# Electroencephalographic Signal Based Clustering of Stimulated Emotion Using Duffing Oscillator


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## ABSTRACT

Duffing Oscillator is well known for its chaotic behaviour. This paper aims at clustering of emotions from the EEG response to external audio-visual stimulus used for excitation of a subject. The EEG signal corresponding to a specific emotive stimulus is used as an excitation input to a non-linear Duffing Oscillator dynamics, and the phase trajectory plots of the two state variables of the oscillator dynamics exhibits significant differences for various emotion-excitatory stimuli. Experimental investigations reveal that injection of Gaussian noise with a Signal-to-Noise Ratio as low as 25 dB retains the results of emotion clustering, indicating robustness of clustering. Further, with different pre-classified audio-visual stimulus responsible for excitation of a specific emotion, the phase-portraits obtained from EEG data of the subject have substantial similarity, indicating accuracy in clustering.



**Keywords:** Duffing oscillator, EEG, Emotion clustering and Gaussian Noise.


## 1. INTRODUCTION

Perception involves interpreting sights, sounds, smells and touch. Perception is relatively younger discipline in Artificial Intelligence, and we are afraid that there are fewer works on perception about emotions. Researchers, however, are keen to develop new models and techniques to understand and recognize emotions from external manifestations, such as crying, laughing etc. this paper deals with classification of emotion aroused with audio-visual stimulus from electroencephalographic (EEGs) signals. Biologists believe that most of our high level understanding process involving emotions is due to the interaction of neural and hormonal activities. EEGs that represent neural activities of the brain might help us in better understanding human emotions than other widely used modes including facial expression [1], [8], [17] and voice [2], [6], [17].

In recent times, researchers have started paying attention to Electroencephalography (EEG) [19], functional Magnetic Resonance Imaging (fMRI) [11], [18], Positron Emission Tomography (PET) [18], and Magnetoencephalography (MEG) [7] –based information to correctly determine the emotional response to external stimulus. However, unfortunately, very little of brain functioning could be identified until this time, and consequently, almost no interesting results have been reported so far on emotion clustering from the above modes of information extraction. The primary objective of this paper is to classify emotion of a subject from his/her EEG signal, obtained through audio-visual excitation of the subject. In our early research [4], [5], we classified the input stimulus based on their power of excitation on a specific emotion. We used these stimuli in the present experiment, and would like to examine whether stimulus used for excitation of same emotion would ultimately map EEG's to a unique pattern.

In order to examine similarity among EEG patterns corresponding to a specific emotion excitatory stimulus, we employ a Duffing Oscillator and the response of the oscillator to the EEG signal as excitation input is recorded. Phase trajectories are built up with two state variables of the oscillator dynamics, and similarity in the chaotic behavior in the phase trajectory is noted for similar stimulus. This fundamental observation reveals that EEG obtained for arousal of a specific emotion has a unique characteristic. Thus emotion classification by EEG signal should result in good accuracy in comparison to other traditional means of emotion clustering from voice and facial expressions.

The paper is classified into 5 sections. In section 2, we briefly outline the state space representation of a Duffing Oscillator dynamics and how phase trajectories were obtained from the time-response of the oscillator dynamics. In section 3, we represent the experimental results for emotion clustering by noting similarity in phase trajectory. The effect of noise on the EEG signal is studied in section 4. Conclusions are listed in section 5.

## 2. THE DUFFING OSCILLATOR DYNAMICS AND PHASE RESPONSE

In this section, we propose a specialized non-linear oscillator dynamics, which has a proven chaotic behavior [3], [13], [14], [16] in its temporal response. The dynamics of Duffing Oscillator has a similarity with typical spring-mass load system of a conventional mechanical process [12], [15]. However, the spring in the present context, being a non-linear device, has a restoration force proportional to its cubic linear displacement. Naturally, the restoration force of ideal spring that obeys Hooke's Law is also maintained in the Duffing Oscillator dynamics. Consequently, the restoration force has two components, one following Hooke's Law, while the other is due to a high stiffness condition of the spring, represented by a cubic displacement term. The dynamics of Duffing Oscillator is given in equation (1).

$$\frac{d^2x}{dt^2} + \delta\frac{dx}{dt} + \beta x + \alpha x^3 = \gamma cos\,(\omega t) + e(t) \tag{1}$$

where,

x represents the linear displacement,

$\frac{dx}{dt}$ represents the velocity of a unit mass connected in spring-mass load system,

$\beta x$ and $\alpha x^3$ are due to spring restoration force,

$\gamma \cos\,(\omega t)$ is a fixed excitation input to maintain certain level of oscillation in the response of the dynamics, and

e(t) is the disturbance input to the oscillator.

