult yet interesting features of control synthesis for SAMS.

The ball and beam system is a common undergraduate control laboratory experiment [7]. Control of the ball and beam system has been widely discussed in teaching and research literature. However, discovering a control law to stabilize the system remains an active topic of research discussion.

The ball and beam mechanism generally comprises a beam with a ball on it. The ball rolls on the beam according to the changing angle of the beam. A ball moving on a beam is a typical nonlinear dynamic system, which is often adapted to proof-test diverse control methods [8]. Such a system may be adopted as a control-training tool by engineering students to test industrial processes and their applications.

Moreover, the “ball and the beam” system is well documented as an example of a system that requires an active control system to maintain the ball at a desired beam position. Several approaches have been presented to control the ball and beam system during the past decades, for example, input-output feedback linearization [8], robust nonlinear control [9-10] and fuzzy logic control [11-13]. However, all of the above papers are based on the same conventional ball-on-beam plant, in which the ball is rolling on the beam according to the changing angle of the beam. The system lacks a new mechanism for other control purposes. However, the review of the above papers indicates that no researchers have attempted to control a ball-and-beam mechanism by a pneumatic cylinder for actuation.

Furthermore, the paper proposes a new cart-seesaw system is shown in [14]. The seesaw can rotate only in a vertical plane, with one degree of freedom, and the cart slides along the seesaw by applying a force with a pneumatic device. This study investigation serves as a reference of the achievable control behavior for the underactuated

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mechanism, and covers the extension of the curriculum to the control of the underactuated robots.

Pneumatic cylinders are sometimes considered to offer a better alternative to electrical or hydraulic actuators for certain types of applications. Unfortunately, owing to the compressibility of air, highly nonlinear behavior and time delay resulting from the slow propagation of pressure waves, position and force control of these actuators are difficult [15].

This work is the extension of the author's previous work [14]. In literature [14], the ball-and-beam-like mechanism of that research replaces the ball is replaced with a cart, which slides 'frictionlessly' on the pneumatic rodless cylinder. The proposed system consists of only one sliding cart. However, the main objectives of this research are to develop a new SAMS, which is called cart-seesaw-pendulum system. The system consists of two mobile carts, which are coupled via rack and pinion mechanics to two parallel tracks mounted on pneumatic rodless cylinders. The cylinder was double-acting. One cart carries an inverted pendulum, which is free to fall. The other cart serves as a counterbalance. Moreover, an adjustable counterweight mass is installed underneath the seesaw to serve as a damping mechanism to absorb impact and shock energy. All the elements are mounted on the seesaw platform. The seesaw is jointed and frees to rotate in unison about the pivot point. The goal is to control the seesaw angle and the pendulum-carrying cart position while keeping the pendulum upright. The proposed work will serve us as a reference of the achievable control behavior for the underactuated mechanism.

II. SYSTEM CONFIGURATION

Figure 1 indicates the conceptual model of a cart-seesaw-pendulum system. Figure 2 depicts the visualization of this cart-seesaw system, and indicates some important devices in such a system. The system consists of two mobile carts, which are coupled via rack and pinion mechanics to two parallel tracks mounted on pneumatic rodless cylinders. The cylinder was double-acting. One cart carries an inverted pendulum, which is free to fall. The other cart serves as a counterbalance. Forces are applied to each cart. The seesaw is jointed and frees to rotate in unison about the pivot point. Moreover, a linear potentiometer was utilized to measure the position of the sliding cart, and rotary potentiometers were adopted to measure the seesaw and inverted pendulum angles respectively. Additionally, the cart position was measured from the seesaw center, and was positive if the cart was on the right side of the seesaw. Similarly, the rotary potentiometer was adopted to measure the seesaw and inverted pendulum angle. The seesaw angle was positive if the seesaw rotated counterclockwise from horizon.

Furthermore, the seesaw falls to the definite direction instantaneously if the cart runs to a particular place, thus creating instantaneous dynamic instability. Hence, in dynamical improvement aspect, adjustable counterweight mass is installed underneath the seesaw to serve as a damping mechanism to absorb impact and shock energy. This mechanism is called dynamic balance apparatus.

