The paper deals with the possibilities of using mathematical apparatus of a stochastic time series for stochastic systems identification and modeling, especially mechanical ones. Its purpose is to briefly characterize fundamental terms and equations of mathematical apparatus of time series, to describe the algorithm of a statistically adequate discrete model of a stochastically dynamic system to develop relationship between parameters of discrete and continuous models and to show some possibilities of developed identification strategy for solution of selected modal characteristics of mechanical structures.

Key words: Time Series, Autoregressive Models, Algorithm of Adaptive Identification, Stochastically Loaded Parts, Mechanical System Modal Parameters.

Introduction
One of possible way of complex systems analysis without loss of accuracy and without necessity of complicated mathematical apparatus utilization is observation of system during its working and utilization of produced data to its analyses. Such analyzed system we call Data Dependent Systems (DDS). It means that we do not need to know anything as-some coherence to the better known apparatus of physical principles of followed processes [1,2,7].

A general type of ARMA dependence is a model of n-th orders named AR(n) as

\[ X_t = a_1 X_{t-1} + a_2 X_{t-2} + \ldots + a_n X_{t-n} + \varepsilon_t. \]  

(1)

The basic presumption of adequacy of AR (n) model is the independence of stochastic values \( \varepsilon_t \) which must form an independent series.

If this presumption does not apply it means that \( \varepsilon_t \) depends on \( \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots \) etc. The pure autoregressive models change to Autoregressive Moving Average Models - ARMA, generally of (n, m) order. With use of ARMA models one can express more complex types of internal dependencies, and as it will be shown further, their parameters have a very narrow dependence with the physical principles of followed processes [1,2,7].

A general type of ARMA dependence is a model ARMA (n, m) described by

\[ X_t - a_1 X_{t-1} - a_2 X_{t-2} - \ldots - a_n X_{t-n} = \varepsilon_t - b_1 \varepsilon_{t-1} - b_2 \varepsilon_{t-2} - \ldots - b_m \varepsilon_{t-m}, \]  

(2)

one can suppose, that residual deviations \( \varepsilon_t \) are of normal distribution of probability with zero mean and dispersion of \( \sigma^2_N(\varepsilon_t = N(0, \sigma^2_N)) \).

The condition of stability of ARMA(n, m) model is generally in form \( |\lambda_k| < 1 \), for \( k = 1,2,\ldots,n \), where \( \lambda_k \) are roots of characteristic equation on left-hand side of equation (2) in form of

\[ \lambda^n - a_1 \lambda^{n-1} - a_2 \lambda^{n-2} - \ldots - a_n = 0. \]  

(3)

The basic characteristics of ARMA models are impulse response function - so called Greens function which can express conditions of stability of models and inverse function describing dynamics of models by expression if influence of former values of the process on the present ones.
Using Greens function one can develop an implicit expression of discrete values of autocorrelation function (ACF), which get the form [1,7] of

\[ R_0 = a_0 R_0 + a_1 R_1 + \ldots + a_n R_n + \ldots + b_0 - b_1 \ldots - b_{n-1} \cdot G_{n-1} \cdot \sigma_z^2 \]

\[ R_t = a_0 R_0 + a_1 R_1 + \ldots + a_n R_n + \ldots + b_0 - b_1 \ldots - b_{n-1} \cdot \sigma_z^2 \]  

(4)

\[ R_{n-1} = a_1 R_{n-1} + a_2 R_{n-2} + \ldots + a_n R_{n-n} + b_{n-1} \cdot \sigma_z^2 \]

\[ R_k = a_1 R_{k-1} + a_2 R_{k-2} + \ldots + a_n R_{k-n} \]

for all \( k \geq n \)

Power spectral density (PSD) can be determined using Fourier transform of ACF or in a simpler way as [7] 

\[ S(\omega) = \left( e^{i(n-1)\omega} - b_1 e^{i(n-2)\omega} - \ldots - b_{n-1} \right)^2 \]

\[ \left( e^{i(n-1)\omega} - a_1 e^{i(n-2)\omega} - \ldots - a_n \right)^2 \]

(5)

which holds for \( \omega \) in \([(-\pi/\Delta t) \leq \omega \leq (\pi/\Delta t)]\) and \( \Delta t \) is a basic sampling interval. Better estimate of power spectra through the whole frequency band one can get from continuous model as it is shown in [6].

