Genetic Algorithm for the Portfolio Selection Problem on the Romanian Capital Market

Prof. Stelian STANCU PhD
Department of Economic Cybernetics
Academy of Economic Studies, Bucharest, ROMANIA
Address: str. Dumitru Secareanu, no.5, district 2, Bucharest, Romania

and

PhD Candidate Oana Mădălina PREDESCU
Academy of Economic Studies, Bucharest, ROMANIA
Address: str. Matei Basarab, no.108, district 3, Bucharest, Romania

ABSTRACT

The present paper presents theoretical aspects regarding the model for portfolio selection developed by Markowitz and aspects related to genetic algorithms. Starting from Markowitz's model, an indicator of the risk-aversion of investors is introduced in the initial model, the new optimization problem being solved with the help of the GA Optimization Tool from Matlab. The results obtained lead to the conclusion that genetic algorithms are rightfully considered to be powerful and efficient optimization tools.

Keywords: Portfolio Selection, Markowitz Portfolio Selection Model, Risk Aversion and Genetic Algorithms.

1. INTRODUCTION

On the speed century's background and on the background of a continuously developing society - a knowledge-based society, the access to information, the continuous learning processes, the technological revolution etc. are only a few of the elements that have put a print on the financial system, at national and global level. Like any other science that develops along with the transformations of the surrounding environment, the financial domain has passed also through numerous stages, coming nowadays to pass through the stage of the technological revolution. The rapid growth of financial information volume has transformed financial information processing into a more and more difficult process due to the increase in complexity, forcing both the business and academic domains to resort to more sophisticated information processing technologies in order to better solutions. In order for the business domain to truly benefit from a competitive advantage, advanced computational techniques and high-performance computers had to play more important roles in the business and industrial sectors.
At this moment, the industries and organizations in Romania are also influenced by the technological developments that take place at global level. Information management has become in our country an activity necessary and important. Managing information, mostly in the financial domain represents a central element in strategic and tactic planning, having an even more pronounced influence in the business sector. Financial performance and operational efficiency can improve through adequate information management, the efficient usage of technological tools and combining knowledge from different domains representing in fact thorough information management.
In recent years, at global level has been noticed a growing inclination of applying biologically inspired systems (like neural networks, natural immune system, DNA, genetic algorithms etc.) [4] to resolve the complex problems. In the financial domain, until recently, the problems regarding optimum portfolios have been resolved through linear or quadratic programming models.
The Markowitz portfolio selection model is a quadratic programming model and it represents the cornerstone of modern portfolio theory. Markowitz's model [6] uses the mean and variance of returns to measure the expected return and risk of the portfolio under the assumption that investors are risk averse and the returns are normally distributed. Hence, a risk-averse investor seeks to obtain a portfolio with the largest possible expected return being given the risk, and with the smallest possible risk being given its expected return. Over the past 50 years, and enormous volume of research has been done to study the model developed by Markowitz and to improve it.
Among the biologically inspired systems, genetic algorithms represent one of the most frequently used methods for improving the results of the model developed by Markowitz. The genetic algorithm represents a method for solving both constrained and unconstrained optimization problems. Based on a natural selection process, the genetic algorithm repeatedly modifies a population of individual solutions, randomly selecting at each step individuals from the current population to be parents and using them to produce the children for the next generation, after successive generations being obtained an optimal solution [8].
Taking into consideration these aspects, Section 2 of the present paper focuses on the theoretical aspects of the model developed by Markowitz, introducing into the model a new factor that denotes the risk-aversion of the investor [5]. Section 3 of the paper concentrates on the theoretical aspects regarding genetic algorithms. In Section 4, using Matlab GA Toolbox and the model proposed in Section 3, the portfolio selection problem on the Romanian Capital Market is solved through a genetic algorithm optimization. The conclusions of the paper are outlined in Section 5.

2. THE MARKOWITZ PORTFOLIO SELECTION MODEL

Harry Markowitz is rightfully regarded to be the founder of Modern Portfolio Theory due to his extended research on portfolio selection, publishing in 1952 a formal portfolio selection model [1] that emphasized diversification principles and identified the *efficient frontier* for risky securities, paving his way to the Nobel Prize for Economy (1990).

Eugene F. Fama refers to the model developed by Markowitz as the *Two-Parameter Portfolio Model* [2]. In this two-parameter model the distributions of returns on any portfolio are normal and can be described from knowledge of portfolio mean and portfolio standard deviation. So, with normal return distributions, a risk-averse investor only considers a portfolio if it has the largest possible expected return being given the standard deviation of return, and if it has the smallest possible standard deviation of return being given its expected return. The portfolio with these two properties is considered to be *efficient*, and the collection of portfolios with these two properties is called *efficient set*. These two portfolio properties can be summarized into a single one which states that for a portfolio to be efficient there must not be another portfolio with the same or higher expected return that has lower standard deviation of return.