In this present context, we use the EEG signal as the disturbance input e(t). We took $\alpha$=1, $\beta$=-1, $\gamma$=0.826, $\delta$=0.5 and the gain of the EEG signal to be 5. The basic Duffing Oscillator dynamics (1) can equivalently be represented by (2) and (3).

$$\dot{x} = y \tag{2}$$

$$\dot{y} = \gamma cos(\omega t) + e(t) - \delta\frac{dx}{dt} - \beta x - \alpha x^3 \tag{3}$$

At first, the EEG signal, which was obtained in sampled version, was passed through a First-Order-Hold circuit, whose transfer function is given by:

$$G_h(s) = \left[\frac{1+Ts}{T}\right] \times \left[\frac{1-e^{-Ts}}{s}\right]^2 \tag{4}$$

where

T= sampling time,

s= Laplace-domain operator.

The hold-circuit is used to get a continuous version of discrete EEG signal. Then, a Runge-Kutta algorithm was used to solve the coupled-differential equations (2) and (3), and phase portraits for x against y at different time slots are plotted. One typical phase portrait for an initial value of x(0)= 2 and y(0)= 20 is given in Fig. 2.a for convenience. Since Duffing Oscillator has a non-linear dynamics, as shown in the block diagram in Fig. 1, it is apparent that for varying initial conditions, the phase portraits could have been different shapes. However, experimental instances reveal that a chaotic response of the dynamics prevails even for redefining new initial conditions. Fig. 2.b and 2.c illustrate this behavior with different initial conditions.

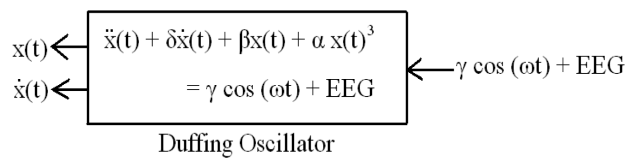



Fig 1: Block diagram of a Duffing Oscillator

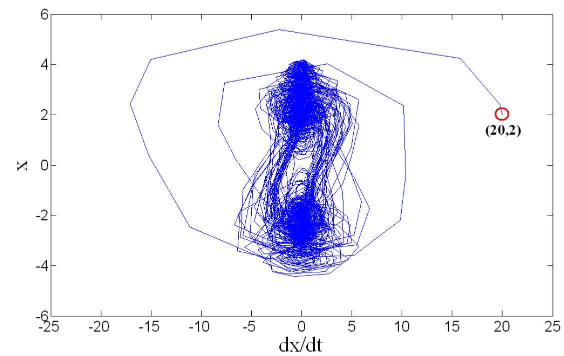



Fig. 2.a: Phase trajectory of anger with initial condition at x(0)= 2, y(0)= 20.

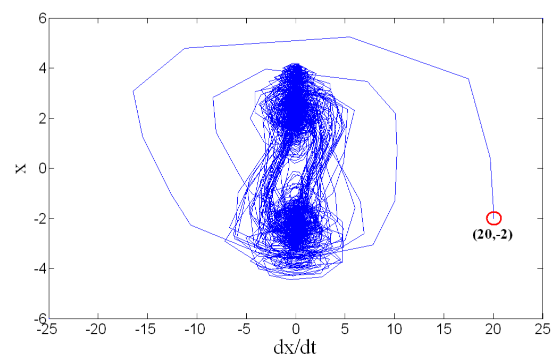



Fig. 2.b: Phase trajectory of anger with initial condition at x(0)= -2, y(0)= 20.