2. Relationship between continuous and discrete model

Because most systems used in technical practice and dynamics of mechanical systems are of continuous nature, continuous mathematical model in form of differential equation eventually of differential equations set is more suitable as the discrete one. Differential equations are formed on base of physical laws concerning studied systems and their parameters (coefficients) have a straigntway relationship to the most important characteristics of mechanical systems (natural frequencies, damping, stability etc.). Relationships between a discrete model, which one gets by sampling of continuous signal in constant time interval and an original continuous system means relationships between parameters of differential and difference equations. Describing will be developed on the simplest one-mass damped mechanical dynamic system – 1 D.O.F. System excited by stochastic force \( F(t) \) with normal distribution \( N(0,\sigma_\xi^2) \) (Fig.1).

Fig.1 The simple mechanical dynamic system

Well-known differential equation in form

\[ m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx(t) = F(t) \]

(6)

or after adjustment

\[ \frac{d^2x}{dt^2} + 2\xi \Omega \frac{dx}{dt} + \Omega^2 x(t) = \frac{F(t)}{m} \]

(7)

is possible simply arrange into operator form as

\[ (D^2 + \alpha_1 D + \alpha_0) x(t) = Z(t) \]

(8)

This equation described continuous autoregressive model of second order – CAR(2) and exciting functions \( Z(t) \) is in continuous white noise form, for which applied \( E[Z(t) Z(t+n)] = \sigma_\xi^2 \delta(n) \) and where \( \delta(n) \) is Diracs \( \delta \)-function [1].

Values of \( \alpha_1 \) and \( \alpha_2 \) which have the meaning of important values of un-damped natural frequency and relative damping of the system, one can get as

\[ \alpha_1 = 2 \frac{\xi}{\Omega} \land \alpha_0 = \Omega^2 \]

(9)

where

\[ \xi = \frac{c}{m \omega_0} = \frac{c}{2\sqrt{k \cdot m}} \land \Omega = \sqrt{k \over m} \]

For the solution, it is decisive form of impulse response function that is possible to determine using characteristics of Diracs \( \delta \)-function as

\[ G(t) = \frac{e^{\mu_1 \cdot t} - e^{\mu_2 \cdot t}}{\mu_2 - \mu_1} \]

(10)

where \( \mu_1 \) and \( \mu_2 \) are roots of characteristics equation (8). Relationship between continuous and discrete models can be determined using comparison of Autocorrelation Function (ACF) forms for continuous and discrete models. Using the ACF form, which is a linear combination of two exponential functions one knows that a continuous second order system can be (by constant interval sampling) expressed by a difference equation of ARMA (2,1) model in form

\[ X_t - a_1 X_{t-1} - a_2 X_{t-2} = \xi_t - b_1 \xi_{t-1} \]

(11)

where

\[ a_1 = \lambda_1 + \lambda_2 = e^{\mu_1 \cdot \Delta t} + e^{\mu_2 \cdot \Delta t} \land a_2 = -\lambda_1 \cdot \lambda_2 = -e^{(\mu_1 + \mu_2) \cdot \Delta t} \]

Parameters of \( b_1 \) and \( \sigma_\xi^2 \) are functions of characteristic equation roots, but their expression is considerably complicated [7]. The basic assumption is the continuous and discrete models have the same ACF. From expression of them, one can get a system of two equations in form

\[ \frac{\sigma_\xi^2}{2 \cdot \mu_1 (\mu_1^2 - \mu_2^2)} = \frac{\sigma_\xi^2}{2 \cdot \mu_2 (\mu_1^2 - \mu_2^2)} = \frac{e^{\lambda_1 \cdot \Delta t} (\lambda_1 - \lambda_2)}{(\lambda_1 - \lambda_2)^2 (1 - \lambda_1 \cdot \lambda_2)} \]

\[ \frac{\sigma_\xi^2}{2 \cdot \mu_2 (\mu_1^2 - \mu_2^2)} = \frac{e^{\lambda_2 \cdot \Delta t} (\lambda_2 - \lambda_1)}{(\lambda_2 - \lambda_1)^2 (1 - \lambda_1 \cdot \lambda_2)} \]

(12)

which implicitly contains searched parameters of right-hand side of differential equation (11) \( b_1 \) and \( \sigma_\xi \).