The experimental pneumatic system was composed of two pneumatic rodless cylinder 510 mm long, four controlled proportional valves, and a source of compressed air. The air supply to the cylinder was manipulated by an electro-pneumatic transducer that provided an air pressure proportional to the supply voltage. Each direction of motion was selected by appropriate actuation of the 3/2-way electro-valves (model type: SMC VEF 3121-1), which converted the electrical signal to proportional airflow. Fig. 3 displays the pneumatic control circuit.

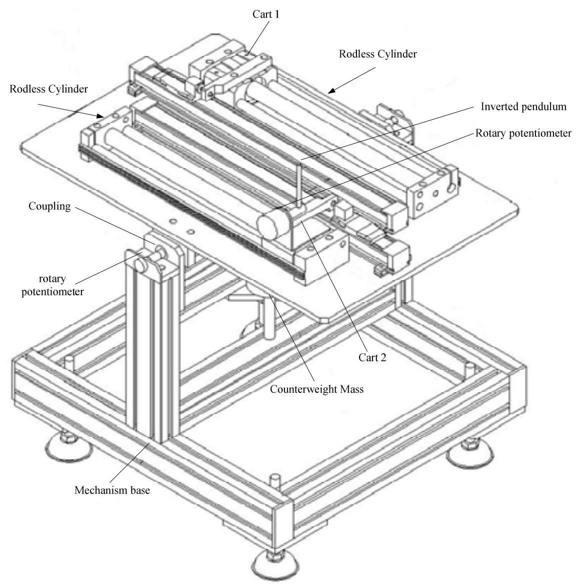

Fig. 1 Conceptual model of the proposed system

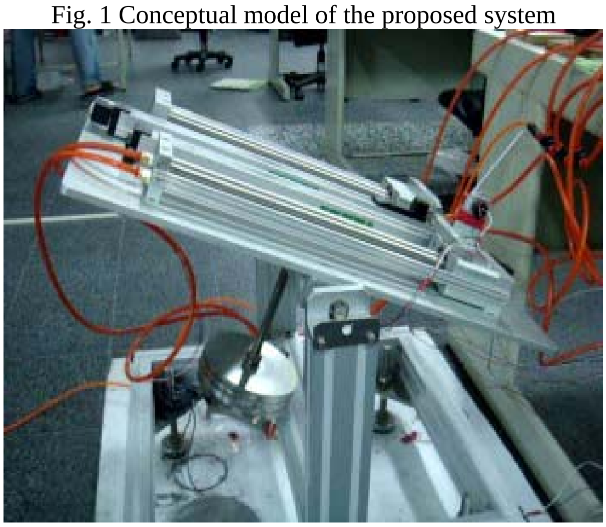

Fig. 2 Visualization of Cart-Seesaw System

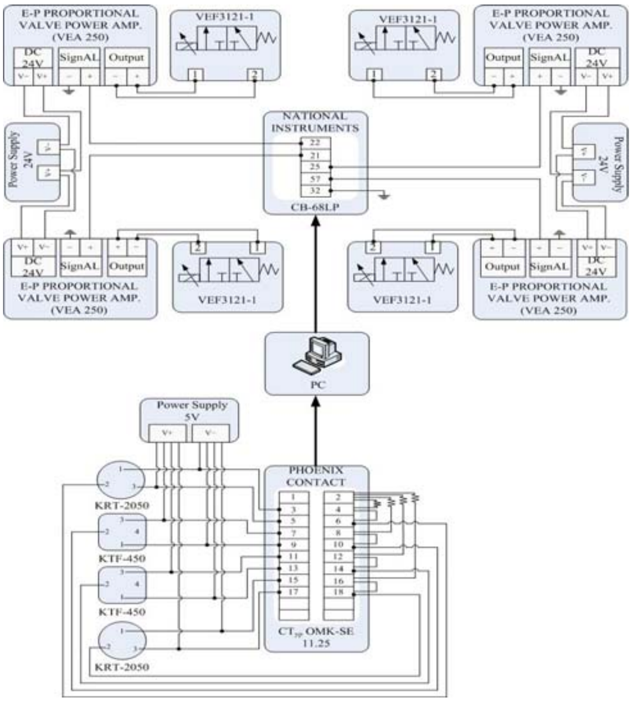

Fig. 3 Control circuit for the proposed system

III. DYNAMIC MODELING

In this section, a mathematical model of the pneumatic cart-seesaw-pendulum system is obtained from independently known dynamics. Consider the cart-seesaw system illustrated in Fig. 4. The cart-seesaw-pendulum system brings the cart from any initial position with any initial speed to a desired position on the seesaw by applying an appropriate force to the cart, and thus adjusting the angle of the seesaw. The cart 1 and cart 2 sliding on the seesaw indicates the first and second degree of freedom respectively, which is actuated by a pneumatic proportional control valve, and the angle of a seesaw and inverted pendulum represents the third and fourth degrees of freedom, which are not actuated.

