Graph Representation of the Variable Chain Mechanisms with Sequential Movement

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ABSTRACT

A mechanism that encounters a certain change in the degree of freedom during sequential movement will also result in the variation of topological structure in every stage. Since the mechanisms with variable chain in different stages have different topologies, this will complicate the representation of the topology thoroughly. However, the applications of this kind of mechanisms are very extensive. Although some approaches were proposed to represent mechanisms with variable chain by graphs according to the topology of each stage, they can be hardly represented by a mathematical formula. This paper proposes an approach to represent the variable chain mechanism with sequential movement. According to the operation of the variable chain mechanisms, the movement of the mechanisms can be classified into parallel system movement and sequential system movement. Parallel movement mechanisms are the mechanisms which operate multiple links concurrently when giving an input; on the contrary, sequential movement mechanisms can operate only one link and transfer the movement of the next one step by step. In this paper, we study an ancient Chinese padlock mechanism and propose both the graph representation and a composite function representation for the movement of each stage. The representation is verified by applying it on some existing examples.

Keyword: Ancient Chinese padlock, composite function, graph representation, sequential movement, variable chain

1. INTRODUCTION

There are few studies about variable chain mechanisms. There are rarely few studies dedicated to the conceptual design and topological representations of variable chains [1-4]. Yan and Liu proposed a definition of the mechanisms of variable chain as "one with variable chains if it has variable topologies which are characterized by the topological variability of kinematic pairs and it thereby cannot be represented by a sole mechanism schematic diagram."[1] Wohlhart [5] exhibits the mechanisms with the property of the variety position namely "Kinematotropic Linkages". Afterward, Galletti et al. [6-8] extended Wohlhart's study systematic. Dai and Jhones [9-11] suggested a new opinion on the nature of variable mobility. Besides, Lee and Herve [12,13] also proposed the concept of discontinuous mobility mechanisms that can be regarded as those of the generalized kinematotropy, which the mobility of the joints may be discontinuous, and the active or inactive pairs may change the number of global or local degrees of freedom. Broadly speaking, kinematotropic linkages or discontinuous mobility mechanisms perform unusual function depending upon the singular kinematic geometry which leads to the change of the linkages mobility and can be categorized into variable chain mechanisms since the variable mobility may lead to topological structure changes [14]. However, all these approaches only seek for graph representation of mechanical structures. To give good illustration to the change of the number or connectivity of the links in each stage, we propose a new manifestation for the sequential movement of variable chain mechanisms by composite functions in addition to the graph representation.

2. SEQUENTIAL MECHANISM

Graph theory is a branch of mathematics that focuses on the studies of *graphs.* In graph theory, a graph is a collection of vertices and edges that connected the vertices. These graphs can illustrate the specific relation between the parts: the vertices usually stand for the mechanical parts and the edges that connect the vertices stand for the relation between these parts. Some of the studies about the mechanisms usually represent the topology and the movement by graphs. In this paper, we shall use the vertices to represent the mechanical parts and the edges to represent the joints that connect the parts. In addition to the original topology, the topological change of a mechanism can also be represented by contraction and subdivision (node expansion). In a graph G, **contraction** of edge e with endpoints u, v is the replacement of u and v with a single vertex whose incident edges are the edges other then e that were incident to u or v. The resulting graph G · e has one less edge than G, as shown in Figure 1.

Figure 1. Contraction

The operation **subdivision** of an edge (u,v) is to replace (u,v) with a path u-w-v, where w is a new vertex which subdivides the edge, as shown in Figure 2.

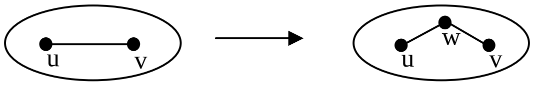
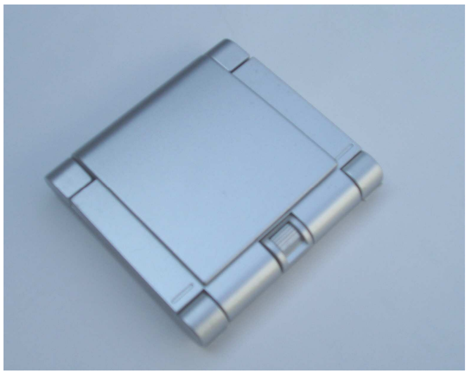


Figure 2. Subdivision (node expansion)

Illustrating the topological change during the movement of the mechanisms by graphs is clear but not efficient enough especially on the representation of the variable chain mechanisms. Take a foldable electronic clock as an example, as shown in Figure 3. This kind of electronic clock is almost the same as the general one, but easy to carry and pack by folding the body.