By division of both equations, one another by some modifications one can get for \( b_1 \) expression in form

\[ b_1 = -P \pm \sqrt{P^2 - 1} \]

(13)
\[ p = -\mu_0 \left( 1 + \lambda_1^2 \right) \left( 1 - \lambda_1^2 \right) + \mu_1 \left( 1 + \lambda_2^2 \right) \left( 1 - \lambda_2^2 \right) \]

\[ = \frac{2 \mu_0 \lambda_1 \left( 1 - \lambda_1^2 \right) - \mu_1 \lambda_2 \left( 1 - \lambda_2^2 \right)}{2 \mu_0 \lambda_1 \left( 1 - \lambda_1^2 \right) - \mu_1 \lambda_2 \left( 1 - \lambda_2^2 \right)} \]

From two of determined values we take in account this one for that holds the condition of invertibility in form \( |p| < 1 \). In similar way is possible to express the searched value of dispersion too. Formulas (11,12) determine unambiguous parameters of the discrete auto regressive moving average model ARMA (2n,2n-1) from known continuous model parameters.

Real procedure of identification process is a reversed one. It means, that using a procedure shown in a former chapter, one determines parameters of discrete model \( a_1, a_2, b_1 \) and \( \sigma^2 \), roots of its characteristic equation \( \lambda_1, \lambda_2 \) and with their help parameters of continuous system \( \mu_1, \mu_2, \sigma^2 \), eventually \( a_1 \) and \( a_0 \) are determined. For defined 1-D.O.F. system with sub-critical damping for values of eigen-frequency and relative damping holds

\[ \Omega = \frac{1}{A} \left[ \ln(-a_1) \right]^2 + \left[ \frac{\ln(-a_2)}{2 \sqrt{-a_2}} \right]^2 \]  

(14)

\[ \xi = \frac{[\ln(-a_1)]^2}{[\ln(-a_2)]^2 + 4 \left[ \frac{\ln(-a_2)}{2 \sqrt{-a_2}} \right]^2} \]  

(15)

The shown procedure can be expanded and applied for general continuous systems of n-th order described by differential equation as (in operator form)

\[ (D^n + \alpha_{n-1} D^{n-1} + \ldots + \alpha_1 D + \alpha_0) X(t) = (\beta_m D^m + \beta_{m-1} D^{m-1} + \ldots + \beta_1 D + \beta_0) Z(t) \]  

(16)

which determines the continuous autoregressive moving average model - CARMA(n, m). From this results that discrete representation of continuous process - CARMA(n,m) is always ARMA(n, n-1) model type [2]. It means that order of right-hand side of differential equation of continuous system representation has no influence on his discrete representation. These relations are possible to exploit for discrete model parameters determination, if we know coefficients of differential equation. Because the subject of our interest is a reverse one it, means parameters of continuous are determined from a discrete uniform sampled model with interval of sampling. At the characteristic equation of left-hand side of ARMA (2n,2n-1) model is sufficient for mechanical systems synthesis in form

\[ \lambda^{2n} + \sum_{i=1}^{2n} a_i \lambda^{2n-i} = \prod_{j=1}^{n} (\lambda - \lambda_j)(\lambda - \overline{\lambda}_j) \]  

(17)

where \( \lambda_j, \overline{\lambda}_j = e^{\frac{\pi^2}{L_f} \sqrt{\lambda_j^2 - \lambda_j^2}} \).

From that then for natural frequencies and relative dampings are

\[ \Omega_j = \frac{1}{A} \left[ \frac{[\ln(\lambda_j, \overline{\lambda}_j)]^2}{4} + \arccos \left( \frac{\lambda_j + \overline{\lambda}_j}{2 \sqrt{\lambda_j, \overline{\lambda}_j}} \right)^2 \right] \]  

(18)

which are similar formulas that were developed in a former part for 1-D.O.F system (14,15).

3. An algorithm of optimum model determination

The aim of identification is to determine order \( n \) of statistically adequate model ARMA(n,n-1), coefficients on left and right-hand side \( (a_1, a_2, \ldots a_n, b_1, b_2, \ldots b_{n-1}) \) and sum of residual deviations squares \( \sum e^2 \) or their dispersion \( \sigma^2 \).

