Uncertainty model based on partial linearization and robust control

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ABSTRACT
A an uncertainty model is presented that is useful to the robust control design. Under-actuated state affine systems are considered. A partially linear model is gotten taking into account the second and third order partial derivatives of the Taylor series expansion around a constant equilibrium point. This model includes the linear model and constant matrices multiplied by the quadratic and cubic of the state. These constant matrices are known and are included in the stability and performance criterions and the quadratic and cubic terms of the state are proposed as an uncertainty model. The finite gain of the uncertainty model is obtained on line and the Small Gain Theorem is used to guarantee stability. A stabilizing controller is designed for the nominal plant and its free parameter is actualized on line solving a mixed sensitivity problem. A robust controller is designed for this type of uncertainty and is applied to the under-actuated non-linear model of a Pendubot. The considered equilibrium point is at top position and the uncertainty model has only a cubic state term. A non-linear control is used to go from low position to around the top position when this control commutes to the robust controller getting the desired performance. The results are shown in simulations.

Keywords: Uncertainty, finite gain, mixed sensitivity, Taylor series, stabilizing controllers, Pendubot.

I. INTRODUCTION
A physical system cannot be modeled exactly, due to the presence of uncertainties that are the consequence of unpredictable system dynamics or dynamics difficult to model. Identification procedures gives accurate and precise models only at low frequencies. Uncertainty [14] implies that the output of a real physical system cannot be predicted exactly, even if the input and the initial conditions are known. Often the models that represent the behavior of a physical system are non-linear and time variant, which are represented by very complex differential equations or by difficult structures. Therefore, a perfect model cannot be proposed, however, an effective and manageable model can be build representing the essential dynamics of the system.

Frequently, a non-linear control is designed to control the steepest changes of the system, and this control commutes to a linear control (based on a linear model of the system) that hold the system within its linear regime. A “good” choice for the linear control is a robust control [14] that preserves stability and performance for systems subject to uncertainties and external disturbances, extending the operation region of the system around the equilibrium point, that is, around the linear regime of the system. Stability is guaranteed by the Small Gain Theorem [14] for a given uncertainty level. Due to this becomes the interest to get a linear model with associate uncertainty (partially linear model).

In section II a partially linear model of the system, that includes an uncertainty model, obtained from a non-linear model of the plant is presented. This model is based on the Taylor series expansion around a constant equilibrium point. The constant matrices of the second and third order partial derivatives are included in the transfer function from the output to the input of the uncertainty. So, the additive uncertainty model has only the quadratic and cubic of the state. The finite gain of the uncertainty or uncertainty level is measured on line in section IV, and is used to get the free parameter of the stabilizing controller [14]. The Small Gain Theorem is used to guarantee stability. Thus an on-line improvement of the robust control algorithm presented by [8], [9] and [10] is proposed, in which a mixed sensitivity problem is solved, i.e., robust stability and robust performance are guaranteed. The analytic expression of [10] for the free parameter is used. This expression preserves stability in high frequencies under uncertainties and the regulated output tracks the reference signal in low frequencies.

The considered control scheme is a direct adaptive control scheme. Applications of adaptive control to under-actuated systems have been developed, see for instance [12] for an underwater autonomous system or [6] for under-actuated mechanical systems. An application of passivity control to a double inverted pendulum can be see in [13] and an energy approach is used in [2]. The double inverted pendulum, the Acrobot [3] and the under-actuated planar robot [1] are similar to the two degrees of freedom planar rotational robot (Pendubot).

A robust controller is designed and is applied to the
under-actuated non-linear model of a Pendubot in section V. The considered equilibrium point is at top position and the uncertainty model has only a cubic state term. A feedback linearizing non-linear control is used to go from low position to around the top position when this control commutes to the robust controller getting the desired performance. The results are shown in simulations.