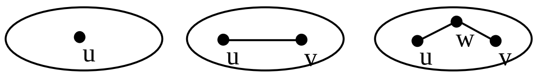
After folding, as shown in Figure 3(a), it looks like a square without extended parts. To use the clock, users may press the sliding button and make the body of the clock extended consequently, as shown in Figure 3(b). After expanding the parts totally, it becomes an open chain of 3 links with 2 joints. During the process of expansion, the degrees of freedom of the foldable electronic clock changes from zero to two and to zero again, as the clock becomes upright and stable. Figure 3(c) illustrates how the movement is represented by a graph. Without loss of generality, we can use a vertex to represent the folded body of the clock, and subdivided after pressing the sliding button. An edgee can represent the relation between the body and the extended part. The process of the expansion can also be represented by the same manifestation. To reverse the process, the graph represents the movement by contractions.

The mechanisms of foldable electronic clock can be taken as a typical variable chain mechanism. The movement of the foldable clock is sequential. The lid of the clock can be released by pressing the sliding button first; this will tigger the movement of the base of the clock. The mechanism of the clock changed during the expansion of the body, and the topology also changed step by step. To illustrate the variable topology, we always represent each stage with a graph. Therefore, the variable chains mechanisms with sequential movement can be manifested by a series of graphs.



(a) Folding

(b) Expanding



(c) Graph Representation

Figure 3. Folding clock

3. GRAPH REPRESENTATION

The movement of variable chain mechanisms can be classified into parallel movements and sequential movements. There are variable designs of the mechanism; the different input can also result in the same output from the design of the mechanisms. If we input a force into the mechanisms that can operate multiple parts simultaneously, then the movement is a parallel movement. In contrast, if an input into the mechanisms can only enable one movement of the mechanisms, and transmit the movement one by one, it is a sequential mechanism, as shown in Figure 4. In this paper, we focus on the representations of variable chain mechanisms with sequential movements.

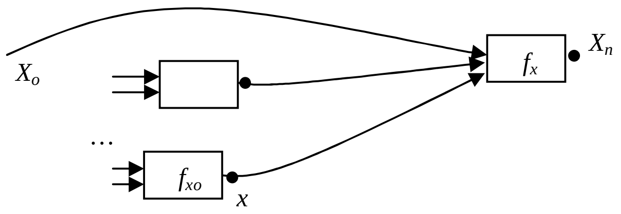


Figure 4. Sequential movement of variable chain mechanisms

Here we would like to apply and develop some early ideas which was introduced by Werbos [15,16] to show that the mechanisms with sequential movements can also be represented by a composite function. A sequence of movements for a variable chain mechanism can be described as a composite function, such as a family of functions as follows:

$$f_1 = x_o \mapsto x_1$$
$$f_2 = x_o \times x_1 \mapsto x_2$$
$$\ldots$$
$$f_n = x_o \times x_1 \times \ldots \times x_{n-1} \mapsto x_n$$

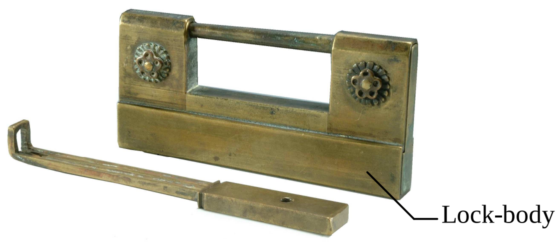
Assume this family of functions satisfy the following conditions

$$x_1 = f_1(x_o)$$
$$x_2 = f_2(x_o, x_1)$$
$$\ldots$$
$$x_n = f_n(x_o, x_1, \ldots, x_{n-1})$$

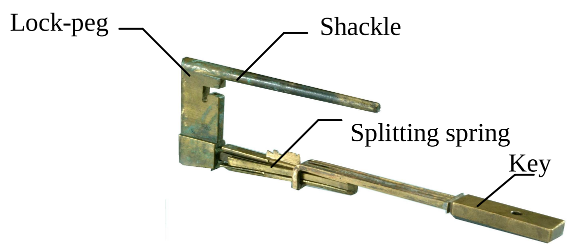
where x_n is a composite function of only one variable x_o. This function can be regarded as a composite function. Consequently, the original input argument x_o can transmit the movement through each output sequentially, and also effect the movement of next stage to finally obtain the required output.

