Multi-Pulse Chaotic Dynamics of Functionally Graded Materials Rectangular Plate

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Abstract
Multi-pulse chaotic dynamics of a simply supported functionally graded materials (FGMs) rectangular plate is investigated in this paper. The FGMs rectangular plate is subjected to the transversal and in-plane excitations. The properties of material are graded in the direction of thickness. Based on Reddy’s third-order shear deformation plate theory, the nonlinear governing equations of motion for the FGMs plate are derived by using the Hamilton’s principle. The four-dimensional averaged equation under the case of 1:2 internal resonance, primary parametric resonance and 1/2-subharmonic resonance is obtained by directly using the asymptotic perturbation method and Galerkin approach to the partial differential governing equation of motion for the FGMs rectangular plate. The system is transformed to the averaged equation. From the averaged equation, the theory of normal form is used to find the explicit formulas of normal form. Based on normal form obtained, the extended Melnikov method is utilized to analyze the multi-pulse global bifurcations and chaotic dynamics for the FGMs rectangular plate. The analysis of global dynamics indicates that there exist the multi-pulse jumping orbits in the perturbed phase space of the averaged equation. From the averaged equations obtained, the chaotic motions and the Shilnikov type multi-pulse orbits of the FGMs rectangular plate are found by using numerical simulation. The results obtained above mean the existence of the chaos for the Smale horseshoe sense for the simply supported FGMs rectangular plate.

Keywords: Functionally graded materials rectangular plate, multi-pulse orbit, chaotic dynamics, the extended Melnikov method.

1 Introduction
Functionally graded materials (FGMs) are new engineering composite materials, which are being widely applied in large space stations, shuttles, aircrafts and many others in recent years [1,2]. Functionally graded materials are microscopically inhomogeneous composites usually made from a mixture of metals and ceramics. By gradually varying the volume fraction of constituent materials, their material properties exhibit a smooth and continuous change from one surface to another.

Due to the big advantage of the FGMs, FGMs plates will be used in more and more engineering fields. Therefore, researches on the complicated dynamics of FGMs plates play a significant role in engineering applications. In the past ten years, several researchers have focused their attention on the investigation of the dynamics for the FGMs plates. Praveen and Reddy [3] gave the nonlinear transient thermoelastic analysis of functionally graded ceramic-metal plates subjected to pressure loading. Senthil and Batra [4] gave a three-dimensional exact solution for the free and forced vibrations of simply supported FGMs rectangular plates. Huang and Shen [5] studied the nonlinear vibrations and dynamic responses of FGMs plates in thermal environment. Hao et al. [6] utilized the asymptotic perturbation method to investigate the nonlinear periodic oscillations, bifurcations and chaos of a FGMs plate. Zhang and Yao [12] used the extended Melnikov method to study the global bifurcations and multi-pulse chaotic dynamics of the non-autonomous system. Zhang and Yao [12] used the extended Melnikov method to analyze the multi-pulse chaotic dynamics for the nonplanar nonlinear oscillations of a cantilever beam.

This paper is aiming at studying the multi-pulse chaotic dynamics of a FGMs rectangular plate subjected to the in-plane and transversal excitations simultaneously. Transversal nonlinear oscillations of the FGMs plate are only considered, the equations of motion for the FGMs rectangular plate can be reduced into a two-degree-of-freedom nonlinear system using the Galerkin’s method. The extended Melnikov method developed by Camassa et al. [10] is employed to show the existence of multi-pulse chaotic dynamics for the FGMs rectangular plate. The results of...
numerical simulations indicate that there exist multi-pulse chaotic responses for the FGMs rectangular plate.

2 Equations of Motion and Perturbation Analysis

Based on research obtained by Hao et al. [6], we begin by giving a brief description of the model of the FGMs rectangular plate whose equations are derived by Hao et al. [6] using the Reddy’s third-order shear deformation plate theory and the Hamilton’s principle.

A simply supported at four-edge FGMs rectangular plate with total thickness \( h \), length \( a \), width \( b \) is considered. The plate is subjected to transversal excitation, in-plane excitation and thermal stress simultaneously, as shown in Figure 1. The in-plane excitation for the FGMs rectangular plate is distributed along the \( y \) direction at \( x = 0 \) and \( x = a \) and is of the form \( p_0 - p_1 \cos \Omega t \). The transverse excitation subjected to the FGMs rectangular plate is represented by \( F(x, y) \cos \Omega t \). Here \( \Omega_1 \) and \( \Omega_2 \) are the two frequencies of the transversal and in-plane excitations, respectively.