**Notation.** diag\{a_1, a_2, \ldots, a_r\} a r \times r diagonal matrix whose elements are a_i, i = 1, \ldots, r.

**II. PARTIALLY LINEAR MODEL**

The considered class of non-linear models is described by the following nonlinear differential equation,

\[
\begin{align*}
\dot{x}(t) &= f(x(t), u(t)) \\
y(t) &= g(x(t), u(t))
\end{align*}
\]

where \(f\) and \(g\) are continuous and differentiable non-linear functions, \(x(t) \in \mathbb{R}^{n \times 1}\), \(u(t) \in \mathbb{R}^{m \times 1}\) and \(y(t) \in \mathbb{R}^{p \times 1}\) are the state, the input and output, respectively.

In the following Proposition it is proposed to cut off the Taylor series expansion arriving to a partially linear model that approach to the behavior of the non-linear model.

**Proposition 1.** Suppose that the non-linear system given by Eq. (1) is not chaotic. Let \((x_o, u_o)\) be a constant equilibrium point of Eq. (1), \(f\) and \(g\) are continuous and differentiable functions of time, and defining the Jacobian Matrices,

\[
A := \frac{\partial f}{\partial x}(x_o, u_o), \quad B := \frac{\partial f}{\partial u}(x_o, u_o)
\]

\[
C := \frac{\partial g}{\partial x}(x_o, u_o), \quad D := \frac{\partial g}{\partial u}(x_o, u_o)
\]

the Hessian matrices,

\[
F := \text{diag}\left\{\frac{\partial^2 f}{\partial x^2}, \ldots, \frac{\partial^2 f}{\partial x^3}\right\}(x_o, u_o)
\]

\[
G := \text{diag}\left\{\frac{\partial^2 f}{\partial u^2}, \ldots, \frac{\partial^2 f}{\partial u^3}\right\}(x_o, u_o)
\]

and,

\[
H := \text{diag}\left\{\frac{\partial^3 f}{\partial x^2 u}, \ldots, \frac{\partial^3 f}{\partial x^3 u}\right\}(x_o, u_o)
\]

\[
K := \text{diag}\left\{\frac{\partial^3 f}{\partial u^2 x}, \ldots, \frac{\partial^3 f}{\partial u^3 x}\right\}(x_o, u_o)
\]

Then, a Linear Model and Uncertainty, that is, a partially linear model of the nonlinear class given by Eq. (1) is,

\[
\begin{align*}
\dot{\bar{x}}(t) &= A\bar{x}(t) + Bu(t) + \frac{1}{2!} F\bar{x}(t)^2 + \frac{1}{3!} G\bar{x}(t)^3 + H\bar{x}(t)u(t) + \frac{1}{3!} K\bar{x}(t)^3u(t) \\
y(t) &= C\bar{x}(t) + Du(t)
\end{align*}
\]

in which the proposed uncertainty model are the quadratic and cubic powers of \(\bar{x}(t)\) and \(u(t)\) that are gotten using \(x_p(t) := [x_1^p(t) \quad x_2^p(t) \quad x_3^p(t)]^T\) where \(p = 1, \ldots, 3\).

**Proof.** Suppose that \(u(t)\) and \(x(t)\) change slightly, becoming \(u_o + \bar{u}(t)\) and \(x_o + \bar{x}(t)\), respectively, where \(\bar{u}(t)\) and \(\bar{x}(t)\) are small \(\forall t\). Since the non-linear system is not chaotic and \((x_o, u_o)\) is a constant equilibrium point, then, \(f(x_o, u_o, t) = 0\) and the Taylor series expansion of Eq. (1) is [5],

\[
\begin{align*}
\dot{\bar{x}}(t) &= f(x_o + \bar{x}(t), u_o + \bar{u}(t)) = A\bar{x}(t) + Bu(t) + \frac{1}{2!} \frac{\partial^2 f}{\partial x^2}\bar{x}(t)\bar{x}(t) + \frac{1}{3!} \frac{\partial^3 f}{\partial x^3}\bar{x}(t)^3 + \frac{1}{3!} \frac{\partial^3 f}{\partial x^2 u}\bar{x}(t)\bar{u}(t) + \frac{1}{3!} \frac{\partial^3 f}{\partial u^3}\bar{u}(t)^3 + \frac{1}{3!} \frac{\partial^3 f}{\partial x^3 u}\bar{x}(t)^3\bar{u}(t)
\end{align*}
\]

