Sequential Metamodeling Approach for Optimum Design of Contact Springs Used in Electrical Connectors

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ABSTRACT

In this paper, optimum designs of a contact spring used in an electrical connector are achieved using a sequential metamodeling approach, with the objective function of the optimization problem being defined as the maximum von Mises stress in the contact spring. In order to ease the computational burden of the optimization process, the procedure is split into two stages, each with four design variables. This two-stepped scheme utilizes the Face-centered central composite experimental design concept, performs non-linear contact finite element analysis on every design point, builds response surface models with regression analysis, and uses the quadratic programming technique to optimize the approximated models.

Keywords: Electrical Connector, Contact Spring, Optimum Design, Metamodeling and Response Surface.

1. INTRODUCTION

With ever increasing demands for portable electronic devices, the reliability of their rechargeable power systems has become an important issue. A portable device connects to a battery or an electronic charger through an electrical connector. An electrical connector serves to couple two circuit devices in an electronic system. A basic electrical connector consists of four elements [1]: contact interface, contact finish, contact spring and connector housing. The contact interface can be categorized into two groups, i.e., the separable interfaces and permanent interfaces, while the contact spring performs three functions in a connector: supplying an electrical path between two subsystems, producing the normal contact force that establishes and maintains the separable interfaces, and permitting the formation of the permanent connections. A separable-interface connector may have many varieties and different shapes and sizes depending on a given set of requirements for a particular application. Figure 1 shows a common cell phone battery, several types of electrical connectors, and a connector embedded in a cell phone.

One of the most important factors affecting the reliability of a high-cycle electronic connector is their mechanical performance, which includes contact forces, deformation and stresses, etc., in the contact springs. Localized damage to the Au plating of a connector caused by a simple, manual coupling operation could lead to a high rate of functional failure [2]. Due to growing demands for smaller connectors with higher mechanical performance, a proper design of the contact spring to achieve the required mechanical performance is increasingly difficult. Weight *et al.* [3] modeled and optimized the contact spring of a constant force electrical connector (CFEC) used in a personal digital assistant docking station, with the ratio of the minimum force to maximum force calculated over the mechanism displacement as the objective function, which provides a good measure about how invariable the contact force of the mechanism truly is. Hsu *et al.* [4] parameterized the geometry of a contact spring pair of a board-to-board connector, and minimized the insertion force while kept the contact normal force and resulting stress within specified ranges. The mating of a contact pair usually involves nonlinear contact force and large deformation, and even plastic theories. Manninen *et al.* [5] and Deshpande and Subbarayan [6] studied the press-fit connector of a printed circuit board and a land grid array (LGA) connector, respectively, using nonlinear finite element (FE) analysis with plastic deformation consideration.

In this paper, the shape and size of a contact spring used in an electrical connector (a separable-interface connector) are analyzed to meet a particular set of constraints using the finite element method with a non-linear contact model and large deformation theories. Further, structural optimization on the contact spring is attained using a two-stepped, sequential metamodeling approach to simplify the optimization procedure. The structural optimization problem for the contact spring is solved to minimize the maximum von Mises stress occurred when the contact spring is engaged with the contact plate. The two-stepped procedure integrates the experimental design concept with a faced-center central composite design, non-linear contact finite element analysis on every design point, regression analysis for building response surface model, and optimization on the approximated model using the quadratic programming technique. Meanwhile, during the optimization procedure, design space reduction scheme is adopted to improve the accuracy. Finally, simulation of a contact plate approaching to and then moving away from the contact spring is rendered to examine the relation between the normal contact force in the contact spring and the traveling distance of the rigid contact plate.