Because of the necessity of recurrent determination of deviations \( e \), from the start, the result is from the point of view of coefficients non-linear. For more information on the algorithm of statistically adequate discrete model determination, prediction of starting guess of parameters, criterion of ARMA(n,n-1) model adequacy are published in the works [1,4,6,7].

Using former shown dependencies and formulas one can describe an optimum ARMA(n,n-1) model getting algorithm in words as follows [4]:

1. Calculation of ARMA (2n, 2n-1) model parameters for \( n=1 \) and its sum of squares in form \( A_0 = \sum e^2 \).
2. Increase of order \( n \rightarrow (n+1) \) and calculation of model parameters and sum of squares \( A_1 \).
3. Testing of statistically significance of sum of squares decrease \( A_1 - A_0 \). In case that decrease is statistically significant, go to step 2. In the other case the former model was statistical adequate.
4. Test of \( a_{2n}, b_{2n-1} \) parameters if their value is near zero or if their internal of confidence contains zero. When not, ARMA (2n, 2n-1) model is suitable.
5. When \( a_{2n}, b_{2n-1} \) are zeros or near zero, calculation of ARMA (2n-1, 2n-2) model parameters.
6. Testing of moving average parameters \( b_l \) of ARMA (2n-1, 2n-2) model. If some of them are near zero, create of ARMA (2n-1, m) model for \( (m<2n-2) \) and calculation of its parameters, eventually of pure AR(2n-1) model.

Shown algorithm was used by developing of program Arma-Get, developed on authors department (Fig.2).
The heart of the program Arma-Get is submenu "Identification", by means which is possible to make selection of the identification method and way of chosen time series, whereupon it is possible to use either adaptive algorithm of time series identification or make identification using non-linear least squares method. Identification by means of higher presented non-linear (respectively linear for AR model) least square method is available in sub-menu NLINLS. Here are four options. First two - Model ARMA - after orders and Model ARMA - complete calculation give as results of identification ARMA model. Next item Model VARMA - after orders gives coefficients of beforehand selected order of VARMA (n,n-1) models determination. It means that we it is necessary beforehand to determine required order (known number of coefficients) of autoregressive part - a0 and moving average part - b0 which principally determine number of former values the calculated value depends on. The last important item is Model VARMA - complete calculation, which aim is to find an optimal VARMA (n,n-1) model. This model is the best describe of system, which outputs are a time series. Item "Simulation" enables adjustment and conversion of incompatible input files of time series to compatible ones and simulation (generation) of time series basing on given AR, ARMA or VARMA models order and parameters with possibilities of mean and dispersion selection of simulated series. This developed software is able to create an adequate mathematical model for describing a matrix model of a tested stochastic loaded mechanical system.

4. Possibilities of identification procedure for basic modal parameters determination

Algorithm for determination of adequate autoregressive models of stochastic systems and developed relationships between parameters of discrete and continuous models were successfully verified on case of identification of mechanical dynamic system of machine tool during cutting with its application by adaptive geometrical control in real time [6,7] and by analysis of feed-back system (machine tool – cutting) [7].

Shown connections can be utilized by determination of modal characteristics of mechanical dynamics systems too. There is the mostly used procedure of experimental investigation of dynamic characteristics of different systems and structures at present the application of dynamic compliance matrices determination and analysis of structure modes. Due to digital analyzers of spectra extension working on the principle of Fast Fourier Transform the most frequently used procedure is the one based on spectra determination of excitation and output of the system [3]. Using them one can get transfer functions of the system and connected information of natural frequencies and structure modes. Due to presented connections it shows as possible procedure of some modal information obtaining use of experimentally got time series of structure vibrations measured simultaneously in multiple points of structure. It means to generalize involved information to Vector Autoregressive Moving Average Models - VARMA, which are in general case described by matrix formulas in operator form as

\[
A_0 - A_1.B^1 - \ldots - A_n.B^n \cdot X_t = (1 - D_1.B^1 - \ldots - D_{n+1}.B^{n+1}) \cdot e_t
\]

for which holds similarly as in scalar case attached \(E(e_t) = 0; E(e_t, e_t) = \delta_0 \sigma^2 \), where \(X_t\) and \(e_t\) are vectors of measurements and white noise series, \(A_0\) and \(D_0\) are matrix of system parameters, \(B\) is vector of back-shift operators, \(\sigma^2\) is matrix of dispersion and reciprocal correlation’s and \(\delta_0\) is Kronecker delta function. It means, if one analyses a mechanical dynamic system with a numerical technique and its vibrations and exciting forces measure in uniform sampling intervals \(\Delta t\), it is possible to developed discrete models to describe the relationship between output (vibration) and input (exciting forces).

