Modeling in the Air-Jet Texturing and Twisting (AJT²) Machine

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ABSTRACT

System identification is a general term to describe mathematical tools and algorithms that build dynamical models from measured data. A dynamical mathematical model in this context is a mathematical description of the dynamic behavior of a system or process in either the time or frequency domain. This paper presents the modeling yarn tension of the air jet texturing and twisting machine. The system model shows the tension change in the twisting process. All computer simulations were completed in MIDSYS toolbox and MATLAB Simulink.

Keywords: Yarn, Air-Jet Texturing and Twisting, System Identification, ARX, Yarn Tension, Cross-Correlation Test.

1. INTRODUCTION

In this study, after improvement and manufacturing of Air-Jet (AJT²) Texturing and Twisting Machine supported by TUBITAK (Research Grant No:105M134 patented as TPE Document Code:69065, Registration No: 2007/02344), we purposed modeling the yarn tension in the Air-Jet Texturing and Twisting Machine.

Yarn is a general material that is long and easy to bend. The yarn transfer system is the device which moves the yarn in certain conditions. It is used in textile industry for various transferring systems. Generally, this system tends to bend and change from easily. Modeling the tension of the yarn is difficult but very important in the textile machine in order to design control systems. Various researches about tension modeling of the yarn have been conducted. This study presents air jet texturing and twisting machine’s twisting process for the yarn transfer system. This system model shows in the roll radius during winding and unwinding of the yarn in the twisting process.

2. SYSTEM DESCRIPTION

The AJT² machine composed of three main processes. These processes are:
- Drawing process,
- Air-Jet Texturing and
- Twisting processes
in the sequence of the yarn processing.

Drawing process

The drawing process is implemented by two units of ceramic plated cylinder-rubber cylinder pair (FC-feed cylinders) and two units of heated cylinder-separator roller pair (DC-drawing cylinders). The yarn transferred from creel is wrapped around the DC and heated until the glass transition temperature of the polymer. The yarn has to be fed by the DC at lower speed than the FC in order to create a draw between the DC and FC. After the drawing operation, the yarn is then ready as finished yarn to be fed into the yarn channel of the air-jet nozzle by the FC.

Texturing process

The air-jet texturing process forms a yarn with tightly convoluted, entangled and looped filaments resembling yarns spun from staple fibres such as cotton and wool. In this process of air-jet texturing, multi filament yarn is fed into a narrow channel where it meets with a flow of compressed air and taken away from that channel at a lower speed than the feeding speed (called overfeed) and makes a right turn just after the narrow channel. At the exit of the narrow channel a supersonic, highly turbulent air jet is formed by the compressed air flow that pushes the freely available filaments in any
direction in such a way that they entangle, convolute and loop with each other (Figure 1). Such converted yarn is much softer, bulkier and gives warmer feeling to wearer and possesses natural look and appearance than the supply yarn which may be composed of one or many filament yarns be thermoplastic, organic or metallic.

![Figure 1. Air-Jet texturing principle](image1)

**Twisting process**

Twisting is a very essential process in the production of staple fiber yarns, twines, cords and ropes. Twist is inserted to the staple yarn to hold the constituent fibres together, thus giving enough strength to the yarn, and also producing a continuous length of yarn. The twist in the yarn has a two-fold effect; firstly the twist increases cohesion between the fibers by increasing the lateral pressure in the yarn, thus giving enough strength to the yarn. Secondly, twist increases the helical angle of fibres and prevents the ability to apply the maximum fibre strength to the yarn. Due to the above effects, as the twist increases, the yarn strength increases up to a certain level, beyond which the increase in twist actually decreases the strength of staple yarn. In the air-jet texturing and twisting machine, in order to obtain twisted yarn, DirectTwist twisting method is used (Figure 2).

![Figure 2. The twisting unit of the air-jet texturing and twisting AJT² machine](image2)

**3. PROBLEM STATEMENT**

In these textile machines, the main problem is yarn tension modeling. In order to get perfect manufacturing, the tension should be controlled. Before control algorithm, the mathematical model which represents the air-jet texturing and twisting machine has to be identified. For useful solution, only texturing and twisting processes are considered in the effects of tension. After some experiments, it was decided to be four main reasons which affect the tension of the system:

- Yarn’s thickness (dtex),
- Yarn’s type (Polyester, Polypropylene, etc.),
- Number of Yarns in the process,
- System’s velocity (m/sec).

![Figure 3. Air-jet texturing and twisting machine yarn transfer system](image3)

**4. PROCESS MODEL**

Yarn transfer with twisting process can be considered as SISO system. The tension is obtained as a result of yarn transfer between winder and unwinder in the air-jet texturing machine (Figure 3).
Dynamical model of system

The yarn between the winder and the unwinder can be modeled as a spring and damping element as shown in Figure 4. After any angular displacement \( \theta \), the tension is obtained. After 40-50 cN tension value, the tension in the yarn reaches the dangerous value. In this model \( k \) and \( b \) parameters can be only found from system identification.

\[
\begin{align*}
(J_M + J) \cdot \ddot{\theta} &= (-b \cdot R^2 - B_M) \cdot \dot{\theta} + (-k \cdot R^2) \cdot \theta \\
&+ (K_M \cdot U)
\end{align*}
\] (1)

In above equation, \( J \): Moment of inertia of winder, \( J_M \): Moment of inertia of motor, \( R \): Radius of the winder, \( B_M \): Damping coefficient of motor, \( K_M \): Motor Constant, \( k \): stiffness of yarn, \( b \): Damping coefficient of yarn.