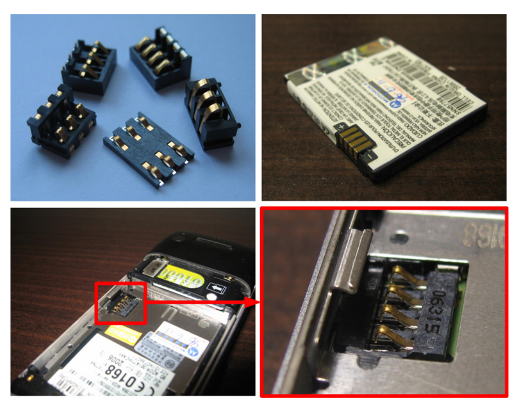

Fig. 1. Cell phone battery and electrical connectors

2. RESPONSE SURFACE APPROXIMATION

The response surface methodology [7], which was originally intended as an empirical modeling approach, is a collection of procedures including design of experiments (DOE), model selection and fitting, and optimization on the fitted model. The methodology has long been expanded to include simulation modeling and approximations. In particular, RSM has been employed by many authors, e.g. [8-11], to solve design optimization problems, especially in the area of multidisciplinary design optimization. A response surface approximation (RSA), usually in the form of a simple polynomial function, can be built from DOE (with numerically simulated experiments) and model fitting. Once a polynomial RSA is created, the optimization on the function can be easily accomplished by most optimization techniques. The most attractive features of RSA are less number of repeated response evaluations and optimization without needing the sensitivity information. In recent years, applications of RSA or RSM-based design optimization in microelectronics have increased quite dramatically. In the present work, with the help of RSA, a minimum stress design, which minimizes the von Mises stress in the contact spring after connection, will be presented. The optimum design maximizes the reliability of the contact springs, as far as reducing the stress is concerned.

Response surface approximation plays a crucial role in RSM. A response surface is a functional expression for a relationship between a response and a set of dependent variables. A complex function (or a response) y can be approximated by a response surface approximation $\hat{y}$ with k independent variables (or factors) $x_1, x_2, \ldots, x_k$ as

$$y = \hat{y}\left(x_1, x_2, \cdots, x_k\right) + \varepsilon \tag{1}$$

where ε is the error between the approximated and the exact values of y. The approximating function $\hat{y}$ usually takes on the form of a polynomial whose coefficients can be determined by the least squares method using data from a chosen set (decided by design of experiments) of the independent variables and the resulting responses. For a second order polynomial expression, the approximating function $\hat{y}$ has the form

$$\hat{y} = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \beta_{ij} x_i x_j \tag{2}$$

where the βs are the regression coefficients to be determined, and there are $(k+1)(k+2)/2$ such coefficients.

A successful application of RSA is greatly dictated by a proper choice of sampling points in design space, i.e., design of experiments. A face-centered central composite design (FCCD) with its independent variables confined within certain upper and lower bounds belongs to a family of central composite designs, which are the most popular second-order designs. An FCCD consists of 2^k factorial points, $2k$ face-centered configurations, and one center point, for a total of 2^k+2k+1 design points. Figure 2 demonstrates two FCCDs with k=2 and k=3. When performing RSM-based design optimization, response surface approximations are often repeatedly executed, and since a good RSA result may only be valid within certain distance around the center design point, the design space can be reduced after each iteration. In general, either increasing the number of experimental trials (design points) or downsizing the design space can effectively enhance the accuracy of RSA. However, the added number of experimental runs can also significantly raise the experimental or computational cost. Therefore, the design space reduction method seems to be a preferable choice, and this technique is done by introducing a pair of move limits on every design variable. After each iteration, both limits in every pair are moved closer to the other by the same ratio, resulting in a much smaller design space centering at the optimizer obtained from the previous iteration. Then a new response surface is subsequently constructed and a new optimum sought. If some portion of a new design space exceeds the boundary of the previous one, the portion is excluded before performing DOE and RSA.