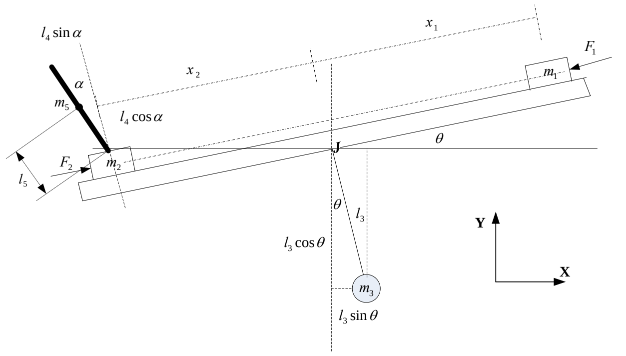

Fig. 4 Dynamic model for Cart-Seesaw-Pendulum system

Let the moment of inertia of the seesaw be *J*; the gravity acceleration *g*, and the mass of the sliding carts be m_1 and m_2, respectively. The counterweight mass m_3 and its length l_3 is attached on the underneath of the seesaw to serve as the vibration damper in order to absorb impact and shock energy. Moreover, the mass of inverted pendulum is m_5 and assumes that the mass is concentrates at the midpoint of the pendulum. l_5 is the half length of the inverted pendulum. Furthermore, θ is the seesaw angle and α is the pendulum angle. F_1 and F_2 are the applied force to each cart, respectively. The main goal is to control the seesaw angle and the pendulum-carrying cart position while keeping the pendulum upright. Hence, the whole system can be divided into five subsystems – cart 1, cart 2, counterweight mass, seesaw, and inverted pendulum. Therefore, we can compute the kinetic energy and potential energy for these subsystems and then derive the global dynamic equations. Selecting the slide carts position x_1 and x_2, the seesaw angle θ, and the pendulum angle α as generalized coordinates for the system, by using the Lagrangian formulation, the dynamic equations can be derived as the following procedure.

First, we compute the kinetic energy of cart 1

$$K_1 = \frac{1}{2} m_1 \left(\dot{x}_1^2 + x_1^2 \dot{\theta}^2\right). \tag{1}$$

Then, the kinetic energy of cart 2 can be written as

$$K_2 = \frac{1}{2} m_2 \left(\dot{x}_2^2 + x_2^2 \dot{\theta}^2\right). \tag{2}$$

Similarly, the kinetic energy for the counterweight mass is

$$K_3 = \frac{1}{2} m_3 l_3^2 \dot{\theta}^2 , \tag{3}$$

and the kinetic energy of the seesaw

$$K_4 = \frac{1}{2} J \dot{\theta}^2 . \tag{4}$$

Furthermore, in order to derive the kinetic energy of the inverted pendulum, we assume that the entire mass of the pendulum is concentrated on the point of the center of mass. Then the position vector in the **X** and **Y** direction can be defined as

$$\begin{aligned} x_5 &= \left(l_5 \sin\alpha + x_2\right)\cos\theta + l_5 \cos\alpha \sin\theta \\ y_5 &= \left(l_5 \sin\alpha + x_2\right)\sin\theta + l_5 \cos\alpha \cos\theta \end{aligned} \tag{5}$$

The velocity in the **X** and **Y** direction is

$$\begin{aligned} \dot{x}_5 &= \left(l_5 \dot{\alpha} \cos\alpha + \dot{x}_2\right)\cos\theta + \left(l_5 \sin\alpha + x_2\right)\dot{\theta}\left(-\sin\theta\right) \\ &\quad + l_5 \dot{\alpha}\left(-\sin\alpha\right)\sin\theta + l_5 \cos\alpha \dot{\theta} \cos\theta \end{aligned} \tag{6a}$$

$$\begin{aligned} \dot{y}_5 &= \left(l_5 \dot{\alpha} \cos\alpha + \dot{x}_2\right)\sin\theta + \left(l_5 \sin\alpha + x_2\right)\dot{\theta} \cos\theta \\ &\quad + l_5 \dot{\alpha}\left(-\sin\alpha\right)\cos\theta + l_5 \cos\alpha \dot{\theta}\left(-\sin\theta\right) \end{aligned} \tag{6b}$$

