Analyzing Historical Changes in the Airline Transportation Network from the 1920's to Present Day

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Abstract

This research effort considers air transport network design as a complex systems optimization problem. Specifically, a top-down approach is taken to identify design metrics associated with scale-free networks that directly impact Federal Aviation Agency and passenger performance metrics. A historical review of the U.S. airline industry illuminates fundamental changes within the air transport network structure. The effects of changing network metrics (connectivity, synchronicity, etc) through basic network design (direct flights verses hub-and-spoke, etc.) and how criteria such as late arrivals and customer satisfaction are impacted will be examined.

Keywords: Airline Transportation Network, Scale-free Networks, and Geo-spatial Network Representation.

1 Introduction

Scale-free networks comprise a relatively new field of study in the academic world. Common applications include social networks, Internet router networks, biological networks, and physical infrastructures. Li, et al.[3] provide a list of basic properties that most researchers assume these networks possess: scale-free networks have scaling (power law) degree distributions; scale-free networks can be generated by certain random processes (the foremost being preferential attachment); scale-free networks have highly connected "hubs" which "hold the network together" and give the "robust yet fragile" feature of error tolerance but also attack vulnerability; scale-free networks are generic in the sense of being preserved under random degree preserving rewiring; scale-free networks are self-similar; and scale-free networks are universal in the sense of not depending on domain-specific details [3]. Simply stated, the nodes within a scale-free network may be categorized into two basic classifications; nodes having a small number of arcs or links connecting them to other nodes within the network (comprises the predominant type in the network) or nodes having an extraordinarily large number of links connecting them to numerous other nodes in the network (demonstrated by only a relatively small proportion of the network).

The current degree distribution of the airline transportation network is believed to be scaling and, therefore, exhibit characteristics of a scale-free network. If the degree distributions are scaling, then the conjecture follows that affecting global metrics associated with the network (such as synchronization and/or connectivity) should alter the network structure and affect the performance metrics that the Federal Aviation Agency (FAA), the commercial airline industry, and the customers are ultimately desiring to optimize. This "top-down" approach may provide a tool for the FAA to make centralized changes to better optimize travel in the United States by providing flexibility to resolve major interruptions in flight schedules due to adverse weather, mechanical problems, or even terrorist activities.

2 Scale-free Networks

In a scale-free network, the mathematical probability of a node connecting to k other nodes is roughly $P(k) = k^{-\gamma}$. This inversely proportional relationship is characteristic of scaling distributions. A prominent characteristic of scaling degree distributions is the vast majority of nodes only have a few connections, while a select few nodes exhibit a very large number of connections. Barabási called nodes with a high number of connections "hubs" and demonstrated that these networks follow a scaling distribution instead of a normal or Poisson distribution.

2.1 Graph Theory Background

We assume the reader is familiar with basic graph theoretic definitions and concepts. For those who are not, please see Chartrand [6]. We begin our discussion with the adjacency matrix. Of note, Fiedler states that the second smallest (first non-negative) eigenvalue of this matrix measures the algebraic connectivity of the graph [31]. Later work will consider the eigenvalues associated with the Laplacian matrix found by subtracting the degree matrix from the adjacency matrix, $L = D - A$. This will provide a metric that indicates the connectivity of the network. The second smallest eigenvalue of the Laplacian is tied to a network's ability to synchronize[27].

2.2 Scale-free Network Metrics

There are general metrics associated with scale-free networks that will assist in understanding the magnitude of changes in an airline transportation network over time. Relatively simple measures include the order or number of airports in the network, the average degree or scheduled destinations, and the average path length or number of hops between airports. Other more involved calculations include the clustering coefficient and transitivity. The clustering metric of a network is found by averaging the clustering coefficients of each node. The clustering coefficient measures the density of triangle subgraphs in a network. Transitivity measures the percentage of edge pairs $(i, j), (j, k)$ in the network that also include (i, k) as an edge in the network.