So, in a two-parameter world, the portfolio risk is measured by its standard deviation, or its variance of return. A risk-averse investor is averse to dispersion of portfolio return, dispersion that can be completely summarized by variance [2,6] in the case of normally distributed portfolio returns. The risk of a security is determined by the contribution of the security to the variance of the return on the portfolio.

For a portfolio to be efficient it must have the property that no other portfolio with the same or higher expected return has lower standard deviation of return. Hence, if a portfolio is efficient then (a) it has the maximum possible expected return given the variance of its return, and (b) it has the smallest possible variance of return given its expected return. Any portfolio that satisfies condition (b) is called a *Minimum Variance Portfolio* and it is considered to be the solution of the following optimization problem:

$$\underset{\substack{x_{ip} \\ i=1,\ldots,n}}{Min}\ \sigma^2(R_p) = \sum_{i=1}^{n}\sum_{j=1}^{n} x_{ip}x_{jp}\sigma_{ij} \qquad (1a)$$

with the following restrictions:

$$\sum_{i=1}^{n} x_{ip}E(R_i) = E(R_e) \qquad (1b)$$

$$\sum_{i=1}^{n} x_{ip} = 1 \qquad (1c)$$

$$x_{ip} \geq 0, i = 1,\ldots,n \qquad (1d)$$

where n is the number of securities available for inclusion in the portfolio, x_{ip} is the proportion of portfolio funds invested in security i in portfolio p, R_i represents the return on security i, $E(R_i)$ is the expected return on security i, $\sigma_{ij} = \text{cov}(R_i, R_j)$ is the covariance between returns on securities i and j.

Eq. (1a) minimizes the total variance associated with the portfolio while Eq. (1b) ensures that the portfolio has a given expected return of $E(R_e)$. Eq. (1c) ensures that the portfolio weights add to one ane Eq. (1d) emphasizes the fact that the weights are nonnegative, meaning that short sales are not allowed. The problem underlined through Eq. (1a) to Eq. (1d) is to choose the proportions $x_{ip}, i = 1,\ldots,n$ invested in individual securities that give a *minimum variance portfolio*. Solving this optimization problem for different levels of the expected return, the *efficient frontier* is computed - the curve that lies between the global minimum risk portfolio and the maximum return portfolio.

In order to enrich the model a new factor [5] $w\ (0 \leq w \leq 1)$ that denotes the risk-aversion of investors is introduced, transforming the above optimization problem into the following one:

$$\underset{\substack{x_{ip} \\ i=1,\ldots,n}}{Max}\ (1-w)\sum_{i=1}^{n} x_{ip}E(R_i) - w\sum_{i=1}^{n}\sum_{j=1}^{n} x_{ip}x_{jp}\sigma_{ij} \qquad (2a)$$

with the restrictions:

$$\sum_{i=1}^{n} x_{ip}E(R_i) = E(R_e) \qquad (2b)$$

$$\sum_{i=1}^{n} x_{ip} = 1 \qquad (2c)$$

$$x_{ip} \geq 0, i = 1,\ldots,n \qquad (2d)$$

When $w = 1$ [5] investors are extremely risk-averse being preoccupied only with the risk of the investment and disregarding expected return. Risk-neutral investors [1] have $w = 0.5$ and judge risky prospects only on the basis of the expected returns, the risk level being irrelevant. For $w = 0$ the investors are risk lovers [1] and they adjust the expected level of return in order to ″enjoy″ the confrontation with the risky prospects.

3. GENETIC ALGORITHMS

Genetic algorithms are heuristic search methods inspired by nature, being used to find solutions to optimization problems when deterministic search methods do not succeed. Principles of evolutionary biology, such as natural selection and reproduction serve as guidelines to find a population of solutions to a given problem. Holland (1975) following the Darwinian principle ″the fittest survive″ in nature, developed the genetic algorithm, a stochastic searching technique that has rapidly become the most used evolutionary technique.

In genetic algorithms, an initial population containing constant number of chromosomes is generated randomly (regarding portfolio optimization, each chromosome represents the weight of an individual security) and an evaluation function is formed to evaluate the fitness of each chromosome, which defines if the chromosome represents a good solution. Using crossover, mutation and natural selection, the population will evolve towards a population that contains only the chromosomes with good fitness. The larger the fitness value is the better objective function the solution has. The basic steps in genetic algorithms [3] are:

Step 1: Initialize a randomly generated population.
Step 2: Evaluate fitness of individual in the population.
Step 3: Apply elitist selection: carry on the best individuals to the next generation from reproduction, crossover and mutation.

Step 4: Replace the current population by the new population.
Step 5: If the termination condition is satisfied then stop, else go to Step 2.