Hence, the kinetic energy of the inverted pendulum is

$$\begin{aligned} K_5 &= \frac{1}{2} m_5 (\dot{x}_5^2 + \dot{y}_5^2) \\ &= \frac{1}{2} m_5 [(l_5 \dot{\alpha} \cos\alpha + \dot{x}_2)^2 + (l_5 \sin\alpha + x_2)^2 \dot{\theta}^2 + l_5^2 \dot{\alpha}^2 \sin^2\alpha \\ &\quad + l_5^2 \dot{\theta}^2 \cos^2\alpha - 2(l_5 \dot{\alpha} \cos\alpha + \dot{x}_2) l_5 \dot{\alpha} \sin\alpha \sin 2\theta \\ &\quad + 2(l_5 \dot{\alpha} \cos\alpha + \dot{x}_2) l_5 \dot{\theta} \cos\alpha (\cos^2\theta - \sin^2\theta) \\ &\quad + 2(l_5 \sin\alpha + x_2) l_5 \dot{\theta} \dot{\alpha} \cos\alpha (\sin^2\theta - \cos^2\theta) \\ &\quad - 2(l_5 \sin\alpha + x_2) l_5 \dot{\theta}^2 \cos\theta \sin 2\theta] \end{aligned} \tag{7}$$

Next, we calculate the potential energy for each subsystem.

Potential energy of cart 1

$$P_1 = m_1 g x_1 \sin\theta \tag{8}$$

Potential energy of cart 2

$$P_2 = m_2 g x_2 \sin\theta \tag{9}$$

Potential energy of the seesaw

$$P_3 = -m_3 g l_3 \cos\theta \tag{10}$$

Potential energy of the inverted pendulum

$$P_5 = m_5 g[(l_5 \sin\alpha + x_2)\sin\theta + l_5 \cos\alpha\cos\theta] \tag{11}$$

Then, the total kinetic energy of the whole system is

$$K = K_1 + K_2 + K_3 + K_4 + K_5 \tag{12}$$

Similarly, the total potential energy of the whole system is

$$P = P_1 + P_2 + P_3 + P_5 \tag{13}$$

Now, we derive the equation of motion by using Lagrange's formulation:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau\ , \tag{14}$$

where L=K-P, $q_i = \begin{bmatrix} x_1 & x_2 & \theta & \alpha \end{bmatrix}$, and $\tau = \begin{bmatrix} F_1 & F_2 & 0 & 0 \end{bmatrix}$.

Therefore, the dynamic equation can be shown in the following matrix form:

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ 0 \\ 0 \end{bmatrix} \tag{15}$$

where M_{ij} denote the element of inertia matrix, N_i is the Coriolis/centrifugal term, and G_i means the gravity term.

Finally, the detail symbolic term of Eq. (15) are indicated as below

$$M_{11} = m_1$$

$$M_{12} = M_{21} = 0$$

$$M_{13} = M_{31} = 0$$

$$M_{14} = M_{41} = 0$$

$$M_{22} = m_2 + m_5$$

$$M_{23} = M_{32} = m_5 l_5 \cos\alpha\cos 2\theta$$

$$M_{24} = M_{42} = m_5 l_5 (\cos\alpha - \cos\alpha\cos 2\theta)$$

$$\begin{aligned} M_{33} &= m_1 x_1^2 + m_2 x_2^2 + m_3 l_3^2 + J + m_3 l_5^2 + m_5 x_2^2 + 2 m_2 l_5 \sin\alpha x_2 \\ &\quad - m_5 l_5^2 \sin 2\alpha \sin 2\theta - 2 m_5 l_5 x_2 \cos\alpha \sin 2\theta \end{aligned}$$

$$M_{34} = M_{43} = m_5 l_5^2 \cos 2\alpha \cos 2\theta - m_5 l_5 x_2 \sin\alpha \cos 2\theta$$