4. EXAMPLE

Lock is a typical variable chain mechanism with sequential movements. We take an ancient puzzle mechanical lock as an example, and apply the approach in the previous session to describe the movement. As an ancient Chinese lock shown in Figure 5, most of ancient Chinese locks are equipped with a splitting-spring and consists of a lock-body, a lock-peg, and a key. The lock-body provides the keyhole for inserting the key and guiding the lock-peg to move. The lock-peg includes splitting-springs, a beam for bonding one end of each splitting-spring, and a shackle for hanging the lock. The keyhead is designed corresponding to the shape of the keyhole and the configuration of the splitting-springs. By inserting the correct key, the keyhead will squeeze the splitting-springs and the lock-peg will be moved out along the lock-body appropriately, then the lock will be unlocked [17-20].



(a) Lockbody and key

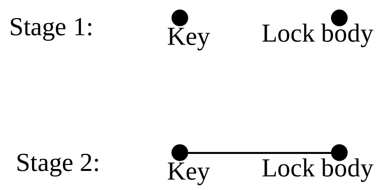


(b) Internal structure of the lock

Figure 5. Ancient Chinese lock

After centuries of evolvement, puzzle mechanical locks have incorporated delicate design on the original simple mechanism. However, graph representation can be utilized to show that all these variations are basically new designs on the splitting springs or the active panels.

A set of a general splitting-spring padlock can be taken as two independent parts before the key inserts into the lock. After the operation of the key, the lock subdivides (node expansion) another part from the lock body which is attached with the locks body originally. The movement of the attached part with the lock body turns the lock from structure to mechanism. At the end of the operation, there are two remaining parts, one is the lock body, and the other is the slider composed by the lock-peg and the key. Then the movement of the lock-peg and the key can be represented by contracting the verteices to illustrate the movement, as shown in Figure 6.



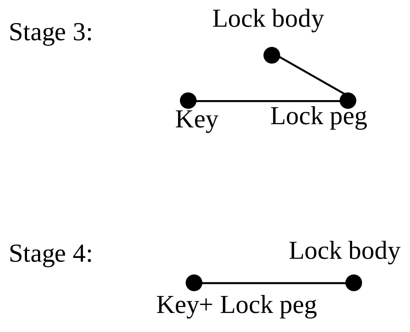
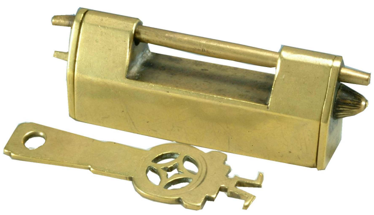


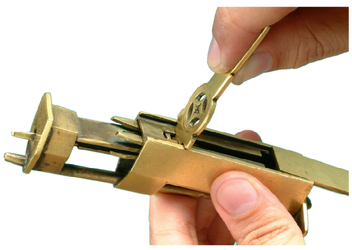
Figure 6. Graph Representation of Ancient Chinese Padlock

5. CHINESE FOUR-STAGE PADLOCK

A four-stage open lock, as shown in Figure 7, is a puzzle mechanical lock with hidden keyhole and can be opened by four steps. On the side of the lock body is a button for opening the hidden keyhole that can also release the obstacle by rotating the side panel and by sliding the bottom panel to reveal the keyhole. The button on the side of the lock body is the extension of the splitting spring of the lock-peg. By pressing the button, the splitting spring will be squeezed to release the lock-peg. According to the different types and lengths of the splitting springs, the lock-peg can only be pulled out partly in the first stage. This movement will certainly changes the topology of the lock body at the same time. Pulling out the lock-peg partly eliminates the obstacle of the shackle for preventing the rotation of the side panel. That also eliminates the function of the side panel that obstructs to slide the panel on the bottom of the lock body. This will reveal the keyhole so that the appropriate key can be inserted to open the lock.