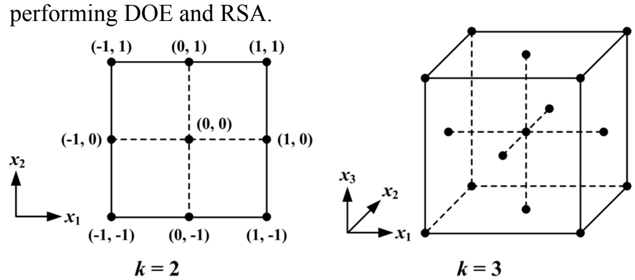

Fig. 2. Face-centered central composite designs

Following the construction of an RSA, the quality of the approximation can be assessed by some statistical testing functions. The coefficient of multiple determination R^2 is a measure of the amount of predictability for the response y by the approximating function $\hat{y}$, and the coefficient is defined as

$$R^2 = \frac{S_r}{S_t} = 1 - \frac{S_e}{S_t} \tag{3}$$

and

$$S_r = \sum_{i=1}^{N} \left(\hat{y}_i - \bar{y}\right)^2 \tag{4}$$

$$S_e = \sum_{i=1}^{N} \left(y_i - \hat{y}_i\right)^2 \tag{5}$$

$$S_t = S_r + S_e \tag{6}$$

where S_r, S_e and S_t are called the sum of squares due to regression, sum of squares due to residual and total sum of squares, respectively; y_i is the ith observation, $\bar{y}$ the average value of all observations, $\hat{y}_i$ the response surface approximation evaluated at the ith set of independent variables, and N the total number of observations. R^2 takes on a value between 0 and 1. A larger value of R^2 does not necessarily indicate a closer fit of the approximation to the response since adding a variable will always raise the value of R^2. The adjusted coefficient of multiple determination, which will not increase if an added variable is not statistically significant and therefore is a better indicator than R^2, is defined as [7]

$$R_{adj}^2 = 1 - \frac{S_e/(N-p)}{S_t/(N-1)} = 1 - \left(\frac{N-1}{N-p}\right)\left(1 - R^2\right) \tag{7}$$

where p denotes the number of the regression coefficients.

3. PROBLEM DEFINITION AND FORMULATION

The contact spring of an electrical connector under study is shown in Fig. 2. Most part of the contact spring, except the contact head, is enclosed in the connector housing. During a connection process, a contact pad from the other subsystem moves in to touch the contact head and complete an electrical path between the two subsystems. As the contact pad continues to move in further, the spring is furthermore compressed, producing an increasing contact normal force. A greater contact force has positive effects on contact electrical resistance and the mechanical stability of the interface, an increasing normal force leading to a decreasing contact resistance and to a better ability to withstand disturbances. However, countering these positive effects is the effect of rising contact force on the stresses in the spring. Higher stresses inevitably reduce the fatigue life of the contact spring, and even produce excessive plastic deformation that could result in a reliability problem on the connector. One of the objectives of this study is to minimize the von Mises stress in a shape design of the spring after the connector is mated. The contact spring is assumed to be made of beryllium copper, whose properties are listed in Table 1. A plastic hardening relation for the material, adopted from [6], is also assumed and given in Table 1.

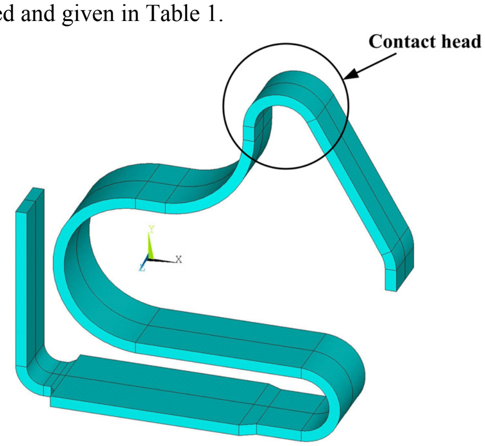

Fig. 2. Contact spring of an electrical connector

Table 1. Properties of beryllium copper

Young's modulus (GPa)	Poisson's ratio	Plastic hardening	
		Plastic strain	Stress (MPa)
113.85	0.3	0.0	621
		0.09454	759

The structural shape optimization of the contact spring requires repeated nonlinear elastic-plastic contact analysis, which could lead to a prohibitively high computation cost and a convergence difficulty if a conventional first-order mathematical programming technique is used to solve the 3-dimentional finite element model in this research. To ease the computational burden and reduce the risk of no convergence, a metamodeling scheme, the response surface methodology, is adopted to solve the problem.