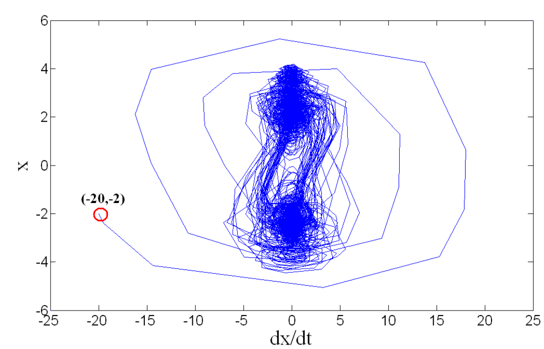



Fig. 2.c: Phase trajectory of anger with initial condition at x(0)= -2, y(0)= -20.

## 3. EMOTION CLUSTERING FROM EEG SIGNALS USING DUFFING OSCILLATOR

The time-continuous EEG signal obtained through first order hold circuit is used to excite the Duffing Oscillator, [9],

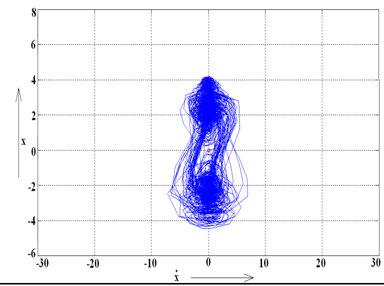
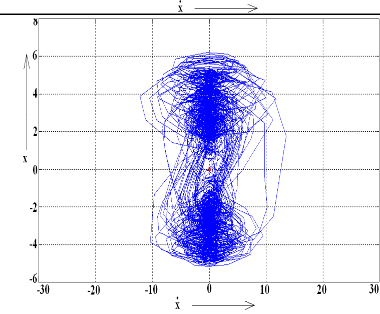
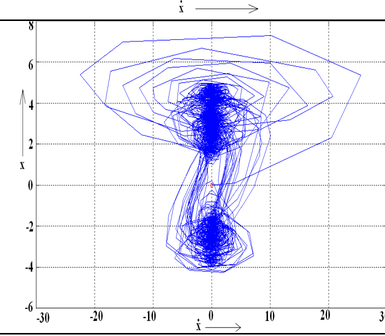
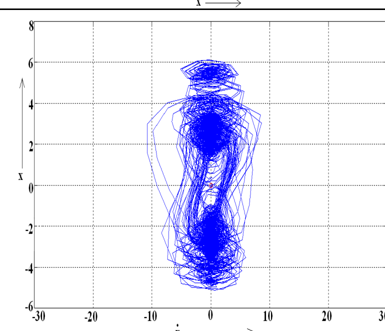
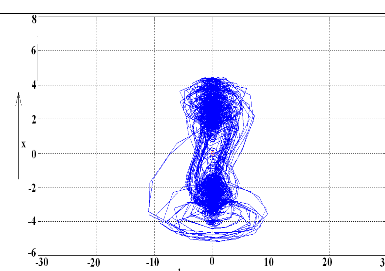
[10], [14], [19] and the response of the oscillator is obtained by solving the differential equation using Runge-Kutta method. The experiment was conducted with 15 audio-visual stimuli, each 3 of which correspond to exciting a specific emotion. The principles of automatically identifying the best audio-visual stimulus, appropriate for excitation of a given emotion is briefly outlined below.

To identify the right audio-visual stimulus responsible for arousal of a given emotion, we classified the stimulus manually with the help of 50 observers, most of whom are University students and faculties. Each observer was asked to classify a given audio-visual stimulus into 5 emotion-arousing classes - anger, fear, joy, relaxation and sadness. He/she used a 100-point scale, and assigned individual score to the entire possibility space of 5 emotions, such that sum of the scores assigned to a given audio-visual stimulus is equal to 100. For 50 observers, we determine the mean and variance of their assignments to a particular emotion-prone category, and evaluate the ratio of mean/variance for each of the 5 emotions. The emotion having the largest mean/variance ratio is considered the best category for a given stimulus. The experiment was repeated for 50 such stimuli, and the mean/variance ratio of the winning emotion for each stimulus is identified. A sorting algorithm is then applied to rank them in descending order of their mean/variance measure in the specific emotion category. The first 3 stimuli for each category of emotion are then identified from the list. The entire experiment was performed with these 3 stimuli responsible for excitation of a specific emotion. Consequently, for 5 emotions, we have 5×3=15 best-selected audio-visual samples. Table I gives the tabular representation of the results obtained by responses of 50 subjects, each of whom was shown 60 audio-visual stimuli. It is apparent from the Table that the row-sum in Table I is always 100.