Dynamics of the mechanical system is after determined by a discrete transfer function in form

\[
H(B) = \frac{A(B)}{A(B)} = \frac{-\alpha_{20} + \alpha_{12}B + \alpha_{12}B^2 + \ldots + \alpha_{20}B^n}{1 - a_{10}B^1 - a_{12}B^2 - \ldots - a_{20}B^n}
\]

where \(B\) is back-shift operator \((X_{t+1} = B \cdot X_t)\) and \(a_{ik}\) are searched unknown parameters [7]. Similarly, it’s possible dynamics to noise expression by a discrete transfer function in form

\[
N(B) = \frac{\Delta(B)}{A(B)} = \frac{1 - d_{11}B - d_{12}B^2 - \ldots - d_{1n-1}B^n}{1 - a_{10}B^1 - a_{12}B^2 - \ldots - a_{20}B^n}
\]

where coefficients \(d_{ik}\) is needed to determine. Supposing non-existence of feed back between vibrations of structure can be expressed in its excitation (which holds for structures tests) from equations (21) and (22) one gets a resulting model of structure dynamics, where attached assumptions shown in formula previous, in form

\[
(1 - a_{11}B - a_{12}B^2 - \ldots - a_{1n}B^n) \cdot X_{t+1} = \left( \alpha_{20} + \alpha_{21}B + \ldots + \alpha_{2n}B^n \right) \cdot X_t + \left( 1 - d_{11}B - d_{12}B^2 - \ldots - d_{1n-1}B^{n-1} \right) \cdot e_t
\]

Physical meaning of such a procedure is in that we are trying to substitute the system with a model with the lowest number of statistically significant modes of vibrations. During this procedure, each increase of model order by two introduces (a further degree of freedom). If its contribution is not significant, the former model is taken as statistically adequate. Then resulted discrete model obtained as statistically adequate and from it developed model continuous means the expansion of system response into minimum number of statistically significant modes which are obtained directly without any prior subjective judgment [8]. After adequate ARMA model determination for curves of measured vibrations it is possible with using of equation (21) and substitution for B=excit to determine frequency characteristics (transfer functions in frequency area).

A summary of the procedure of the structure modal parameters identification is expressed as follows [1]:
1. Recording of excitation and vibrations (output) in uniform discrete time interval.
2. Acquiring of statistically adequate models ARMA (n, n-1), respectively VARMA.
4. Calculation of modal characteristics: eigen-frequencies $\Omega_i$ and relative damping $\xi_i$ (18), (19).
5. Expression of system transfer function (21) in partial fractions form and to determine complex residuals $A_i$.
6. Determinations of structure vibrations transfer function (22) by using to obtained complex residuals $A_i$.
7. Drawing of transfer functions, modes of vibrations etc.

6. Application of the identification procedure

The shown theoretical approach was verified on more examples of applications. To illustrate this developed approach is demonstrated on a simple 3 degree of freedom (3 D.O.F.) mechanical system and the model of a real crane jib.

The 3 D.O.F. model of mechanical system was excited by random force $F(t)$ with normal distribution which acts in direction after Fig.3.

$$\begin{bmatrix}
k_{12} & c_{12} & c_{13} \\
k_{23} & m_3 & c_{23} \\
k_{01} & m_1 & c_{01}
\end{bmatrix} \begin{bmatrix}
x_1(t) \\
x_2(t) \\
x_3(t)
\end{bmatrix} + \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x_1(t) \\
x_2(t) \\
x_3(t)
\end{bmatrix} = F(t)$$

Fig.3 Simple 3 D.O.F. mechanical system

Basic parameters of system: $m_1 = 1$, $m_2 = 1$, $m_3 = 2$, $k_{02} = k_{12} = k_{23} = 100$, $k_{03} = 200$, $c_{01} = c_{02} = 0.8$, $c_{12} = c_{13} = 0.6$, $c_{23} = 1.6$