Figure 1 The model of a FGMs rectangular plate and the coordinate system.

We assume that the plate is made from a mixture of ceramics (Al2O3) and metals (Ti-6Al-4V) with continuously varying such that the top surface of the plate is ceramic rich, whereas the bottom surface is metal rich by following a simple power law in terms of the volume fractions of the constituents. The material properties such as Young’s modulus \( E \), of the volume fractions of the constituents.

\( t_{pp} \) is considered. The plate is subjected to temperature variation occurs in the thickness direction as a function of temperature, see [13-14]. The FGMs rectangular plate whose equations are derived begin by giving a brief description of the model of the plate.

\( \Omega_1 \) is of the form

\[
\Omega_1 = \frac{1}{2} + \sigma_1 \varepsilon + \sigma_2 \varepsilon^2,
\]

where \( \sigma_1 \) and \( \sigma_2 \) are two linear frequencies, \( \sigma_1 \) and \( \sigma_2 \) are the two detuning parameters.

Substituting equations (2) and (3) into equation (1), the approximate solutions \( w_1(t) \) and \( w_2(t) \) of equation (1) are sought in a power series of small parameter \( \varepsilon \)

\[
w_1(t) = \varepsilon^0 \psi_0(t, \varepsilon) + \varepsilon^1 \psi_1(t, \varepsilon) e^{-i \frac{\Omega_1}{2}} + \varepsilon^2 \psi_2(t, \varepsilon) e^{-i \frac{\Omega_1}{2}} + \varepsilon^3 \psi_3(t, \varepsilon) e^{-i \frac{\Omega_1}{2}} + \varepsilon^4 \psi_4(t, \varepsilon) e^{-i \frac{\Omega_1}{2}} + cc + O(\varepsilon^5)
\]

\[
w_2(t) = \varepsilon^0 \Phi_0(t, \varepsilon) + \varepsilon^1 \Phi_1(t, \varepsilon) e^{-i \frac{\Omega_1}{2}} + \varepsilon^2 \Phi_2(t, \varepsilon) e^{-i \frac{\Omega_1}{2}} + \varepsilon^3 \Phi_3(t, \varepsilon) e^{-i \frac{\Omega_1}{2}} + \varepsilon^4 \Phi_4(t, \varepsilon) e^{-i \frac{\Omega_1}{2}} + cc + O(\varepsilon^5),
\]

where the symbol \( cc \) stands for the parts of complex conjugate of the functions on the right hand side of equation (4).

Substituting equation (4) into equation (1), the averaged equation in the Cartesian form is obtained as follows

\[
\frac{dx_1}{dt} = -\mu_1 x_1 + (\sigma_1 + \alpha_1 + \alpha_2 f_1) x_1 + 14 \alpha_4 x_2 (x_1^2 + x_1^2)
\]

\[
+ \alpha_3 x_1 (x_1^2 + x_1^2) + \alpha_4 x_1 (x_1^2 + x_1^2)
\]

\[
+ \alpha_5 x_1 x_3 - \alpha_2 x_2 x_4,
\]

\[
\frac{dx_2}{dt} = -\sigma_1 + \alpha_1 + \alpha_2 f_1 x_1 - \mu_1 x_2 - 14 \alpha_4 x_2 (x_1^2 + x_1^2)
\]

\[
- \alpha_4 x_1 (x_1^2 + x_1^2) + \alpha_5 x_1 x_3 - \alpha_2 x_2 x_4,
\]

where all coefficients can be found in [6], \( f_1 \) and \( f_2 \) are the magnitudes of the forcing excitation.

To consider the influence of the quadratic terms on the nonlinear dynamic characteristics of the FGMs rectangular plate, we need to obtain the second-order approximated solution of equation (1). To guarantee the validity of perturbation analysis, we use the asymptotic perturbation method [15] to obtain the averaged equation of system (1).

Asymptotic perturbation method [15] to obtain the approximate solution of equation (1). To guarantee the validity of perturbation analysis, we use the asymptotic perturbation method [15] to obtain the averaged equation of system (1).