Li, et. al. introduced the metric $s(G)$, a measure of network assortativity. This metric is the sum of the products of d_i and d_j for all edges (i, j) in the network (Equation 1). Dividing $s(G)$ by $s_{\max}$, the maximum value of

$s(G)$, while keeping the network's degree distribution fixed yields a scaled value dentoed $S(G)$. When $s(G)_{min} = 0$, the range of $S(G)$ is between 0 and 1, with high degree nodes connected to low degree nodes having values closer to 0 and high degree nodes connected to other high degree nodes having values closer to 1. Preliminary calculations for $S(G)$ for specific years have been found for 1990, 1995 and 2000.

$$s(G) = \sum_{(i,j)\in\epsilon} d_i d_j \tag{1}$$

$$S(G) = \frac{s(G)}{s_{max}} \tag{2}$$

We note that Li, et. al. [3] explain how $s(G)$ and traditional assortativity metrics are similar. Moreover, they show that $s(G)$ has a larger range than traditional assortativity when graphs are restricted to those without self-loops and without multiple edges.

3 Airline Industry

Technological advances coupled with major shifts in population demographics have contributed directly to the current configuration of today's airline transportation network. In this section, significant events throughout U. S. aviation history are explored and the impacts on the industry are demonstrated. Obvious changes such as increased aircraft speed, longer flight segments, improved maintenance, more ground facilities, and enhanced navigational aides significantly impacted daily operations and the overall structure of the network.

The face of aircraft, and for that matter the airline industry, has changed dramatically since the Wright Brothers first took flight at Kitty Hawk, NC on 17 December 1903. As aircraft technology improved, the distances traveled and the payload capacities both increased dramatically. Initially, during the 1920's, the Air Mail Service of the U. S. Post Office Department proved to be the standard bearer for progress in the airline industry. The only other contributor that operated on a regular schedule covering significant distances (100 miles) during that time was the Model Airway of the U. S. Army Air Service.

Although the United States led the world with the first manned-flight, Europe took the lead in the development and extension to the airline industry. There were several contributing reasons for these two diverging paths. First, in the United States the railroad industry was "king" and had many powerful and influential connections to the government. Trains also offered luxurious Pullman cars and relatively quiet travel compared to the noisy, open-air cabins of airplanes. Slow aircraft speeds compounded the issue, as traveling by rail was often quicker than by air. On the other hand in Europe, the infrastructure (particularly the transportation networks) suffered greatly during World War I (WWI). In 1918, France, Germany and England embraced the newest mode of transportation, modified existing bombers to accommodate passengers and filled the infrastructure void by developing a viable passenger-carrying industry[16]. Airlines quickly established daily cross channel flights between London and Paris.

In the same year Europe was developing the foundation of an air network, the fledgling airline industry in the United States truly began in earnest when $100,000 was awarded to the U. S. Post Office System to start air mail service. Eventually legislation would promote competition between private companies to take over the air routes from the post office. Low public opinion of airlines would not ultimately change until hero Charles Lindbergh made his famous trans-Atlantic flight.

The airline transportation network may have developed more slowly than the Internet, but airport nodes and scheduled flight routes similarly exhibit properties associated with complex networks. In [29] the authors consider growth dynamics associated with networks. They demonstrate that different growth regimes are linked with a node's age and connectivity. Throughout the development of the airline industry, these same characteristics (airport age and connectivity) have impacted air network growth and ultimately its development.

Strogatz [26] attempts to characterize the properties of the associated physical networks of a complex system. He provides a list of several inherent difficulties associated with the networks. These difficulties must be addressed by any mathematical modeling done to represent or explore an existing complex system. The following list explores how the airline industry compares to the criteria of complex networks given by Strogatz.

1. **Structural Complexity.** Air route maps of scheduled airline flights highlight the observation that airline networks are complex. On an average day there are close to 30,000 commercial flights (regional and major airlines)[1]. In 2007, the total number of scheduled airline flights was 88,766[2]. These flights were spread out between 1030 airports in 945 cities.

2. **Network Evolution.** The airline industry began from somewhat humble beginnings (barnstormers and air mail) and evolved into today's complex network. Various influences have shaped the network (aircraft technology, navigational aides, etc.), but ultimately airline transportation is a business. The primary network design has been greatly shaped by simple supply and demand of customers (passenger demand).