where \(\bar{x}(t) = x(t) - x_o(t), \bar{u}(t) = u(t) - u_o(t), \frac{\partial^2 f}{\partial x^2},\frac{\partial^3 f}{\partial x^2 u},\frac{\partial^3 f}{\partial x^3 u}\) are Hessian matrices and \(J(x(t), \bar{x}(t))\) and \(J(u(t), \bar{u}(t))\) are matrices associated to the derivatives or third order. Then, neglecting the cross terms, the Hessian matrices becomes \(F\) and \(G\) given by Eq. (3),and \(J(x(t), \bar{x}(t))\) and \(J(u(t), \bar{u}(t))\) becomes,

\[
\begin{align*}
\text{diag} \left\{\begin{array}{c}
\bar{x}_1(t) \\
\bar{x}_n(t) \\
\bar{u}_1(t) \\
\bar{u}_n(t)
\end{array}\right\} = \begin{pmatrix}
\frac{\partial^2 f}{\partial x^2}(x_o, u_o) & \cdots \\
\frac{\partial^2 f}{\partial x^3}(x_o, u_o) & \cdots \\
\frac{\partial^3 f}{\partial x^2 u}(x_o, u_o) & \cdots \\
\frac{\partial^3 f}{\partial x^3 u}(x_o, u_o) & \cdots 
\end{pmatrix}
\end{align*}
\]

respectively. Thus the result of Eq. (5) follows cutting off the terms associated to the higher order derivatives and tacking a first order approximation for the output function \(y(t) = g(x(t), u(t), t)\).

Note that output function can contain a non-linear part that is not considered.

In general the Jacobian and Hessian matrices are time variant for a particular solution \((x_o(t), u_o(t))\) of Eq. (1), however a constant equilibrium point is considered in order to include these matrices in the robust control design. So, these matrices becomes known information of the proposed non linear uncertainty model, i.e., of the quadratic and cubic powers of \(\bar{x}(t)\) and \(\bar{u}(t)\).

In the following section the partially linear model of Proposition 1 is used to minimize on-line a mixed sensitivity criterion.

**III. ROBUST CONTROL**

It is shown an improvement of the controller proposed by [7], taking advantage of the information of the uncertainty model of Proposition 1 (see Eq. (5)), and proposing an auto-tuning of the control parameters thanks to the fixed structure of the controller.

From Proposition 1 let consider in what follows a
A partially linear model of the form,
\[
\begin{align*}
\dot{x}(t) &= \begin{bmatrix} a + r \end{bmatrix} + Bu(t) + H \bar{x}^3(t) \\
y(t) &= C \bar{x}(t)
\end{align*}
\] (8)

that is useful for the application to the Pendubot. Suppose that the linear part of the partial linear model of Proposition 1 is a controllable and observable subsystem and,
\[
B = \begin{bmatrix} 0 \\
B_1 \end{bmatrix}
\] (9)

where \(B_1 \in \mathbb{R}^{m \times m}\) is a non-singular matrix. Since the inputs are independent, without loss of generality a changes of basis exist such that \(B\) has the form of Eq. (9).

The structure of the controller (proper, stable and stabilizing) proposed by [7] is,
\[
K(s) = A + \frac{(a + r) s + a^2}{s + a - r} I_n
\] (10)

where \(a\) and \(r\) are free control parameters that are used to tune it. Note that the stability condition of the controller is \(a < r\). The controller \(K(s)\) is designed for a plant model of the form,
\[
\dot{x} = Ax(t) + v(t)
\] (11)

where,
\[
v(t) := Bu(t)
\] (12)

Since \(B\) is full column rank then there exists a left inverse of \(B\), getting,
\[
u(t) = B^L v(t)
\]

in which \(B^L B = I\). Considering the structure of \(B\),
\[
B^L = \begin{bmatrix} G \\
B_1 \end{bmatrix}
\] (13)

where \(G\) includes the free parameters that are used to tune the controller diminishing the error generated by \(B B^L\).