TABLE I: ASSESSMENT OF THE AROUSAL POTENTIAL OF SELECTED AUDIO-VISUAL MOVIE CLIPS IN EXCITING DIFFERENT EMOTIONS

| Subjects used to access the emotion aroused by the audio-visual clips | Title of audio-visual clips | Percentage arousals of different emotions by a clip | | | | |
|---|---|---|---|---|---|---|
| | | Anger | Relax | Joy | Sad | Fear |
| Subject 1 | Clip 1 | 0 | 20 | 80 | 0 | 0 |
| Subject 2 | Clip 1 | 0 | 25 | 75 | 0 | 0 |
| … | | | | | | |
| Subject 50 | Clip 1 | 0 | 12 | 88 | 0 | 0 |
| Subject 1 | Clip 2 | 0 | 82 | 0 | 9 | 9 |
| Subject 2 | Clip 2 | 0 | 80 | 0 | 12 | 8 |
| … | | | | | | |
| Subject 50 | Clip 2 | 0 | 84 | 0 | 10 | 6 |
| … | | | | | | |
| Subject 1 | Clip 60 | 78 | 10 | 0 | 0 | 12 |
| Subject 2 | Clip 60 | 80 | 16 | 0 | 0 | 4 |
| Subject 50 | Clip 60 | 84 | 8 | 0 | 0 | 8 |

TABLE II: EXPERIMENTAL RESULTS ON THE CLUSTER-SHAPES

| Emotions and remarks on related phase plots | Experimentally obtained cluster shapes (without noise) |
|---|---|
| Anger<br>The phase trajectory covers the least area and width, confined within $\dot{x}$ = -8 to $\dot{x}$ = 8. |  |
| Fear<br>An extension is visible to the right side of the main phase trajectory. |  |
| Happiness<br>The upper lobe is highly dispersed, less dense, thus covering the maximum area. |  |
| Relaxation<br>Above the upper lobe, a thick lobe is formed. An extension is formed nearer to the left side of the original upper lobe. |  |
| Sadness<br>Below the lower lobe, another lobe is formed, which is sparse in nature. |  |

### Clustering from phase trajectory

We performed two different experiments for clustering of emotions in the EEG space. First, different audio-visual stimuli were used to excite a specific emotion of a subject, and response of the Duffing Oscillator having initial condition x= 0 and y=0 from his/her EEG signal was obtained. Table II above gives a comparative study of the phase portraits of x against y, formed due to one of the selected audio-visual stimuli for each of the emotions. We noted that for three stimuli responsible for exciting the same emotion, the phase trajectories looked almost similar, indicating the fundamental truth that similar excitations for arousal of a given emotion is responsible for excitation of similar brain-activities, pertaining to similar EEG response. These EEG, when fed to a Duffing Oscillator, thus maintains similarity in these portraits of the oscillator state variables.

Figures 3.a, 3.b, 3.c, 4.a, 4.b, 4.c, for example, demonstrate similarity in the phase portrait for excitation of emotions fear and relaxation from its stipulated stimulus list.