The state space representation of this system can be easily obtained as in below:

\[
\begin{align*}
\dot{x} &= A \cdot x + B \cdot u \\
y &= C \cdot x + D \cdot u
\end{align*}
\] (2)

State variables:

\[
\begin{align*}
x_1 &= \theta \\
x_2 &= \dot{\theta} \\
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{-k \cdot R^2}{(J_M + J)} \cdot x_1 + \frac{(-b \cdot R^2 - B_M)}{(J_M + J)} \cdot x_2 \\
&+ \frac{K_M}{(J_M + J)} \cdot U
\end{align*}
\] (3)

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
\frac{-k \cdot R^2}{(J_M + J)} & \frac{(-b \cdot R^2 - B_M)}{(J_M + J)}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
0 \\
\frac{K_M}{(J_M + J)}
\end{bmatrix} \cdot U
\]

\[
y = [1 \ 0] \cdot \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
\] (4)

System identification

System identification is a general term to describe mathematical tools and algorithms that build dynamical models from measured data. A dynamical mathematical model in this context is a mathematical description of the dynamic behavior of a system or process in either the time or frequency domain. There are four steps for system identification;

- Experimental Design,
- Model Structure Selection,
- Parameter Estimation,
- Model Validation.

This section addresses the identification of the observability indices \( n_i \) of a single-input/single-output model. The method based on the RLS loss function is used to find these indices for parameterizing the model.

Model structure estimation

The recursive least square method: The discrete-time transfer function of any system is shown below;

\[
y(t) = \frac{q^{-d} \cdot B(q^{-1})}{A(q^{-1})} \cdot u(t)
\] (5)

Polynomial \( A \) and \( B \) are;

\[
A(q^{-1}) = 1 + a_1 \cdot q^{-1} + \ldots + a_{n_A} \cdot q^{-n_A}
\] (6)

\[
B(q^{-1}) = b_1 \cdot q^{-1} + \ldots + b_{n_B} \cdot q^{-n_B}
\] (7)

\[
y(t+1) + a_1 \cdot y(t-1) + \ldots + a_{n_A} \cdot y(t-n) = b_1 \cdot u(t-1) + \ldots + b_{n_B} \cdot u(t-n) + \varepsilon(t)
\] (8)

or

\[
A(q^{-1}) \cdot y(t) = B(q^{-1}) \cdot u(t) + \varepsilon(t)
\] (9)
Equation (9) is named AutoRegressive with eXogamous input (ARX) and $\varepsilon(t)$ represents the error. After rewriting equation (8) again;

$$
y(t + 1) = -\sum_{i=1}^{n_a} a_i \cdot y(t + 1 - i) 
+ \sum_{i=1}^{n_b} b_i \cdot u(t - d + 1 - i) + \varepsilon(t)
= \theta^T \cdot \phi(t) + \varepsilon(t) 
$$

(10)

where

$$
\theta^T = \begin{bmatrix} a_1, \ldots, a_{n_a}, b_1, \ldots, b_{n_b} \end{bmatrix}
$$

(11)

is the parameter vector and

$$
\phi(t)^T = [-y(t), \ldots, -y(t-n_A+1), u(t-d), \ldots, u(t-d-n_B+1)]
$$

(12)

is the vector of measures. The adjustable prediction model (a priori) is described by

$$
\hat{y}(t+1) = -\sum_{i=1}^{n_A} \hat{a}_i \cdot y(t+1-i) 
+ \sum_{i=1}^{n_B} \hat{b}_i \cdot u(t-d+1-i) = \hat{\theta}(t)^T \cdot \phi(t)
$$

(13)

where $\hat{y}^o(t+1)$ represents the a priori prediction depending on the values of the parameters estimated at instant $t$ and

$$
\hat{\theta}(t)^T = \begin{bmatrix} \hat{a}_1(t), \ldots, \hat{a}_{n_A}(t), \hat{b}_1(t), \ldots, \hat{b}_{n_B}(t) \end{bmatrix}
$$

(14)

is the estimated parameter vector. The a priori prediction error is given by

$$
\varepsilon^o(t+1) = y(t+1) - \hat{y}^o(t+1) 
= y(t+1) - \hat{\theta}(t)^T \cdot \phi(t)
$$

(15)

The main aim is to find any recursive parameter estimation algorithm in order to minimize the least square index.

$$
\min_{\hat{\theta}(t)} J(t) = \sum_{i=1}^{t} \left[ y(i) - \hat{\theta}(t)^T \cdot \phi(i-1) \right]^2
$$

(16)