Design Variables and Constants

Before commencing a finite element analysis, the structural model of the contact spring is parameterized. The geometrical parameters, shown in Figs. 3 and 4, establish the shape and size of the structure and are employed as inputs, i.e. design variables and constants, to the analysis and optimization. Figure 3 illustrates the geometrical parameters on the x-y plane for the contact spring, including design variables r_1-r_4 and d_1-d_3, constants C_{d1}-C_{d6}, C_{r1}-C_{r2} and t, and geometrical constraints H_h, V_h, V_t and H_g. Figure 4 shows the parameters on the z axis, which are the beam width parameters (in z axis) at various locations, consisting of design variables w_1-w_4 and constants C_{w1}-C_{w4}. The beam thickness t is set constant in consideration of manufacturability, and geometrical constraints H_h, V_h, V_t and H_g are required to satisfy either assembly or functionality restraints. The rest of the constants are set due to their less significant influence on the analysis results. Every cross section perpendicular to the centroidal axis of the bended and curved beam is assumed to have a rectangular shape. The z dimensions of the solid model are formed by linearly (in z) connecting adjacent beam width parameters. In order to reduce the complexity of the optimization process, the procedure is split into two stages. The first stage treats only r_1-r_4 and d_3 as the design variables and the rest as constants. By considering the geometrical constraints H_h, V_h and V_t, the parameters r_4, d_1 and d_2 may be linked to the other design variables, cutting down the total number independent variables in this stage to four (r_1-r_3 and d_3). When the first-round optimization is completed, the second stage will begin, based on the results obtained from the former, by setting four new design variables w_1-w_4 and the rest constant. The initial values of the design parameters and the constants are given in Table 2. The invariant C_{d5} may be regarded as a function of other constants by noticing the constraint Hg, and therefore it is removed from Table 2.

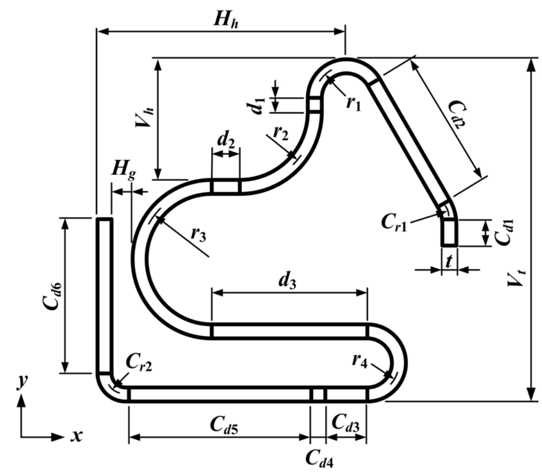

Fig. 3. Geometrical parameters on x-y plane for the contact spring

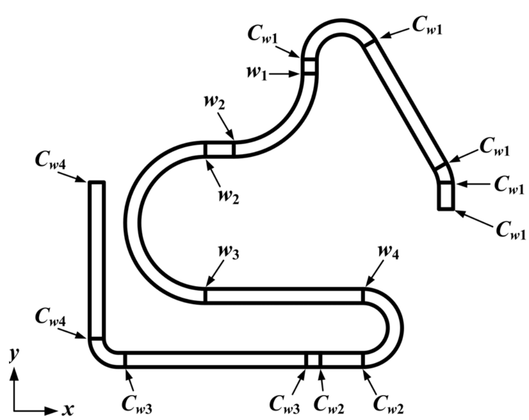

Fig. 4. Geometrical parameters on z axis for the contact spring

Table 2. Initial values for the geometrical parameters of the contact spring (all units in mm)