3. **Connection diversity.** Air traffic safety demands a structured yet diverse air transportation network. An assortment of flights may be achieved via air corridors, operational altitudes, operational times and no-fly zones. The FAA is undergoing an extensive effort to identify and influence the capacity needs of the National Airspace System (NAS) between present day and 2025.

4. **Dynamical Complexity.** Severe weather conditions (fog, heavy rain, snow and ice) can greatly impact specific airports. Undulations are often felt throughout the entire network (this is akin to a cyber attack taking out certain key servers).

5. **Node Diversity.** There are numerous facilities that act as nodes in the airline network. The FAA monitors airports, balloonports, gliderports, heliports,

[1] Based on information from the National Air Traffic Controllers Association website (http://www.natca.org/mediacenter/bythenumbers.msp).

[2] BTS-RITA, T-100 Domestic data.

seaplane bases, stolports and ultralight ports, both public and private. Often facilities operate multiple modes. Facilities also have unique requirements based on physical restrictions of the surrounding terrain and population footprint.

6. **Meta-complication.** Interactions between the above mentioned mentioned traits.

4 History

There have been numerous eras associated with the airline industry since the Wright Brother's biplane glider took flight. The initial years (1903 to early 1920's) were dominated with great advances in technology associated with aircraft, airports and navigation. Mail routes and military operations dominated activities during the later part of the 1920's and trunk lines became the accepted method of air travel. Post WWII travel saw continued expansion and the arrival of the jet engine. The 1970's and 1980's saw airline deregulation and the rise of airlines using the hub-and-spoke model. In the most recent era, concerns about National Airspace System Capacity, a return to direct flight or point-to-point models and the events of 9-11 have influenced the existing system. The following section looks at the key events within specified eras and determines their impact on the current air transportation network.

4.1 Initial Years, 1903—1920's

The preponderance of the civil air transportation in the United States during the initial start up was comprised of non-scheduled flights less than 100 miles[3]. Airplanes were primarily seen as curiosities without many practical applications as they were relatively slow (averaging 75 mph), had open compartments, and were severely limited by weather and limited visibility (navigation was primarily conducted using available landmarks). The train industry, in comparison, was close to its zenith during this period. Passengers selected traveling in a "plush" environment offered by Pullman cars in lieu of the windy, loud and potentially oily airplane flights.

The outbreak of World War I (WWI) further obstructed air passenger services, as the industry focused on developing military warplanes and not the advancement of personnel transportation. Some scheduled services would begin operation, typically borne in locations rife of other transportation options. The first scheduled passenger flight service did occur prior to the United States involvement in the war. Thomas Benoist set up regularly scheduled flights in 1914 between St. Petersburg and Tampa. The only transportation options at the time included a once-daily, two hour boat trip, a twelve hour train ride, or an almost day-long car ride over unimproved dirt roads. The twenty to twenty-five minute plane ride (depending on direction of travel) proved to be popular with the local denizens. Over the course of a four month period, the airline would carry a total 1,204 passengers without incident and only saw services canceled eight days due to bad weather or mechanical breakdown[18]. Air transportation for the general population would not occur until a later date. Few craft available were reliable enough to maintain any type of scheduled services. To compound matters, there were few logistic facilities established that could facilitate the type of support that general aviation required. Up to this point, aircraft would typically fly from a home base of operations, land at a suitable flat, obstruction free field, then return to the point of origin. Formal air travel in America would begin after the United States government opened contracts for air mail in 1917.

1. **Mail Routes/Military.** The primary outlet for fledgling airlines became the mail service routes sponsored by the government. Brief experiments conducted prior to WWI validated the feasibility of carrier services. The first mail flight was undertaken by Earle Ovington on September 23, 1911. Given the success the the government formally introduced U. S. Air Mail Service on May 13, 1918[17]. As WWI concluded, the U. S. Postal Service began acquiring war surplus aircraft (primarily de Haviland DH-48 biplanes) which made it possible for the mail service to become transcontinental. The air routes slowly expanded between 1918 and 1921 (See Figure 1).