4. GENETIC ALGORITHM PORTFOLIO OPTIMIZATION

Many optimization problems are very complex and genetic algorithms are often used to solve this type of problems. Therefore, the model proposed through Eq. (2a) to Eq. (2d) can be solved through genetic algorithm optimization for different levels of the risk-aversion w factor. Using the GA Optimization Tool from Matlab, for a sample of 15 stocks quoted on the first category of the Bucharest Stock Exchange (BSE) [9] between January 2002 and December 2009 we computed the values of the objective function and the portfolio weights. We have chosen these stocks because they meet the conditions of financial performance, liquidity and supplemental dimension in comparison to the rest of the stocks quoted on the exchange rate.

Stock	Portfolio Weights
ALR	6,66%
ATB	0,00%
AZO	0,00%
BRD	0,00%
IMP	0,00%
OIL	65,29%
OLT	1,81%
SIF1	0,00%
SIF2	0,00%
SIF3	0,00%
SIF4	0,00%
SIF5	0,00%
SNP	0,00%
TBM	1,00%
TLV	25,24%
Objective Function	-0,17%

Table1. Investment proportion with risk-averse coefficient $w = 1$

For a risk-averse investor ($w = 1$), the optimum combination is to invest 6.66% of the portfolio funds in the ″Materials″ business sector, 65.9% in the ″Services and Energetic Equipments″ sector, 1.81% in the ″Chemistry″ sector, 1.00% in the ″Equipments″ sector and 25.4% in the ″Banks, Insurances and Financial Services″ sector. As it can be seen the optimum solution for the risk-averse investor is to distribute the portfolio funds across securities. For this situation the value of the objective function is -0.17%.

Stock	Portfolio Weights
ALR	0,00%
ATB	0,00%
AZO	0,00%
BRD	0,00%
IMP	0,00%
OIL	72,53%
OLT	0,00%
SIF1	0,00%
SIF2	0,00%
SIF3	0,00%
SIF4	0,00%
SIF5	0,00%
SNP	0,00%
TBM	0,00%
TLV	27,47%
Objective Function	0,05%

Table2. Investment proportion with risk-averse coefficient $w = 0.5$

The risk neutral investor ($w = 0.5$), invests 72.53% in the ″Chemistry″ sector and 27.47% in the ″Banks, Insurances and Financial Services″ sector and the value of the objective function is 0.05%.

Stock	Portfolio Weights
ALR	0,00%
ATB	0,00%
AZO	0,00%
BRD	0,00%
IMP	0,00%
OIL	0,00%
OLT	0,00%
SIF1	0,00%
SIF2	100,00%
SIF3	0,00%
SIF4	0,00%
SIF5	0,00%
SNP	0,00%
TBM	0,00%
TLV	0,00%
Objective Function	0,33%

Table3. Investment proportion with risk-averse coefficient $w = 0$

In contrast, the risk lover ($w = 0$) invests all his funds into the ″Banks, Insurances and Financial Services″ business sector. This result shows that this type of investor prefers to make risky investments.

5. CONCLUSIONS

The capital allocation and portfolio selection decisions represent fundamental elements for investors, the model emphasized by Markowitz serving as a basis for the development of modern financial theory during the last five decades. Due to the fact that many researchers consider that investors can not fundament their decisions only on this model, we have introduced in the initial model a factor that indicates the level of the investor's risk-aversion. Using the GA Optimization Tool from Matlab for a sample of 15 stocks we solved the new optimization problem for different levels of the risk-aversion factor. We concluded that the results obtained are consistent with the investment behavior outlined for the values of the risk aversion factor ($w = 1; w = 0.5; w = 0$).Therefore, the optimization of the investment decisions through genetic algorithms represents a powerful tool in the portfolio selection domain.

6. REFERENCES

[1] Z. Bodie, A.Kane, A.Marcus, **Investments**, The McGraw-Hill Companies, Fifth Edition, 2001, pp. 163-265.

[2] E.F. Fama, **Foundations of Finance**, Basic Books, 1976, pp. 3-132, pp. 212-270.

[3] T-J. Chang, S-C. Yang, K-J. Chang, ″Portfolio optimization problems in different risk measures using genetic algorithm″, **Expert Systems with Applications**, Vol.36, 2009, pp. 10529-10537.

[4] V.C.S. Lee, X. Yang, ″Development and Test of an Artificial-Immune-Abnormal-Trading-Detection System for Finanacial Markets″, http://www.bsys.monash.edu.au/index.html.

[5] Y. Xia, B. Liu, S. Wang, K.K. Lai, ″A model portfolio selection with order of expected returns″,

Computers & Operations Research, Vol.2007, 2009, pp. 409-422.
[6] H. Markowitz, "Portfolio Selection", **The Journal of Finance**, Vol.7, No.1, 1952, pp. 77-91.
[7] S. Stancu, C. Huidumac, **Teoria Portofoliilor cu aplicatii pe piata financiara**, Ed. Didactica si Pedagogica, 1999, pp. 53-100.
[8] ***http://www.mathwork.com;
[9] ***http://www.bvb.ro;