r_1	r_2	r_3	r_4	d_1	d_2	d_3
0.45	1.07	1.03	0.45	0.2	0.4	2.22
t	H_h	V_h	V_t	H_g	C_{d1}	C_{d2}
0.15	3.57	1.8	4.91	0.4	0.3	2.0
C_{d3}	C_{d4}	C_{d6}	C_{r1}	C_{r2}	w_1	w_2
0.6	0.2	2.2	0.45	0.3	0.7	1
w_3	w_4	C_{w1}	C_{w2}	C_{w3}	C_{w4}	
1	1	0.7	1	1.2	0.7	

Parameterized Finite Element Models

Finite element software ANSYS is used for computational modeling of the contact spring. Following a preliminary convergence analysis, the finite element mesh shown in Fig. 5 is proven satisfactory. The finite element model is consisted of 10,638 three dimensional elements, mostly bricks and a few tetrahedron elements. In addition, a finite element contact pair is also defined over portions of the contact pad and spring surfaces that may potentially engage contact, and a friction coefficient of 0.1 between the contact pair is assumed in the analysis. The contact pad, whose contacting surface is assumed to be rigid, has an initial position of 0.075 mm (half of the beam thickness) from above the contact head. Downward compression distances of 1 mm and 1.5 mm on the contact head are realized and studied by moving down the contact pad 1.075 mm and 1.575 mm, respectively.

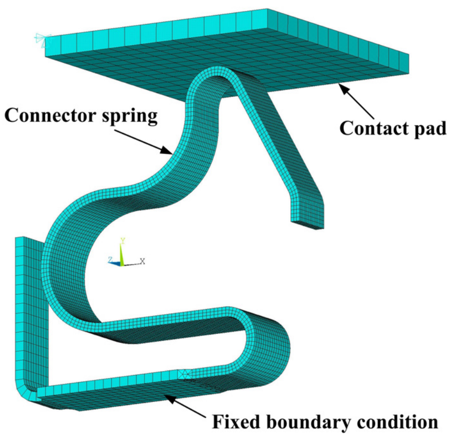

Fig. 5. Finite element mesh of the contact spring model

Optimization Problem Formulation

In this study, the objective function of the optimization problem is the maximum von Mises stress in the contact spring during the entire coupling and decoupling processes between the contact spring and contact pad. And this maximum stress is minimized in an attempt to increase the reliability of the spring and therefore the connector. Mathematically, the optimum shape design problem can be written as

$$\mathit{Minimize} \quad f(\boldsymbol{x}) = \sigma_{\max} \tag{8}$$

Subject to

$$x_i^{\mathrm{L}} \leq x_i \leq x_i^{\mathrm{U}}, \quad i = 1, 2, \cdots, k \tag{9}$$

where $\sigma_{\max}$ denotes the maximum von Mises stress, x_i are the design variables, and the superscripts U and L represent the upper and lower bounds, respectively. Also, the total number of the design variables is symbolized by k, which is equal to four in both stage one and two. For stage one, the lower and upper bounds for the design variables are: (0.3, 0.5) for r_1, (0.5, 1.2) for r_2, (0.5, 1.2) for r_3, and (1.5, 2.5) for d_3, and for stage two, they are: (0.5, 1.2) for w_1-w_4. The selections of the bounds are based on manufacturability and engineering judgment.

To approximate objective function $\sigma_{\max}$ by RSA, repeated finite element analyses are performed on all design configurations, after which a explicit functional relation, also known as the response surface, of $\sigma_{\max}$ with respect to the design variables is created by least squares curve fitting to a polynomial model. The minimization of the multi-variable polynomial function subjected to side constraints can be easily carried out by common optimization routines.