   A transcontinental air postal route was a vast improvement over the existing railroad system. Due to territorial ownership of railroads, there was no single company that went coast to coast. As a result, all mail shipments had to be transfered from one rail company and rail line to another, typically in Chicago. The first successful transcontinental delivery by air took 16,000 letters from New York to San Francisco and was able to average 80 mph, including stops. More importantly, the air service saved almost 22 hours compared to the rail system.

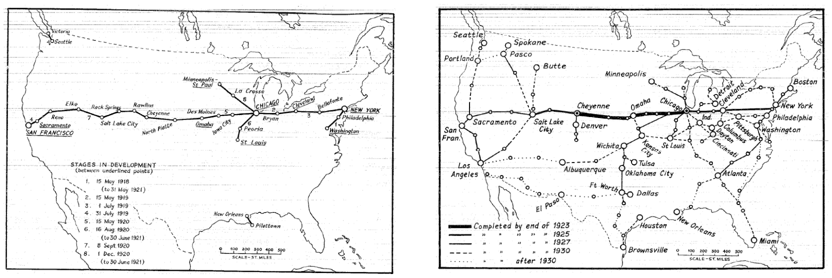
2. **End of the Government Air Mail.** The Air Mail Act of 1925 and the Air Commerce Act of 1926 paved the way for contract carriers to assume the duties of air mail service. In the nine plus years that the Post Office was in charge of air mail, their planes flew over 15 million miles, completed 93% of scheduled flights and delivered over 300 million leters. The cost was high financially and physically. The mail program cost more than $17 million dollars during a time the country was involved in WWI. Additionally, there were 200 airplane crashes at the cost of 43 deaths and 37 serious injuries.

3. **Safety Standards and Equipment Improvements**

   The westward expansion of the airlines was slow due, primarily, to three factors; the lack of quality airfields, poor navigational aides, and limited range of aircraft. Suitable locations existed for airfields (flat, open areas), however re-supply and mechanical assistance were sparse and required more time to become mature. The military created aerial maps along the new western routes to assist with navigation. Until radio beacons, the primary technique used by pilots was visual recognition of land marks[18]. This caused flights to occur primarily during daylight.

   Better aircraft, improved navigational aides and emphasis on pilot training improved air travel. By 1927 the Army had successfully built a night flying system between Dayton and Columbus. Additional lighted air routes (Figure 1) would continue to be built. As mentioned before, the primary influence on network structure was trunk lines operations (planes fly out in one direction, stop at airports along the

[3]One such example during this time period was the Aero Ltd's New York City to Atlantic City, NJ scheduled flight.



(a) Air Mail Routes

(b) Lighted Airways

Figure 1: Early aerial maps.

route, then return back through the same way points. Trunk lines would dominate network structure until the Deregulation Act of 1978, when *hub-and-spoke* networks would become more prevalent.

Airline route origins and destinations developed following a supply and demand model (based on demand for air mail traffic, supplies and ultimately passengers); however, way points were a function of technology limitations. Additional landing fields along these routes were added out of necessity[4].

Early landing fields along mail delivery and military routes were often determined by the ranges of aircraft and the availability of suitable landing space (sufficiently large, open, well-drained fields[22]). The fields were marked with letters or numbers at least 100 feet long, either white vegetable growth, crushed stone or lime being used. Two types of fields existed, permanent and emergency. As more air traffic routes were constructed in the 1930's and 1940's and aircraft technology improved, scheduled flights became demand-driven. The airline transportation network growth begins to exhibit a *preferential attachment*, or a "rich-get-richer" philosophy.

Other significant events prior to WWII that contributed to the continued expansion and improvement in the airline industry include the following: Air Mail Act of 1925, Air Commerce Act of 1926, Western Air Express offering regular passenger service in 1926, and the Civil Aeronautics Act of 1938[18].