It is assumed that the width-to-length ratio of the FGMs rectangular plate is \( a/b = 1 \). Therefore, we only consider the case of 1:2 internal resonance, primary parametric resonance and 1/2-subharmonic resonance for the FGMs rectangular plate. In this resonant case, there are the following relations

\[
\omega_1 = \frac{1}{2} \pi + \sigma_1 \varepsilon + \sigma_2 \varepsilon^2,
\]

\[
\omega_2 = \Omega_1 + \sigma_2 \varepsilon^2,
\]

\[
\Omega_2 = \Omega_1 = \Omega,
\]

where \( \omega_1 \) and \( \omega_2 \) are two linear frequencies, \( \sigma_1 \) and \( \sigma_2 \) are the two detuning parameters.

The scale transformations may be introduced as

\[
a_1 = \varepsilon^2 a_1, a_2 = \varepsilon^2 a_2, f_1 = \varepsilon^2 f_1, h_1 = \varepsilon^2 h_1,
\]

\[
b_2 = \varepsilon^2 b_2, f_2 = \varepsilon^2 f_2.
\]
\[
\frac{dx_3}{dt} = -\mu_2 x_3 + (\sigma_2 + \beta_1 f_1) x_4 + \beta_2 x_4 (x_1^2 + x_2^2) + \beta_3 x_3 (x_1^2 + x_2^2) + \beta_4 x_2 (x_1^2 + x_2^2) - \beta_5 x_3 (x_1^2 + x_2^2),
\]
\[
\frac{dx_4}{dt} = (\beta_1 f_1 - \sigma_2) x_3 - \mu_2 x_4 - \beta_2 x_5 (x_1^2 + x_2^2) - \beta_3 x_5 (x_1^2 + x_2^2) - \beta_4 x_3 (x_1^2 + x_2^2) - \beta_5 x_4 (x_1^2 + x_2^2) + \frac{1}{4} f_2.
\]

In order to conveniently analyze the multi-pulse Shilnikov type orbits and chaotic dynamics of the FGMS rectangular plate, it will be convenient and necessary to reduce averaged equation (5) to a simpler normal form. It is found that there are 22 symmetries which are held in normal form. A linear transformation is introduced as

\[
\cos 2\beta \eta + \eta \mu_1 + \eta \mu_2 + \beta_2 u_2 - \beta_5 f_2 
\]

In order to get the unfolding of equation (6), a linear transformation is introduced as

\[
\left[\begin{array}{c} y_1 \\ y_2 \end{array}\right] = \left[\begin{array}{c} \sqrt{3} \frac{a_6}{b_6} [1 - \sigma_1] \\ \frac{1}{b_6} \mu_1 \end{array}\right] \begin{array}{c} u_1 \\ u_2 \end{array},
\]

Then, normal form (6) can be rewritten as the form with the perturbations

\[
u_1 = \frac{\partial H}{\partial u_2} + e g^{u_1} = u_2,
\]
\[
u_2 = -\frac{\partial H}{\partial u_1} + e g^{u_2} = -\beta_1 u_1 + \eta_1 u_1^3 + \beta_2 u_1^2 - \epsilon \sigma_2 u_2,
\]
\[
\epsilon = \frac{\partial H}{\partial \gamma} = -e \gamma_2 u_1 - e \sigma_2 \gamma_2 u_1^3 + \beta_2 u_1^2 - \epsilon \gamma_2 f_2 
\]

and \(g^{u_1}, g^{u_2}, g^{I}\) and \(g^{\gamma}\) are the perturbation terms induced by the dissipative effects

\[
g^{u_1} = 0, \ g^{u_2} = -\epsilon \gamma_2 u_2, \ g^{I} = -\mu_1 I - \sigma_2 \sin \gamma, \ g^{\gamma} = -\gamma_2 \cos \gamma.
\]

### 3 Unperturbed Dynamics

When \(\epsilon = 0\), it is noted that system (8) is an uncoupled two-degree-of-freedom nonlinear system. The \(I\) variable appears in \((u_1, u_2)\) components of system (8) as a parameter since \(i = 0\). Consider the first two decoupled equations

\[
u_1 = u_2, \quad \nu_2 = -\beta_1 u_1 + \eta_1 u_1^3 + \beta_2 u_1^2.
\]