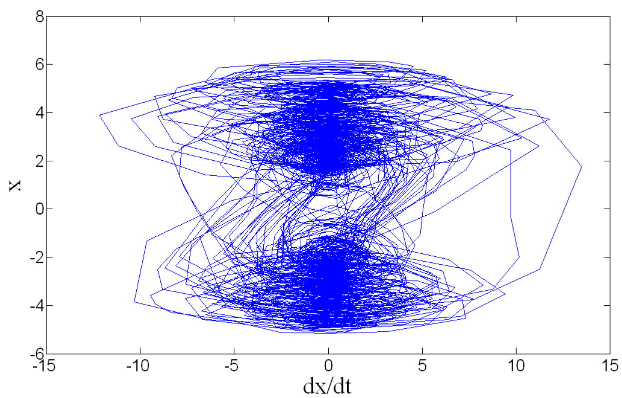


Fig. 3.a: Phase trajectory for Fear due to 1st stimulus of Fear

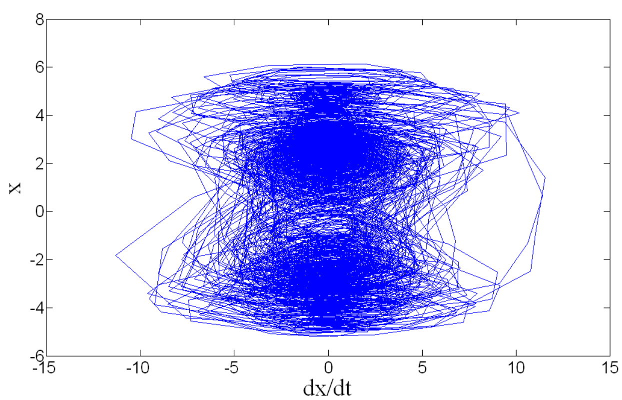


Fig. 3.b: Phase trajectory for Fear due to 2nd stimulus of Fear

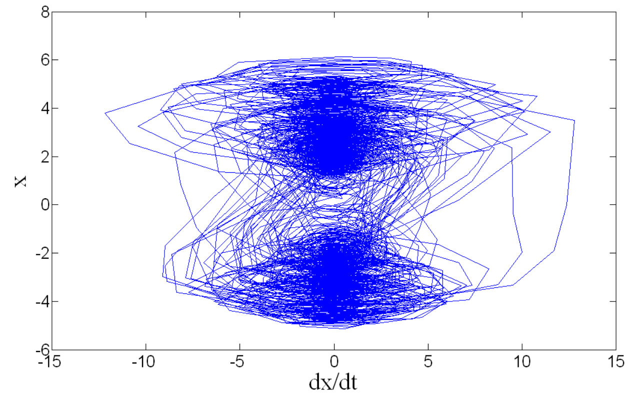


Fig. 3.c: Phase trajectory for Fear due to 3rd stimulus of Fear

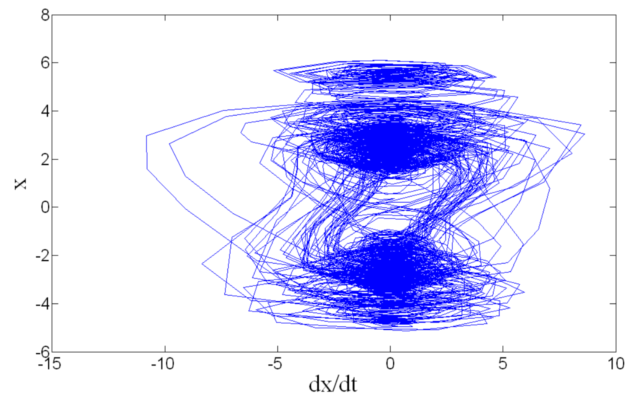


Fig. 4.a: Phase trajectory for Relaxation due to 1st stimulus of Relaxation

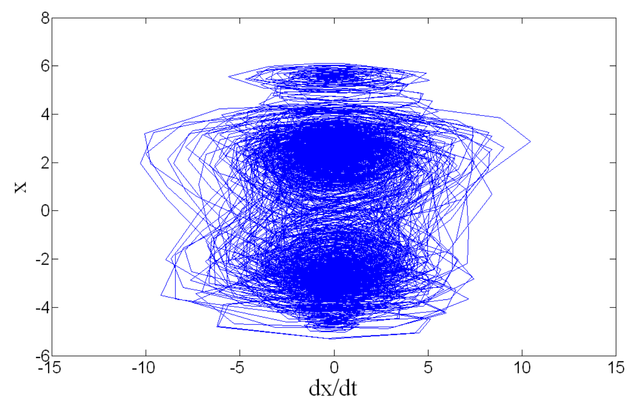


Fig. 4.b: Phase trajectory for Relaxation due to 2nd stimulus of Relaxation

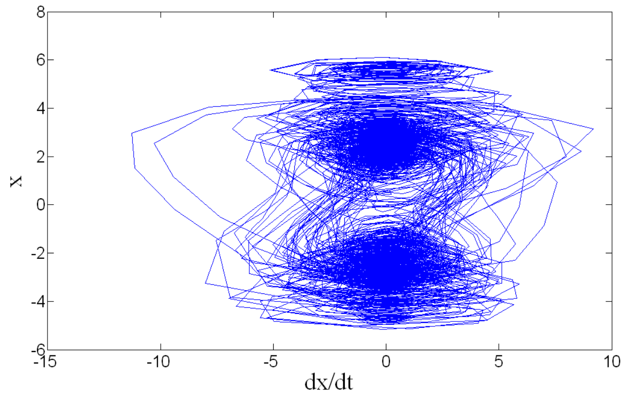


Fig. 4.c: Phase trajectory for Relaxation due to 3rd stimulus of Relaxation

## 4. EFFECT OF NOISE ON EMOTION CLUSTERING FROM DUFFING OSCILLATOR RESPONSE

In this section, we experiment by adding noise to the original signal corresponding to a specific emotion, and note the