4.2 Expansion (Post World War II)

After soldiers returned from WWII, the industry saw a large influx of experienced pilots. Additionally, returning veterans were accustomed to traveling and, consequently, led to increased demand for commercial passenger service. Major airlines continued to grow to support this demand (TWA, United, Northwest, American, Eastern, etc.) of travel to various locations in the United States. The development of the jet engine for combat planes in WWII would find its way into commercial airlines starting in 1959. Jets would alter the transportation network similar to way interstate highways altered ground transportation. Smaller airports necessitated by earlier logistics would now be bypassed in favor of major cities.

4.3 De-Regulation

Until 1978 all interstate flights were regulated by the U. S. government. The Civil Aeronautics Board (CAB) set fares, routes and schedules to promote air travel. Intrastate flights were not subject to the bureaucracy associated with the CAB. The resulting flight schedules were predominantly point-to-point, with many regional airlines "owning" the rights to territorial routes. In the early 1970's, rising energy prices and stagflation[28] led to the Airline Deregulation Act of 1978 (Public Law 95-504)[5] and prompted airlines to switch to the hub-and-spoke flight operations model. Consolidating operations at a hub location reduced airlines operational costs and, therefore, increased profits (or decreased losses). Figure 2 displays the before and after deregulation changes in Western Airlines' routes [28]. The before schedule has more point-to-point flights, while the after schedule has more hub-and-spoke (Salt Lake City as the primary hub). The degree distribution shows the emergence of a network with a few highly connected nodes and many low connected nodes.

The CAB concluded operations on 1 January, 1985. After that time the full results of deregulation could finally be evaluated. Some individuals predicted dire consequences for regional airports with small passenger demand. However, the hub-and-spoke model supported regional airports and competition actually drove down passenger fares by %9. Another result was the reallocation of airplanes to specific flights. Larger aircraft were now assigned to longer routes (hub-to-hub), while smaller aircraft were assigned to shorter (hub-to-regional). This allocation of planes led to an increase in the number of passengers per flight.

4.4 Direct Flights

Recently smaller airlines have found a niche in the airline industry through re-introducing point-to-point flights. These point-to-point flights are primarily found along high demand air routes. Several studies sponsored by the FAA have been initiated to determine the optimal distribution of flights for the future. This will become more important

[4]Planes were slaves to supply requirements and weather conditions. Appropriate grade gas and oil were not readily available at all landing sites. Additionally poor weather conditions forced planes land until conditions improved[22].

[5]The legislation was designed to "encourage, develop, and attain an air transportation system which relies on competitive market forces to determine the quality, variety, and price of air services".

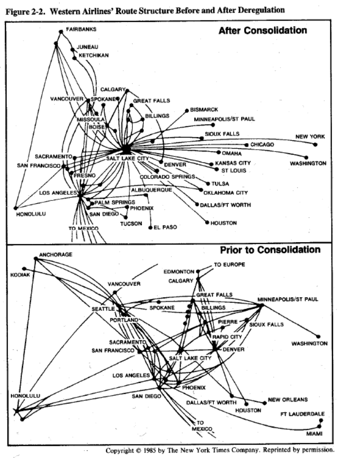


Figure 2: Western Airlines' restructing before and after 1978 Deregulation.

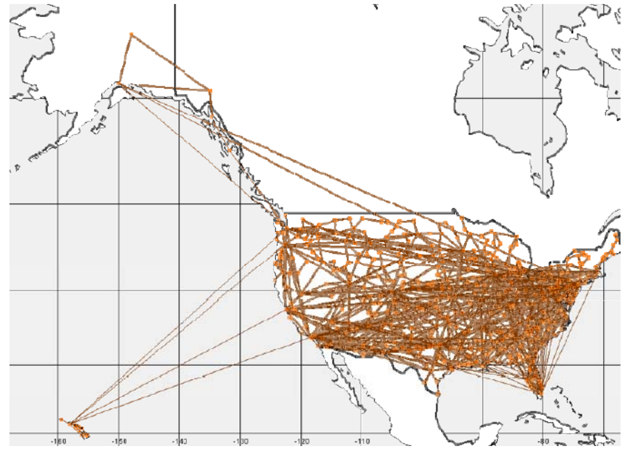


Figure 4: ORA Geospatial Map of Scheduled Airline Flights in 1960.

as airspace becomes increasingly congested. One can easily see in Figure 3 the dramatic increase in the number of flights in recent years.