Since \(\eta_1 > 0\), system (11) can exhibit heteroclinic bifurcations. It is known that the singular points \(q_0(I) = (B, 0)\) are saddle points and the singular point \(q_0 = (0, 0)\) is the center point. There exists heteroclinic loops \(\Gamma_0\) which consists of saddle points \(q_0(I)\) and a pair of heteroclinic orbits

\[
\lim_{t \to \pm \infty} u_0^\mu(T, I) = q_0(I).
\]

So in full four-dimensional phase space the set defined by

\[
M = \left\{(u, I, \gamma) \mid u = q_0(I), I_1 \leq I \leq I_2, 0 \leq \gamma < 2\pi\right\}
\]

is a two-dimensional invariant manifold. From the results obtained in references [8-12], it is known that two-dimensional invariant manifold \(M\) is normally hyperbolic. Two-dimensional normally hyperbolic invariant manifold \(M\) has three-dimensional stable and unstable manifolds which are represented as \(W^s(M)\) and \(W^u(M)\), respectively. The existence of the heteroclinic orbit of system (11) to \(q_0(I) = (B, 0)\) indicates that \(W^s(M)\) and \(W^u(M)\) intersect non-transversally along a three-dimensional heteroclinic manifold denoted by \(\Gamma\), which can be written as

\[
\Gamma = \left\{(u, I, \gamma) \mid u = u_0^\mu(T, I), I_1 < I < I_2, \gamma = \int_0^T D_2 H(u_0^\mu(T, I), I) dx + \gamma_0\right\}.
\]

In order to calculate the phase shift and the extended Melnikov function, we need to obtain the equations of a pair of heteroclinic orbits which are given as

\[
u_1(T_i) = \pm \frac{\epsilon_1}{\sqrt{2\eta_1}} \tanh \left(\frac{\sqrt{2\eta_1}}{2} T_i\right),
\]
\[
u_2(T_i) = \pm \frac{\epsilon_1}{\sqrt{2\eta_1}} \sech \left(\frac{\sqrt{2\eta_1}}{2} T_i\right),
\]

where the parameter is \(\epsilon_1 = I_1 - \beta_2 I_2^2\).

Let us turn our attention to the computation of the phase shift. Substituting the first equation of equation
(13) into the fourth equation of the unperturbed system of equation (8) yields
\[ \dot{\gamma} = \eta \gamma^2 + \frac{e \beta}{\eta_1} \tanh \left( \frac{\sqrt{2} \eta_1}{2} \right). \] (14)

Integrating equation (14), the phase shift may be expressed as
\[ \Delta \gamma = -\frac{2 \beta \sqrt{2 \eta_1}}{\eta_1} \tan^{-1} \left( \frac{\sqrt{2} \eta_1}{2} \right). \] (15)

The geometry structure of the stable and unstable manifolds of \( M \) in full four-dimensional phase space for the unperturbed system of equation (8) is given in Figure 2.

**Figure 2.** The geometric structure in full four-dimensional phase space.

**4 Dissipative Perturbations**

After obtaining detailed information on the nonlinear dynamic characteristics of \((u_1, u_2)\) components for the unperturbed system of (8), the next step is to examine the effects of small perturbation terms \((0 \leq \epsilon \ll 1)\) on the unperturbed system of (8). In this section, the objective of research is to identify the parameter regions where the existence of the multi-pulse orbits is possible in the perturbed phase space. The existence of such multi-pulse orbits provides a robust mechanism for the existence of the complicated dynamics in the perturbed system.

We analyze the dynamics of the perturbed system and the influence of small perturbations on \( M \). Based on the analysis in references [8-12], we know that \( M \) along with its stable and unstable manifolds are invariant under small, sufficiently differentiable perturbations. It is noticed that the saddle point may persist under small perturbations, in particular, \( M \rightarrow M_{\epsilon} \). So, we obtain
\[ M = M_{\epsilon} = \{(u_1, I, \gamma) | u_1 = q_{\epsilon}(I), I_1 \leq I \leq I_2, 0 \leq \gamma \leq 2\pi \}. \] (16)

Considering the later two equations of system (8) yields
\[ \dot{I} = -\mu I - f_2 \sin \gamma, \] (17a)
\[ \dot{\gamma} = \eta \gamma^2 + \beta \mu u_1^2 - \gamma / \cos \gamma. \] (17b)