changes in the phase portrait obtained from the Duffing Oscillator response. It is interesting to note that when Signal-to-Noise Ratio of the EEG signal is maintained to a level of 25 dB, the phase portraits maintains similarity, indicating robustness in emotion clustering.

Figures 5.a, 5.b, 5.c, 6.a, 6.b, 6.c demonstrate the behavior in the phase portrait for different level of Signal-to-Noise Ratio as indicated in the figure caption. It is also noteworthy that when the Signal-to-Noise Ratio goes below a threshold, misclassification starts, by noting differences in the phase portraits for a given emotion.

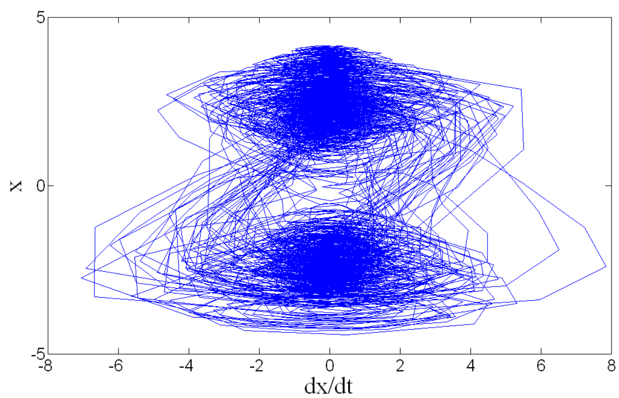


Fig.5.a: Phase trajectory for Anger when the EEG signal is corrupted by a noise of SNR 30dB

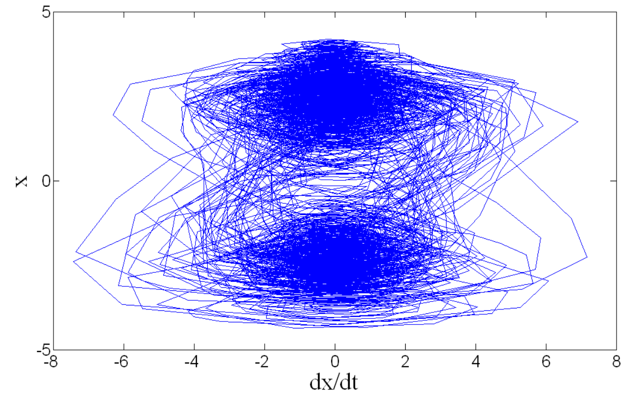


Fig. 5.b: Phase trajectory for Anger when the EEG signal is corrupted by a noise of SNR 25dB