The proposed control scheme is shown in Figure 1, where \(H\) is known and included in the transfer function from the output to the input of the feedback uncertainty model \(\Delta := \bar{x}^3(t)\), the internal scheme of the controller is shown in Figure 2 and the change of basis \(T_N\) is given in Section IV.

\[
\begin{array}{cccccc}
\bar{e}(t) & K(s) & v(t) & B^L & u(t) \\
\end{array}
\]

Fig. 2. Controller.

**Lemma 1.** Consider the nominal plant \(\dot{x}(t) = Ax(t) + v(t)\) and the controller \(K(s) = A + \frac{(a + r) s + a^2}{s + a - r} I_n\) in the feedback configuration of Figure 1. Suppose that the uncertainty model is given by Eq. (8). If,
\[
\frac{s + a - r}{(s + a)^2} \leq \frac{1}{\|H\|\infty} \gamma(\Delta)
\] (14)

then the closed loop system is stable, where \(\gamma(\Delta)\) denotes the finite gain of the uncertainty \(\Delta\).

**Proof.** Since the nominal plant is \(\dot{x}(t) = Ax(t) + v(t)\), the scheme of 1 is analyzed removing \(B\) and \(B^L\), so, the transfer function from the output to the input of the uncertainty is,
\[
u_\Delta(s) = \frac{s + a - r}{(s + a)^2} H y_\Delta(s)
\] (15)

Then, from triangle inequality, an upper bound of the \(H_{\infty}\) norm of \(T_{u_\Delta y_\Delta}(s)\) is,
\[
\|T_{u_\Delta y_\Delta}(s)\|_{\infty} \leq \|s + a - r\|_{\infty} \|H\|_{\infty} \gamma(\Delta)
\] (16)

Thus if Eq. (14) is satisfied then,
\[
\|T_{u_\Delta y_\Delta}(s)\|_{\infty} \leq \frac{1}{\gamma(\Delta)}
\] (17)

and stability follows by the Small Gain Theorem [11].

Because the model proposed for the uncertainty is non-linear, it is proposed that the finite gain of the uncertainty \(\gamma(\Delta)\) is gotten dynamically. So, using the sector criterion [11] for non-linear systems, \(\gamma(\Delta)\) is obtained measuring the energy of the output and input of the uncertainty \(\|y_\Delta(t)\|_2\) and \(\|u_\Delta(t)\|_2\), respectively, that is,
\[
\|y_\Delta(t)\|_2 = \gamma(\Delta) \|u_\Delta(t)\|_2 + \beta
\] (18)

In [8] a mixed sensitivity problem is solved, such that a robust controller guarantees robust stability and robust performance, under the presence of uncertainties and admissible disturbances. Basically, to get robust stability \(\|T_{u_\Delta y_\Delta}(s)\|_{\infty}\) must be diminished in high frequencies and in order to guarantee robust performance \(\|S_o(s)\|_{\infty}\) must be diminished in low frequencies, where \(S_o(s)\) is the transfer function that represents the nominal performance of the controlled system \((S_o(s) := T_{exo}(s))\). So,
\[
\begin{align*}
\|T_{u_\Delta y_\Delta h}\|_{\infty} \\
S_o
\end{align*}
\] (19)

must be minimized, where \(S_o = \lim_{s \to 0} S_o(s)\) and \(T_{u_\Delta y_\Delta h} = \lim_{s \to \infty} T_{u_\Delta y_\Delta}(s)\).

In the following section the auto-tuning of the controller satisfies the stability condition of Lemma 1.
IV. AUTO-TUNING OF THE CONTROLLER

In [10] a procedure for the tuning of the controller \( K(s) \) for the nominal plant is \( x(t) = Ax(t) + v(t) \) appears, given an exact solution of a mixed sensitivity problem for the free parameter \( r \) of \( K(s) \). This solution is [10],

\[
   r = a - \frac{a^2}{w_h \| A \|_\infty} \tag{20}
\]

for feedback uncertainty models as the one of Figure 1.