(a) Key and Lockpad

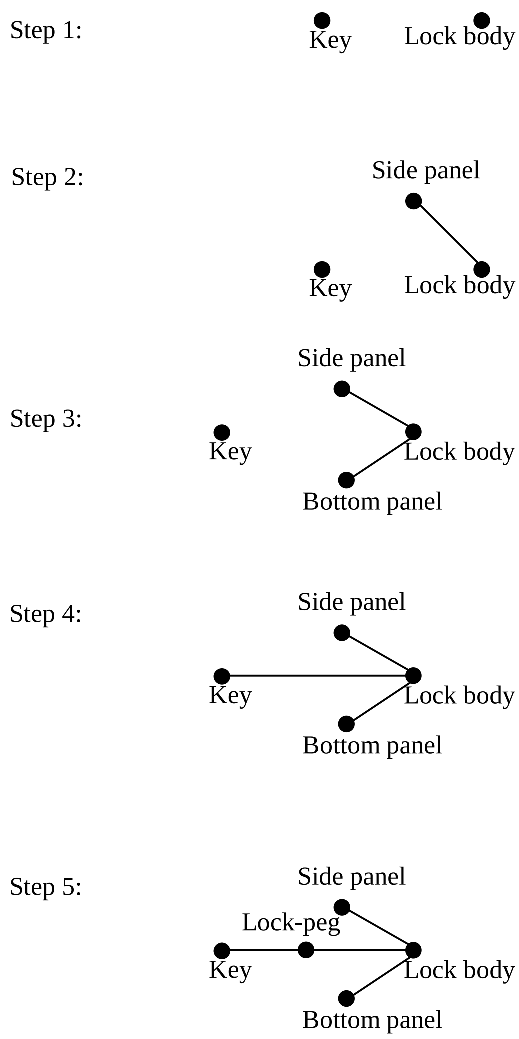


(b) Opening the 4-stage padlock

Figure 7. A 4-stage padlock

Figure 8 illustrates the graph representation of the steps in opening the lock:

Step 1: Before the operation of the four-stage open lock, we can regard the lock body and the key as two independent parts.
Step 2: To operate the lock, we input force to the system, which will release the side panel.
Step 3: The movement of the bottom panel of the lock increases a new part.
Step 4: After inserting the key, the splitting-spring of the lock-peg is squeezed.
Step 5: The topology of the lock changes to two parts, a lock body and a lock-peg
Step 6: The key and the lock-peg are combined to become one part, and can be contracted as a vertex.



Step 6:

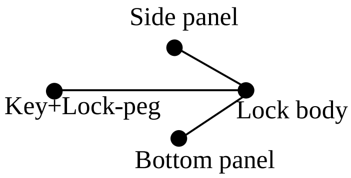


Figure 8. Four-stage open lock

If we use the composite function to represent the movement of the four-stage open locks, it can be formulated as:

$$f_1 = S \mapsto x_0$$
$$f_2 = S \times R \mapsto x_1$$
$$\ldots$$
$$f_n = S \times R \times P \times S \times P \mapsto x_n$$

This family of functions satisfy

$$x_1 = f_1(x_o)$$
$$x_2 = f_2(x_o, x_1)$$
$$\ldots$$
$$x_n = f_n(x_o, x_1, \ldots, x_{n-1})$$

where S represents the squeeze of the splitting springs, and R, P represent the movement of rotation joints and prismatic joints respectively. With this family of functions, the movement can be represented by a composite function x_n directly.

Furthermore, if we can describe the movement of the mechanism assisted by setting the coordinate of the mechanisms, we can manifest the movement of the mechanism more effectively. This can also help to distinguish the similar movement of the mechanisms. The manifestation of the four-open lock can then be modified as,

$$f_1 = S_{k^-} \mapsto x_0$$
$$f_2 = S_{k^-} \times R_{j^-} \mapsto x_1$$
$$\ldots$$
$$f_n = S_{k^-} \times R_{j^-} \times P_{j^+} \times S_{k^+} \times P_{j^+} \mapsto x_n$$

This family of functions satisfy

$$x_1 = f_1(x_o)$$
$$x_2 = f_2(x_o, x_1)$$
$$\ldots$$
$$x_n = f_n(x_o, x_1, \ldots, x_{n-1})$$

Functions of this system can be represented by directed edges that join vertices that correspond to the function arguments.

6. CONCLUSIONS

In this paper, we introduced the graph representation of variable chain mechanisms. We also proposed an approach that helps us to describe the mechanisms step by step with composite functions. This approach elegantly hides the laborious details under graph representations, and shows the difference between the graph representation and the function representation. The goal of our future work is to develop more complex distributed strategies to build the possible representations. With appropriate graph representation, variable chain mechanisms can be manipulated by computer software more efficiently.

7. ACKNOWLEDGMENT

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