$$M_{44} = m_5 l_5^2 - m_5 l_5^2 \sin 2\alpha \sin 2\theta$$

$$N_1 = 0$$

$$\begin{aligned} N_2 &= -[m_5 l_5 \cos\alpha \sin 2\theta(2 l_5 + 1) + m_2 x_2 + m_2 l_5 \sin\alpha + m_5 x_2]\dot{\theta}^2 \\ &\quad + m_5 l_5 (\sin\alpha + \cos\alpha \sin 2\theta)\dot{\alpha}^2 + m_5 l_5 \sin\alpha \cos 2\theta(3 l_5 + 1)\dot{\alpha}\dot{\theta} \end{aligned}$$

$$\begin{aligned} N_3 &= (2 m_1 x_1)\dot{x}_1\dot{\theta} \\ &\quad + [(2 m_2 x_2 + 2 m_5 x_2 + 2 m_2 l_5(\sin\alpha - 2\cos\alpha \sin 2\theta + l_5 \cos\alpha \cos 2\theta)]\dot{x}_2\dot{\theta} \\ &\quad - (2 m_5 l_5 \sin\alpha \cos 2\theta)\dot{x}_2\dot{\alpha} \\ &\quad + 2 m_5 l_5 (\cos x_2 - \mathbf{2} l_5 \cos 2\alpha \sin 2 - x_2 \sin\alpha \sin 2\theta)\dot{\alpha}\dot{\theta} \\ &\quad - m_5 l_5 x_2 (2 l_5^2 \sin 2\alpha \cos 2\theta + 6 x_2 \cos\alpha \cos 2\theta - l_5 x_2 \sin 2\alpha \cos 2\theta)\dot{\theta}^2 \\ &\quad - (m_5 l_5^2 \sin 2\alpha \cos 2\theta)\dot{\alpha}^2 - (m_5 l_5^2 \sin 2\alpha \cos 2\theta)\dot{\alpha}\dot{\theta}^2 \end{aligned}$$

$$\begin{aligned} N_4 &= -m_5 l_5 (\cos\alpha \sin 2\theta - \sin\alpha)\dot{x}_2\dot{\alpha} - (m_5 l_5 \sin\alpha)\dot{\alpha} \\ &\quad - (m_5 l_5 \sin\alpha \cos 2\theta)\dot{x}_2\dot{\theta} \\ &\quad + m_5 l_5 (x_2 \cos\alpha \cos 2\theta + l_5 \sin 2\alpha \cos 2\theta + l_5 x_2 \cos\alpha \cos 2\theta)\dot{\alpha}\dot{\theta} \\ &\quad - m_5 l_5^2 (2\dot{\alpha}^2 \cos 2\alpha \sin 2\theta - \cos\alpha \sin\alpha - \sin 2\alpha)\dot{\alpha}^2 \\ &\quad - m_5 l_5 (2 l_5 \cos 2\alpha \sin 2\theta + 2 x_2 \sin\alpha \sin 2\theta + l_5 \sin\alpha \cos\alpha)\dot{\theta}^2 \end{aligned}$$

$$G_1 = m_1 g \sin\theta$$

$$G_2 = (m_2 + m_5) g \sin\theta$$

$$\begin{aligned} G_3 &= (m_1 x_1 + m_2 x_2) g \cos\theta - m_3 g l_3 \sin\theta - m_5 g x_2 \cos\theta \\ &\quad + m_5 l_5 g \cos\alpha \sin\theta \end{aligned}$$

$$G_5 = -m_5 l_5 g(\cos\alpha + \cos\alpha \sin\theta - \sin\alpha \cos\theta)$$

The inertia matrix of the overall system $\mathbf{M}(x)$ shown in Eq. (15) is symmetric and positive definite. Therefore, the inertia matrix $\mathbf{M}(x)$ which is uniformly bounded both from above and from below, namely, satisfies

$$\underline{m} I_{4\times4} \le \mathbf{M}(x) \le \overline{m} I_{4\times4}\ ,$$

where $\underline{m}$ and $\overline{m}$ are positive constants, and $I_{4\times4}$ is the identity matrix.

The resulting equations above demonstrate that the element of the dynamic equation matrix is very complex and highly nonlinear. Because manual symbolic expansion for such system is tedious, time consuming, and prone to errors, an automated derivation process is highly desirable. Therefore, a symbolic program written in MATLAB to generate the dynamic equations for such system is used in this research.