Analyzing the proposed control scheme of Figure 1 and removing \( B \) and \( B^L \), \( S_{ol} = \frac{\| q - r \|_{\infty}}{\| A \|_{\infty}} \) and \( \| T_{uA\gamma\Delta}h \|_{\infty} = \frac{1}{w_h} \) are gotten. Then, from the inequality proposed in Lemma 1 given by Eq. (14), it is obtained that,

\[
   w_h \geq \| H \|_{\infty} \| A \|_{\infty} \gamma(\Delta) \tag{21}
\]

this inequality is used for the auto-tuning of the controller, and it assures the stability of the closed loop system, assuming that the worst uncertainty is present in high frequencies.

In [9] a change of basis,

\[
   T_a = \begin{bmatrix} I_q & 0 \\ -T_{21} & I_q \end{bmatrix}, \quad T_a^{-1} = \begin{bmatrix} I_q & 0 \\ -T_{21} & I_q \end{bmatrix} \tag{22}
\]

is proposed to assigns a desired dynamics to a part of the system and preserving the structure of \( B \) given by Eq. (9), where \( q \geq m \), and

\[
   A = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix} \tag{23}
\]

be partitioned according with the block partition of \( T_a \).

So, \( \tilde{A} = TAT^{-1} \). Then

\[
   T_{21} = \tilde{A}_{12}^{-1} \left( \tilde{A}_{11} - \Lambda_{11} \right) \tag{24}
\]

is used to assign a desired dynamics to \( \tilde{A}_{11} \), and

\[
   T_{21} = \left( \Lambda_{22} - \tilde{A}_{22} \right) \tilde{A}_{12}^{-1} \tag{25}
\]

is used to assign a desired dynamics to \( \tilde{A}_{22} \). The characteristic polynomials of the matrices \( \Lambda_{11} \) and \( \Lambda_{22} \) have desired dynamics and \( \tilde{A}_{12} \) is supposed to be a non-singular matrix.

So the term \( T_N \) show it in the 1, shall be

\[
   T_N = T_aT_b \tag{26}
\]

where \( T_b \), is the possible first change of basis used to get the structure of \( B \).

The parameters of the matrices \( \Lambda_{11} \) (or \( \Lambda_{22} \)) and \( G \) are used to assure stability and to tune the response of the system.

Finally, Figure 3 shows the general scheme proposed to find the finite gain \( \gamma(\Delta) \) using Eq. (18) and auto-tuning the controller by \( w_h \) such that Eq. (21) is satisfied and using \( r \) given by Eq. (20).

V. APPLICATION EXAMPLE

Consider the two-link under-actuated planar rotational robotic mechanism called Pendubot of Figure 4 where \( q_1 \) is the angle that the link 1 makes with the horizontal, \( q_2 \) is the angle that the link 2 makes with link 1, \( m_1 \) is the mass of link 1, \( m_2 \) is the mass of link 2, \( L_1 \) and \( L_2 \) the lengths of links 1 and 2, respectively, \( l_{c1} \) and \( l_{c2} \) the distance to the center of mass of link 1 and 2, respectively. This robot has an actuator at the shoulder (link 1) and no actuator at the elbow (link 2). The link 2 moves freely around link 1 and the control objective is to bring both links of the mechanism to the unstable equilibrium point of top position.

A control based in feedback linearization is used to arrive near the top position and after that commutes to the robust controller of section III.

It is assumed that the robot moves without friction. So, using Euler-Lagrange formulation, the equations of motion are [4],

\[
   \ddot{q}(t) = M_{q(t)}^{-1} \left( \tau(t) - C_{q(t),\dot{q}(t)}\dot{q}(t) - G_{(q(t))} \right) \tag{27}
\]

where

\[
   q(t) = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}, \quad \tau(t) = \begin{bmatrix} \tau_1 \\ 0 \end{bmatrix} \tag{28}
\]
\[ M(q(t)) = \begin{bmatrix} K_1 + K_2 + 2K_3 \cos(q_2) & K_2 + K_3 \cos(q_2) \\ K_2 + K_3 \cos(q_2) & K_2 \end{bmatrix} \tag{29} \]

\[ C(q(t), \dot{q}(t)) = \begin{bmatrix} -K_3 q_m \sin(q_2) & -K_3 \sin(q_2) (q_m + q_p) \\ K_3 q_m \sin(q_2) & 0 \end{bmatrix} \tag{30} \]

\[ G(q(t)) = \begin{bmatrix} gK_1 \cos(q_1) + gK_3 \cos(q_1 + q_2) \\ gK_3 \cos(q_1 + q_2) \end{bmatrix} \tag{31} \]

where, \( K_1 = m_1 l_1^2 + m_2 l_1^2 = 0.0761 \)

\[ K_2 = m_2 l_2^2 = 0.0662 \]

\[ K_3 = m_2 L_1 l_2 = 0.0316 \]

\[ K_4 = m_1 l_1 l_2 + m_2 L_1 = 0.979 \]

\[ K_5 = m_2 l_2 = 0.383 \]