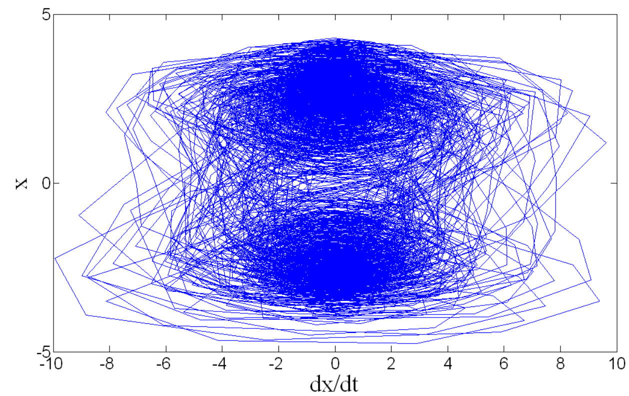


Fig. 5.c: Phase trajectory for Anger when the EEG signal is corrupted by a noise of SNR 20dB

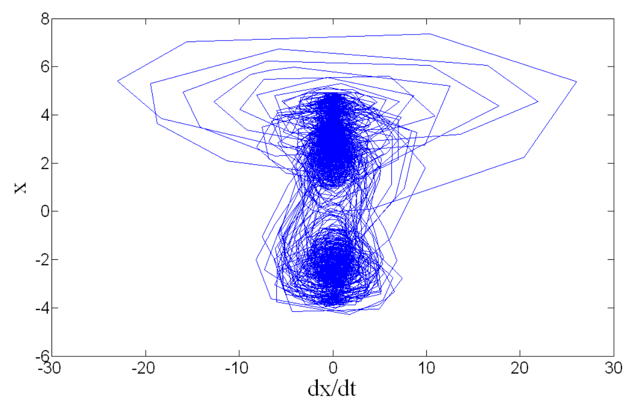


Fig. 6.a: Phase trajectory for Joy when the EEG signal is corrupted by a noise of SNR 30dB

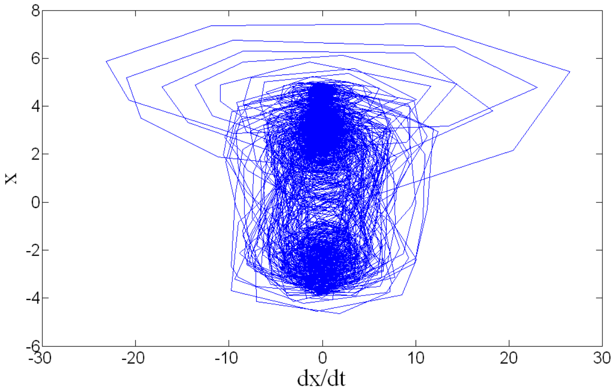



Fig. 6.b: Phase trajectory for Joy when the EEG signal is corrupted by a noise of SNR 25dB

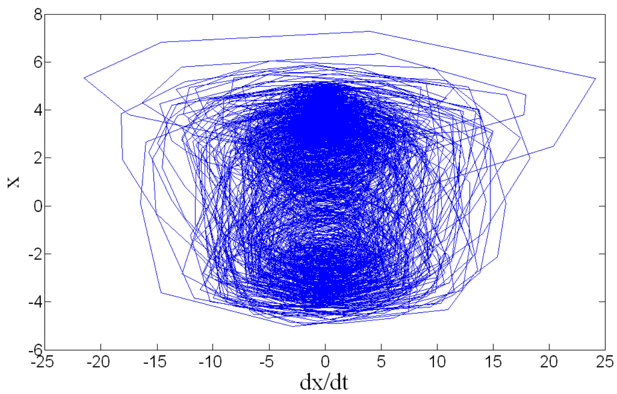



Fig. 6.c: Phase trajectory for Joy when the EEG signal is corrupted by a noise of SNR 20dB

## 5. CONCLUSIONS

The paper attempted to cluster emotions from the stimulated EEG signals using Duffing Oscillator as a medium. EEG signals aroused with specific emotion excitatory stimulus were supplied as an input to the Duffing Oscillator, the phase portrait corresponding to the response of which is plotted. Similarity in phase portraits is considered clustering of EEG in the phase-space. Consequently, clustering of emotions can be undertaken by determining similarity of the EEG signals. A noise analysis undertaken reveals that the clustering of emotions can be clearly visualized in the phase portrait, as long as the Signal-to-Noise Ratio is maintained above a prescribed threshold (25 dB). It is also obtained from the experiments that excitations responsible for arousing specific emotion have similar EEG signals, which can be clustered easily in the phase space from the response of the Duffing Oscillator. In brief, the similarity in chaotic behavior of the phase portraits resembles the similarity in EEG and consequently, similarity in emotions. The paper thus opens up a new methodology of emotion clustering from the EEG signals in the phase-space.