If $F(t)$ is estimation gain matrix, Recursive Least Square parameter estimation formulation is defined as

$$
\hat{\theta}(t+1) = \hat{\theta}(t) + F(t+1) \cdot \phi(t) \cdot \varepsilon^o(t+1)
$$

(17)

$$
F(t+1) = F(t) - \frac{F(t) \cdot \phi(t)^T \cdot F(t)}{1 + \phi(t)^T \cdot F(t) \cdot \phi(t)}
$$

(18)

$$
\varepsilon^o(t+1) = y(t+1) - \hat{\theta}(t)^T \cdot \phi(t)
$$

(19)

Then obtained as in below;

$$
\hat{\theta}(t+1) - \hat{\theta}(t) = F(t+1) \phi(t) \varepsilon^o(t+1)
$$

$$
= F(t) \cdot \phi(t) \cdot \frac{\varepsilon^o(t+1)}{1 + \phi(t)^T \cdot F(t) \cdot \phi(t)}
$$

(20)

Let consider the a posteriori error $\varepsilon(t+1)$,

$$
\varepsilon(t+1) = y(t+1) - \hat{\theta}(t)^T \cdot \phi(t)
$$

$$
= y(t+1) - \hat{\theta}(t) \cdot \phi(t)
$$

$$
= \left[ \hat{\theta}(t+1) - \hat{\theta}(t) \right]^T \cdot \phi(t)
$$

(21)

$$
\varepsilon^o(t+1) = y(t+1) - \hat{\theta}(t)^T \cdot \phi(t)
$$

$$
= \frac{\varepsilon^o(t+1)}{1 + \phi(t)^T \cdot F(t) \cdot \phi(t)}
$$

(22)

$$
\hat{\theta}(t+1) = \hat{\theta}(t) + F(t) \cdot \phi(t) \cdot \varepsilon(t+1)
$$

(23)

$$
F(t+1) = \frac{F(t) \cdot \phi(t)^T \cdot F(t)}{1 + \phi(t)^T \cdot F(t) \cdot \phi(t)}
$$

(24)
The a posteriori prediction error is given by

\[ \varepsilon(t+1) = \frac{y(t+1) - \hat{\theta}(t)^T \cdot \phi(t)}{1 + \phi(t)^T \cdot F(t) \cdot \phi(t)} \quad (25) \]

**Model validation**

In system identification both the determination of model structure and model validation are important steps. An over-parameterized model structure can lead to unnecessarily complicated computations for finding the parameter estimates and for using the estimated model. An under-parameterized model may be too inaccurate.

The Cross-Correlation test, consists of verifying if the output error;

\[ \varepsilon_i(t) = y_i(t) - \hat{y}_i(t) \quad (26) \]

where \( \hat{y}_i(t) \) is the model output. For doing this the normalized cross-correlation functions;

\[ \sqrt{N} \cdot \frac{\Gamma_{\varepsilon_i,\hat{y}_j}(T)}{\sqrt{\sum_{\tau=-K}^{K} \Gamma_{\varepsilon_i}(\tau) \cdot \Gamma_{\hat{y}_j}(\tau)}} \quad T=1,2,\ldots,T_{\text{max}} \quad (27) \]

for \( i = 1, 2, \ldots, p \) and \( j = 1, 2, \ldots, p \) and for large \( K \) (\( K \) is chosen = \( N/2 \)), where

\[ \Gamma_{\varepsilon_i,u_j}(\tau) = \frac{1}{N} \sum_{t=1}^{N-\tau} \varepsilon_i(t + \tau) \cdot u_j(t) \quad (28) \]

The typical value of \( T_{\text{max}} \) ranges from \( n \) to \( 2n \). If \( \varepsilon_i(t) \) is independent of \( \hat{y}_i(t) \) then the random variable in equation (27) is asymptotically Gaussian with zero mean and variance = 1.

The normalization of the cross-correlation function allows determining the zero thresholds corresponding to a predefined confidence interval.

**5. CONCLUSION**

This study consists of system identification of yarn tension. In the experiments, after PRBS signal in Figure 5 is applied to the system, the output data are acquired. With these data sets, the estimated parameter values of ARX-SISO model are obtained and the estimated output was plotted in Figure 6.

\[ (1 + a_i \cdot q^{-1} + \ldots + a_a \cdot q^{-a}) \cdot y(t) \]

\[ = (b_i \cdot q^{-1} + \ldots + b_b \cdot q^{-b}) \cdot u(t) + \varepsilon(t) \quad (29) \]

\( a_0 = 1, \ a_1 = -1.8193, \ a_2 = 0.9543, \)

\( b_1 = 0.0307, \ b_2 = 0.0821 \)

Then, the state-space representation of model is found as;

\[
\begin{align*}
A &= \begin{bmatrix} 0 & 1 \\ -0.9543 & 1.8193 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0307 \\ 0.1380 \end{bmatrix}, \quad C = [1 \ 0], \quad D = [0]
\end{align*}
\]

**Figure 5. System’s input**

**Figure 6. Model and system’s output**
6. ACKNOWLEDGEMENT

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7. REFERENCES