\( g = 9.804 \)

Let,

\[ x = \begin{bmatrix} q_1 \\ q_2 \\ \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}, \quad u = \begin{bmatrix} \tau_1 \\ 0 \end{bmatrix} \tag{33} \]

Then, the partially linear model is given by Eq. (5) and does not have quadratic term of the state and the input, where,

\[ A_o = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 127.9292 & -29.3756 & 0 & 0 \\ -132.2741 & 100.1189 & 0 & 0 \end{bmatrix} \tag{34} \]

\[ B_o = \begin{bmatrix} 0 \\ 0 \\ 16.3891 \\ -24.2124 \end{bmatrix} \tag{35} \]

\[ H_o = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -21.3215 & 26.8458 & 0 & 0 \\ 22.0457 & -56.1250 & 0 & 0 \end{bmatrix} \tag{36} \]

\[ C_o = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{37} \]

The change of basis,

\[ T_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.6769 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{38} \]

is proposed to get the desired structure to \( B \), and \( T_{21} = [20 \ 19 \ 10] \) in Eq. (22) assigns the desired dynamics to the system, so from Eq. (26),

\[ T_N = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.6769 \\ 20 & 19 & 10 & 7.6789 \end{bmatrix} \tag{39} \]

And the partially linear model is:

\[
\begin{align*}
\dot{x}(t) &= \bar{A} x(t) + \bar{B} u(t) + \bar{H} x^3(t) \\
y(t) &= \bar{C} x(t)
\end{align*}
\tag{40}
\]

where \( \bar{A} = T_N A_o T_N^{-1} \), \( \bar{B} = T_N B_o \), \( \bar{H} = T_N H_o \) and \( \bar{C} = C_o T_N^{-1} \).

A partial feedback linearization with initial pump is used. The control parameters of the robust control are \( a = 10 \) and \( B^L = [-8 \ -8.04 \ -1.1 \ -0.0413] \). The parameters \( w_h \) and \( r \) are gotten dynamically as shown in Figure 3. The states are shown using MatLab-Simulink in Figures 5 to 8, for an initial condition \( q_1 = \frac{\pi}{2} \), \( q_2 = 0 \), \( \dot{q}_1 = 0 \) and \( \dot{q}_2 = 0 \) (at low position).

![Fig. 5. Angular position q1](image)

![Fig. 6. Angular position q2](image)

Figures 5 and 6 shows that the feedback linearization balance the Pendubot at the beginning. Also, after commutation to the robust controller, the outputs are well regulated at top position \( q_1 = \frac{\pi}{2} \) and \( q_2 = 0 \). As expected the angular velocities are zero at top position as shown by Figures 7 and 8. Figures 5 to 8 show “good” performance guaranteeing stability in spite of the uncertainties that are
attenuated. This is due to the mixed sensitivity criterion and the tuning and auto-tuning of the control parameters.

VI. CONCLUSIONS

A partially linear model of the system around a constant equilibrium point is presented, that includes an uncertainty model of the quadratic and cubic of the state and input. A robust controller is designed for the linear subsystem minimizing a mixed sensitivity criterion. Stability is guaranteed by the Small Gain Theorem. Also, the constant matrices of the partially linear model and the stability conditions are used for the auto-tuning of the parameters of the robust control algorithm. The auto-tuning of the control parameters guarantees closed loop stability for the robust controller applied to the uncertain plant when the nominal plant has full state information and actuation. The results are applied to the under-actuated system of a Pendubot. The simulation results shows “good” performance in spite of the uncertainties that are attenuated.

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