## REFERENCES


[1] Adolphs R, Damasio H, Tranel D, Damasio AR., “Cortical systems for the recognition of emotion in facial expressions,” Journal of Neuroscience 1996;16:7678±87.

[2] Bezooijen Rv, “The Characteristics and Recognizability of Vocal Expression of Emotions,” Walter de Gruyter, Inc., The Netherlands, 1984.

[3] Cristian Bonatto, Jason A. C. Gallas, and Yoshisuke Ueda, “Chaotic phase similarities and recurrences in a damped-driven Duffing oscillator”, Phys. Rev. E 77, 026217 (2008).

[4] Aruna Chakroborty, “Cognitive Cybernetics – A Study of the Behavioral Models of Human, Interactions”, Ph D Thesis, Jadavpur University.2005

[5] Aruna Chakroborty and Amit Konar, “Emotional Intelligence: A Cybernetic Approach”, Springer 2009.

[6] Dellaert F., Polzin T., Waibel A., “Recognizing emotion in speech,” Spoken Language, 1996. ICSLP 96. Proceedings. Fourth International Conference on, Volume: 3, 3-6 Oct. 1996. Pages: 1970 - 1973 vol.3.

[7] Dolan RJ, Heinze HJ, Hurlemann R, Hinrichs H. Magnetoencephalography (MEG) determined temporal modulation of visual and auditory sensory processing in the context of classical conditioning to faces. Neuroimage 2006;32(2):778–89.

[8] Ekman P., Friesen W. V., “Facial Action Coding System: A Technique for Measurement of Facial Movement,” Consulting Psychologists Press Palo Alto, California, 1978.

[9] P. Holmes, “A nonlinear oscillator with a strange attractor,” Philosophical Transactions of the Royal Society A, 292, 419-448, 1979.

[10]P. Holmes and D. Rand , “Phase portraits and bifurcations of the non-linear oscillator,” International Journal of Non-linear Mechanics, 15, 449-458, 1980.

[11]Tom Johnstone, Carien M. van Reekum1, Terrence R. Oakes and Richard J. Davidson, “The voice of emotion: an FMRI study of neural responses to angry and happy vocal expressions,” Social Cognitive and Affective Neuroscience, vol. 1, number 3, Pp. 242-249, October 20, 2006.

[12]Benjamin C. Kuo, “Automatic Control Systems,” John Wiley & Sons Inc.

[13]H. Nakano, T. Saito, “Grouping synchronization in a pulse-coupled network of chaotic spiking oscillators,” IEEE Trans. Neural Netw., vol. 15, no. 5, pp. 1018-1026, Sep. 2004.

[14]S Novak, RG Frehlich, “Transition to chaos in the Duffing oscillator,” Physical Review A, 1982 – APS

[15]K. Ogata, “Modern Control Engineering,” Fifth Edition, Prentice Hall

[16]E. Ott, “Chaos in Dynamical Systems (2nd edition),” Cambridge University Press, 2002.

[17]James A. Russell,Jo-Anne Bachorowski, and José-Miguel, “Facial and Vocal Expressions of Emotion,” Fernández-Dols Vol. 54: 329-349 (Volume publication date February 2003) (doi:10.1146/annurev.psych.54.101601.145102) First published online as a Review in Advance on October 4, 2002

[18]F. Schneider, W. Grodd, R. E. Gur, U. Klose, A. Alavi and R. C. Gur, “PET and fMRI in the study of emotion,” Psychiatry Research: Neuroimaging, Volume 68, Issues 2-3, 7 February 1997, Pages 174-175

[19]Ye Yuan, Yue Li, "Study on EEG Time Series Based on Duffing Equation," bmei, vol. 2, pp.516-519, 2008 International Conference on BioMedical Engineering and Informatics, 2008.