It is known from the aforementioned analysis that the last two equations of system (8) are of a pair of pure imaginary eigenvalues. Therefore, the resonance can occur in system (17). Also introduce the scale transformations
\[ \mu_{\epsilon} \rightarrow \epsilon \mu_{\epsilon}, I \rightarrow \sqrt{\epsilon} I, \gamma \rightarrow \sqrt{\epsilon} \gamma. \] (18)

Substituting the above transformations into equations (17) yields
\[ \dot{h} = -\mu h - \frac{\delta h^2}{\eta_1} \sin \gamma, \] (19a)
\[ \dot{\gamma} = -\frac{2 \delta}{\eta_1} I h - \frac{\delta^2}{\eta_1} h^2 - \gamma / \cos \gamma, \] (19b)

where \( \delta = \eta_1 / \eta_2 + \beta \epsilon^2 / \eta_1^2. \)

When \( \epsilon = 0 \), equation (19) becomes
\[ \dot{h} = -\mu h - \frac{\delta h^2}{\eta_1} \sin \gamma, \] (20a)
\[ \dot{\gamma} = -\frac{2 \delta}{\eta_1} I h. \] (20b)

The unperturbed system (20) is a Hamiltonian system with the Hamiltonian function
\[ H(h, \gamma) = -\mu h^2 + \gamma + \frac{\delta}{\eta_1} h^2. \] (21)

The singular points of system (20) are given as \( P_0 = (0, \gamma_s) = \left(0, -\arcsin \left[ \frac{\mu}{\gamma_s} \left( f \frac{\gamma_s}{2} \right) \right] \right) \) and \( Q_0 = (0, \gamma_s) = \left(0, \pi + \arcsin \left[ \frac{\mu}{\gamma_s} \left( f \frac{\gamma_s}{2} \right) \right] \right). \) (22)

It is known that the singular point \( P_0 \) is a center. The singular point \( Q_0 \) is a saddle which is connected to itself by a homoclinic orbit. The phase portrait of equation (20) is given in Figure 3(a). The phase portrait of perturbed system (19) is also depicted in Figure 3(b).

**Figure 3.** Dynamics on the normally hyperbolic manifold; (a) the unperturbed case; (b) the perturbed case.
5 The k-pulse Melnikov Function

We use a general method for finding orbits that make several consecutive fast excursions away from a set of hyperbolic manifolds by constructing an extension of the Melnikov method. We reduce the search for multi-pulse excursions to that of finding non-degenerate zeros of a function $M_k(e, I, \gamma_0, \beta_k)$ of certain parameters $e, I, \gamma_0, \beta_k$, which we call the $k$-pulse Melnikov function.

We apply the extended Melnikov method to multi-pulse orbits to resonance bands. We use the new Melnikov method for extending the results to cover multi-pulse orbits with several consecutive fast pulses rather than just one. This is a typical singular perturbation problem in which there are two different time scales, and the dynamics on the hyperbolic manifold is slow. Multi-pulse orbits are constructed by concatenating pieces of slow-time orbits on the hyperbolic manifold and fast-time heteroclinic orbits off of this manifold. Since the motion along the hyperbolic manifold is slow in this problem, this theory simplifies considerably due to the facts that the $k$-pulse Melnikov function does not depend on the small parameter $e$, and that the non-folding condition is automatically satisfied and thus not needed.

The $k$-pulse Melnikov function is the geometric interpretation of a signed distance measured along the normal to a heteroclinic manifold which replaces the estimate of the change of energy computed along unperturbed heteroclinic orbits. In order to show the existence of multi-pulse heteroclinic orbits, it is important to obtain the expression of the $k$-pulse Melnikov function. Firstly, we computed 1-pulse Melnikov function based on equation (23) at the resonance $I = I_M$. The Melnikov function $M(I, \gamma_0, \beta_k)$ on both manifolds $W^s(M)$ and $W^u(M)$ is equal to

$$M(I, \gamma_0, \beta_k, \eta_1, \epsilon_1) = \int_{-\infty}^{\infty} \left( p(p^s(t), g(p^u(t), \mu, 0)) \right) dt$$

$$= \int_{-\infty}^{\infty} \left( \frac{\partial H}{\partial \alpha} g^m + \frac{\partial H}{\partial g} g^n + \frac{\partial H}{\partial \gamma} g^r + \frac{\partial H}{\partial \tau} g^s \right) dt$$

$$= -\frac{2\alpha e_2}{3} e_1^{\gamma_0} + \frac{1}{2} f_I \sin \gamma_{k-1} \sin (\gamma_0 / 2k)$$