IV. PREPLINARY RESULTS AND DISCUSSIONS

In order to test the function of the new mechanism of cart-seesaw-pendulum system, numerous real-time experiments were performed to verify the system. The aim of the experiment was to achieve a desired sliding mass position on the cart subjected to the different location of the counterweight mass. Double solenoid proportional direction control valves were used to drive two double-acting pneumatic rodless cylinders. The air supply was regulated to 6 bar (6kgf/cm²). The controller of the cart-seesaw system is composed of two NI DAQ boards (PCI-MIO-16E-4 and PCI-6174) with a host personal computer. The controller board with its real time software interface permits rapid control prototyping in connection with LabVIEW, thus enabling a quick implementation of the proposed supervisory control approach on the basis of the real-time block model applied in LabVIEW.

To achieve the best possible control of a particular system, as much information about the system as possible should be used when designing a controller. Simple experimental test is typically necessary for comparing the performance of controllers acting on a nonlinear system. In this section, we simplify the system to cart-seesaw system. The inverted pendulum is neglected in this test. Therefore, the main objective of the test is to investigate the stabilization of the equilibrium point for the seesaw.

Figure 5 plots the time response for carts position (Fig. 5a) and seesaw angle (Fig. 5b) with adjustable counterweight mass which locates under the seesaw $l_3 = 9\mathbf{cm}$. It demonstrates that the counterweight mass owns a damping capability to stabilize the dynamic system. Moreover, while the adjustable counterweight mass locates under the seesaw $l_3 = 18\mathbf{cm}$, the seesaw angle limitation will become smaller (Fig. 6). Furthermore, adjusting the location of the counterweight mass will further change the seesaw angle limitation. Figure 7 indicates that the seesaw angle limitation becomes smaller while the counterweight mass device farther from the seesaw ($l_3 = 27\mathbf{cm}$) applied. The farther from the seesaw of the counterweight mass device, the smaller seesaw angle limitation. Consequently, the proposed counterweight mass device produced a substantial improvement in performance of the seesaw equilibrium motion. The counterweight mass device can serve as a damping mechanism to absorb impact and shock energy. This mechanism is called dynamic balance apparatus. Hence, the proposed counterweight mass device is a powerful and efficient way to cope with such a cart-seesaw system.

Due to the time and space limitation, this research only investigates some simple experimental test to verify the performance of the proposed cart-seesaw-pendulum system. However, the development the proposed software/hardware platform can be also profitable for the standardization of laboratory equipment, as well as for the development of entertainment tools. It can provide with the experimental apparatus for the interesting person to explore the performance for such system.

V. CONCLUSIONS

This work is the extension of the author's previous work [14] and modifies to a new SAMS model. The main objective of this research is to develop a model, which is called cart-seesaw-pendulum system. The system consists of two mobile carts, which are coupled via rack and pinion mechanics to two parallel tracks mounted on pneumatic rodless cylinders. The cylinder was double-acting. One cart carries an inverted pendulum, which is free to fall. The other cart serves as a counterbalance. Forces are applied to each cart. The seesaw is jointed and frees to rotate in unison about the pivot point. In order to absorb impact and shock energy, the adjustable counterweight mass is attached on the underneath of the seesaw. The dynamic formulation is the first proposed for control purposes of such new model. The result of the proposed software/hardware platform can be also profitable for the standardization of laboratory equipment, as well as for the development of entertainment tools. Therefore, the future work will demonstrate the controller design and verify the performance for such system.