5 Results

Since the early data does not exist in a digitized form, there may be some minor inconsistencies in the collection effort (primarily interpretation of maps). However, using redundant sources helped to minimize these errors and it is believed that the final data product provides an adequate representation of trends exhibited in the airlines during the time period in question. Airline flight data was gathered using two different approaches. Pre-1990 data had to be collected from various sources to include government maps, Air Commerce Bulletins, Civil Aeronautics Journals, and actual airline brochures (images found at *http://www.airchive.com*). Post-1990 data is maintained at the U. S. Department of Transportation's Research and Innovative Technology Administration's Bureau of Transportation Statistics and is easily accessed. Initially analysis was conducted using a combination of *Excel* spreadsheets, *Matlab*, *SOCNETV*, *R*, and *Python*. After collecting the raw flight data from either source, the flight information was formed into city-pairs. The city pairs were then converted into adjacency matrices and degree distribution matrices. Recently the discovery of the Organizational Risk Analyzer (ORA) greatly reduced the reliance on multiple software applications. ORA is a risk assessment tool developed at Carnegie Mellon University built specifically for use at the Computational Analysis of Social and Organizational Systems center. Although all previous data had to be re-formatted, the analysis and graphical capabilities of ORA exceeded any of the drawbacks. To form a network in ORA, one simply enters the adjacency matrix into the Meta-Network Manager. A second location matrix with airports and the associated longitudes and latitudes would then be entered into ORA and saved as attributes of the network. At this point ORA can provide the user with Network-Level, Node-Level and Location-Level Measures (over one hundred network statistics) and plot the network on geospatial map (See Figure 3).

Data was collected every decade from 1920 to 1990. Beginning in 1990, data was collected annually. Figure 3 shows the changes in the value of the exponent, γ, for the power law distribution. The values were calculated using the empirical distribution of the annual degree distribution of flights. Typically, γ falls somewhere between 2 and 3, but the low values are probably due to poor modeling of tail behavior. Classical scale-free networks will have the majority of nodes with only 1 or 2 links. Rationally, even small, isolated airports will have more than 1 or 2 scheduled flights. Interestingly, the value of γ changes dramatically after the deregulation of airlines occurred in 1978. Another interesting result is seen in Figure 3. The number of scheduled destinations (or average degrees) from airports appeared to be decreasing at a somewhat linear rate. The trend clearly changes in 2001, which reflects the events surrounding the terrorist attacks of 9-11. Other key events such as the Airline Deregulation Act of 1978, Orlando, FL becoming a major tourist attraction and even the recent economic downturn are represented in the statistical data. Now that the data has been collected and the networks have been constructed, further analysis will be conducted to look for additional trends.

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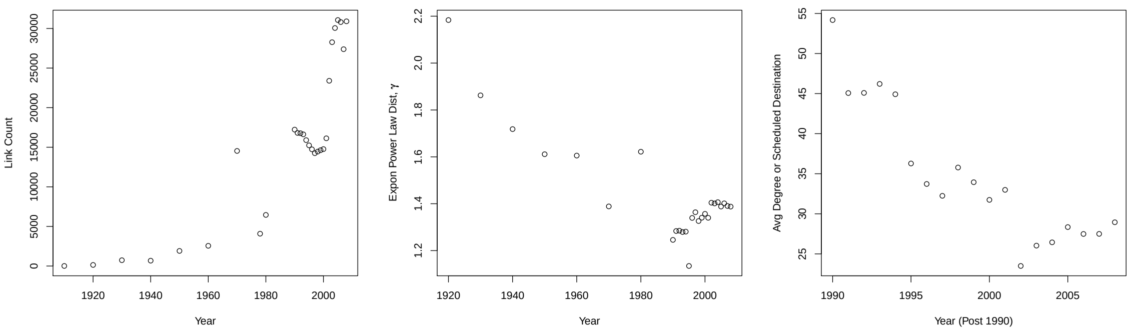

(a) Number Flights. (b) Estimated γ. (c) Flights per Airport

Figure 3: ORA Outputs.

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