Then, the $k$-pulse Melnikov function is obtained as

$$M_k(I, \gamma_0, \beta_k, \eta_1, \epsilon_1)$$

$$= \sum_{j=0}^{k-1} M(I, \gamma_0, j\pi / 2, \beta_k, \eta_1, \epsilon_1)$$

$$= -f_I \sin \gamma_{k-1} \sin (\gamma_0 / 2k) + \frac{2\alpha e_2}{3\beta_k} (\gamma_0 / 2k)$$

where we set $\gamma_{k-1} = \gamma_0 + (k - 1) \pi / 2$.

The main aim of the following research focuses on identifying simple zeroes of the $k$-pulse Melnikov function. Define a set that contains all such simple zeroes

$$Z_k = \left\{ (I, \gamma_{k+1}, \beta_k, \eta_1, \epsilon_1) \mid M_k = 0, D_{\gamma_0} M_k \neq 0 \right\}$$

(25)

There are two simple zeroes of the $k$-pulse Melnikov function in the interval $\gamma_{k+1} \in [0, \pi]$, that is

$$\gamma_{k-1} = \arcsin \left( \frac{1}{2k} \alpha e_2 \right) \left( \beta_k \eta_1 e_1 \right) \sin (\gamma_0 / 2k)$$

$$\gamma_{k-1} = \pi - \gamma_{k+1}$$

(26)

6 Numerical Results of Chaotic Motions

Based on the above qualitative analysis for the multi-pulse orbits and chaotic dynamics of the FGMs rectangular plate, chaotic motion of conditions under the sense of Smale horses are obtained. In order to verify further evidence that the systems exist chaotic motion, we choose averaged equation (5) to do the numerical simulations. Numerical approach through a computer software Matlab is utilized to explore the existence of the Shilnikov type multi-pulse chaotic motions in the FGMs rectangular plate under transverse loads. It is found that different solutions can be obtained when changing the transverse load. Therefore, in the following investigation, we choose that the transverse load $f_I$ as controlling parameter to find the chaotic motions of the FGMs rectangular plate.

Figure 4 illustrates the existence of the chaotic motion for the FGMs rectangular plate when the forcing excitation is $f_I = 7.773$. The parameters and the initial conditions are respectively chosen as $\alpha_1 = 0.12$, $\sigma_2 = 0.12$, $\mu_1 = 2.26$, $\mu_2 = 3.18$, $\alpha_1 = 5.9$, $\alpha_2 = 2.36$, $\alpha_3 = -3.9$, $\alpha_4 = 1.6$, $\alpha_5 = 0.66$, $\beta_1 = 1.8$, $\beta_2 = 4.78$, $\beta_1 = -6.33$, $\beta_4 = 9.16$, $\beta_5 = -5.28$, $f_2 = 405.10$, $x_10 = 0.44$, $x_20 = 1.55$, $x_30 = 2.35$, $x_40 = 5.18$.

![Figure 4 The phase portraits of chaotic motion](attachment:image.png)
7 Conclusions

The Shilnikov type multi-pulse orbits and chaotic dynamics for the FGMs rectangular plate under transverse loads are investigated by using the extended Melnikov method. The study is focused on coexistence of 1:2 internal resonance, primary parametric resonance and 1/2-subharmonic resonance in equation (1). The Matlab software is used to perform numerical simulation. The numerical results show the existence of the Shilnikov type multi-pulse chaotic motions in the averaged equation. It is well known that the Shilnikov type multi-pulse chaotic motions in the averaged equation can lead to the Shilnikov type multi-pulse amplitude modulated chaotic oscillations in the original system under certain conditions. Therefore, it is demonstrated that there are the amplitude modulated chaotic motions of the Shilnikov type multi-pulse for the FGMs rectangular plate under transverse loads. Numerical simulations obtained in this paper indicate that there exist the chaotic responses for the FGMs rectangular plate under certain the transverse load of \( f_j \), parameters and initial conditions. We find that the transverse load of \( f_j \) has important influence on the chaotic motions of the FGMs rectangular plate.

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