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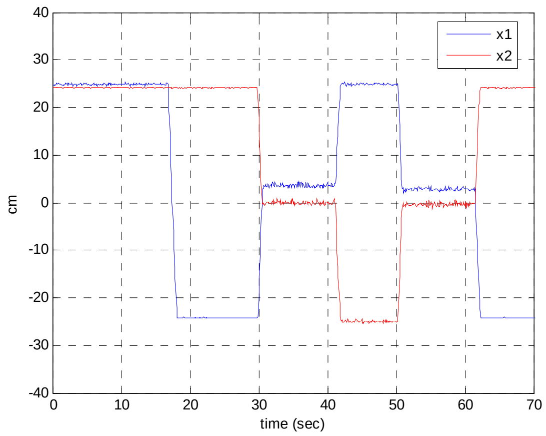
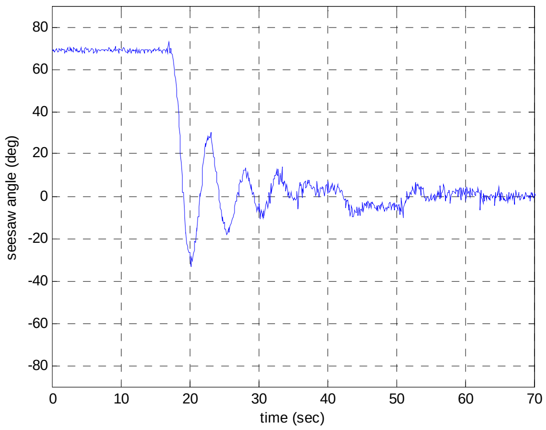
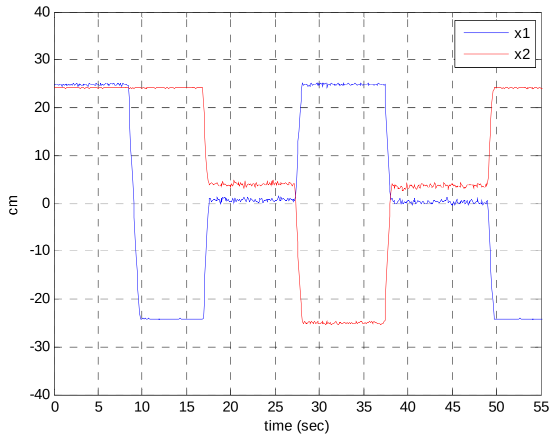


(a) Position of cart 1 and cart 2

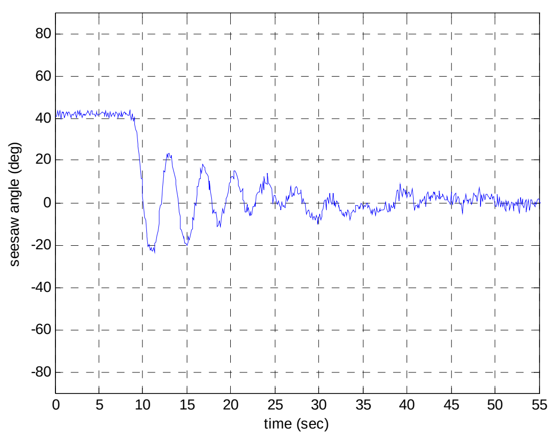


(a) Seesaw angle

Fig. 5 Position response of the cart-seesaw system (counterweight mass $l_3 = 9$**cm**)

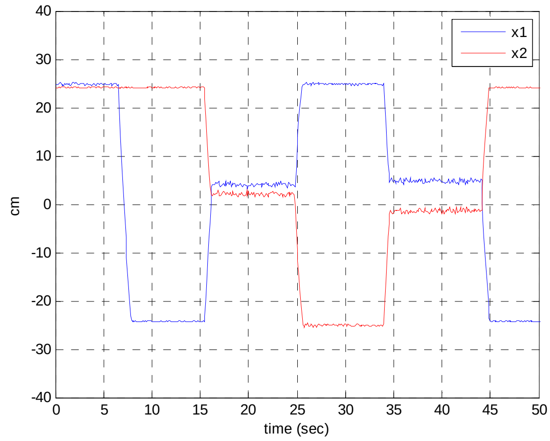


(a) Position of cart 1 and cart 2

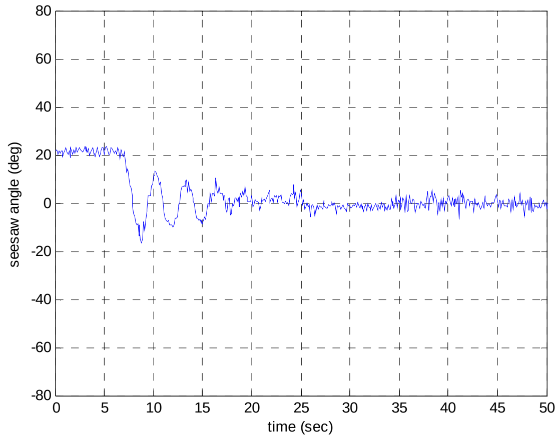


(b) Seesaw angle

Fig. 6 Position response of the cart-seesaw system (counterweight mass $l_3 = 18$**cm**)



(a) Position of cart 1 and cart 2



(b) Seesaw angle

Fig. 7 Position response of the cart-seesaw system (counterweight mass $l_3 = 27$**cm**)