Well-known system of differential equations in form

$$\mathbf{M} \mathbf{x} + \mathbf{C} \mathbf{x} + \mathbf{K} \mathbf{x} = \mathbf{F}(t)$$

in this case acquired form

$$\begin{bmatrix}
1 & 0 & 0 & | & 1.6 & -0.4 & -0.4 \\
0 & 1 & 0 & | & -0.4 & 1.8 & -0.6 \\
0 & 0 & 2 & | & -0.4 & -0.6 & 2.6
\end{bmatrix} \begin{bmatrix}
x_1(t) \\
x_2(t) \\
x_3(t)
\end{bmatrix} = \begin{bmatrix}
1 & -100 & 0 \\
0 & 200 & -100 \\
0 & -100 & 300
\end{bmatrix} \begin{bmatrix}
x_1(t) \\
x_2(t) \\
x_3(t)
\end{bmatrix} = \begin{bmatrix}
F(t) \\
0 \\
0
\end{bmatrix}$$

from its left side were determined in numerical way (using Faddejev and Bairstow methods) theoretical values of eigen-frequencies.

Next the whole system was solved using simulation excitation of force $F(t)$. A sampling interval of $\Delta t=0.1$s was chosen and between each sample were 10 steps of numerical integration performed using Runge–Kutta method [9]. A time series of 100 000 values from vibrations of each mass was made. These samples were started after getting stationary values of vibrations amplitudes. Using identification procedure the ARMA (6,5) model was obtained as statistically adequate (Fig.4).

Fig.4 Results of identification process (XNod5 file)

Its parameters are presented in Tab.1. Calculated values of eigen-frequencies $\Omega_i$ and relative damping $\xi_i$ and their comparison with theoretical values are shown in Tab.2.

Tab.1: Parameters of autoregressive parts of models

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{1t}$</td>
<td>1.5820</td>
<td>-2.8093</td>
<td>2.5325</td>
</tr>
<tr>
<td>$X_{2t}$</td>
<td>1.3915</td>
<td>-2.4370</td>
<td>2.0704</td>
</tr>
<tr>
<td>$X_{3t}$</td>
<td>1.5859</td>
<td>-2.8081</td>
<td>2.4854</td>
</tr>
</tbody>
</table>

Tab.2: Values of eigen-frequencies $\Omega_i$ and relative damping $\xi_i$ of 3 D.O.F.system

<table>
<thead>
<tr>
<th>Modal characteristic</th>
<th>$\Omega_i$</th>
<th>$\xi_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st. mode</td>
<td>1.2958</td>
<td>0.04709</td>
</tr>
<tr>
<td>2nd. mode</td>
<td>1.9647</td>
<td>0.04709</td>
</tr>
<tr>
<td>3rd. mode</td>
<td>2.7945</td>
<td>0.04709</td>
</tr>
</tbody>
</table>

From presented results a very good agreement between experimental and theoretical values of eigen-frequencies $\Omega_i$. |
A relative lower accuracy of relative damping values $\xi_i$ was caused probably by choosing of low values of coefficients $c_{ij}$. After getting of characteristic equations roots the modes of vibration were obtained. Their comparison with theoretical values is shown in Fig.5.

Fig.5: Obtained modes and their comparison with theoretical ones.

7. Conclusions

It introduces problems were proposed and verified in a frame of grant research of Ministry of Education of Slovak Republic, named VEGA # 1/0430/09, where some possible applications of the proposed identification procedure were investigated. It was namely a connection of proposed identification procedure with systems of complicated machine structures solution using Finite Elements Method. The shown theoretical approach will in near future verified on more complicated structure too – a jib of a real crane (was modeled by means of Finite Element Method - Fig.6). The crane jib model was made by means of program Cosmos/M.

Fig.6: FEM Model of a crane jib – future research

It introduces problems were proposed and verified in a frame of grant research, where some possible applications of the proposed identification procedure were investigated. Results of further problems using proposed procedure by dynamic analysis and identification of modal characteristics of mechanical systems showed a relatively good agreement between theoretical and identified modes of vibrations, eigen-frequencies and relative damping. From presented facts one can develop that above shown assumptions and theoretical starting points are correct and developed procedure can reduce number of calculation in an expressive way and improve efficiency of mechanical structures dynamic calculation.

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