4. RESULTS AND DISCUSSION

A typical run of the nonlinear elastic-plastic contact analysis using ANSYS takes approximately 950 CPU seconds on a PC. In either stage of the optimization process, there are N=2^k+$2k$+1=25 sets (k=4) of design points and, therefore, 25 different analyses need to be performed to constitute one iteration. With the help of shrinking design space boundaries, it usually takes several iterations to achieve convergence. For a comparison purpose, FE analyses using the initial values given in Table 2 yield $\sigma_{\max}$=685.524 MPa for the case of 1 mm downward compression and $\sigma_{\max}$=741.917 MPa for 1.5 mm downward compression. The following subsections will present the optimization results of 1 mm and 1.5 mm downward compression on the contact head for both stages and also the analyses of contact normal forces during the contact engagement process.

Optimum Design after Stage One

For the case of 1 mm downward compression on the contact head, finite element analyses are executed repeatedly using 25 sets of input parameters for the first iteration in stage one, then a quadratic model is fitted. Statistical testing is performed on the fitted model, and gives an R^2 value of 0.992 and an adjusted R^2 value of 0.982, which represent a very good fit of the approximations to the responses. After 6 iterations, convergences are apparent. The final optimized result is checked by another ANSYS verification run, and it produces $\sigma_{\max}$=577.055 MPa at r_1=0.5 mm, r_2=1.2 mm, r_3=1.199 mm and d_3=1.5 mm. This optimum design has its parameters located either at their upper or lower bounds. Figure 6 shows the von Mises stress distribution of this optimum model for the case of 1 mm downward compression on the contact head after stage one optimization.

For the case of 1.5 mm downward compression, larger stresses and even permanent deformations are expected. The same procedure as for the 1 mm case but with a downward compression of 1.5 mm on the contact head is executed. The optimization process converges within 10 iterations. The verification run gives $\sigma_{\max}$=652.563 MPa at r_1=0.351 mm, r_2=0.825 mm, r_3=0.951 mm and d_3=1.921 mm, and the von Mises stress distribution of this optimum model is shown in Fig. 7.

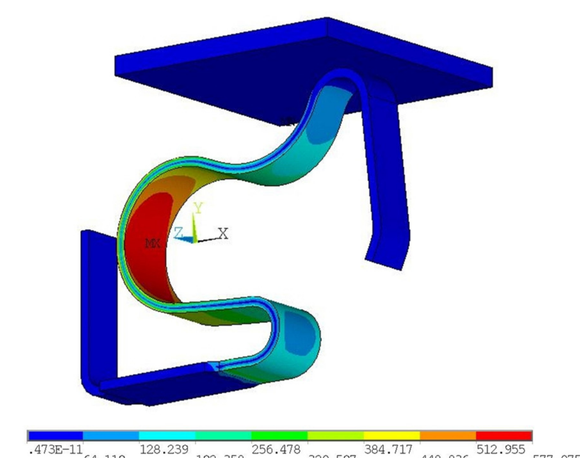

Fig. 6. Stress distribution of the optimum model for the 1 mm compression case after stage one optimization

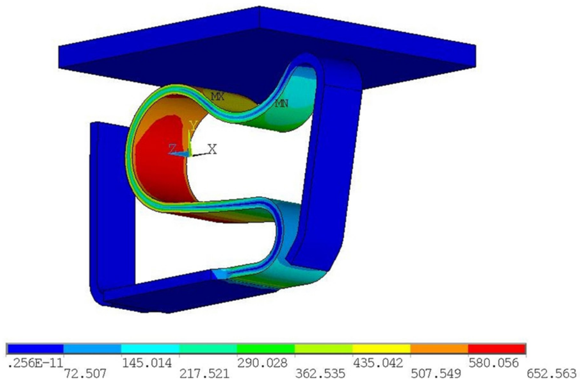

Fig. 7. Stress distribution of the optimum model for the 1.5 mm compression case after <u>stage one</u> optimization

Optimum Design after Stage Two

Based on the optimal parameters acquired from stage one, new design variables, w_1-w_4, are introduced to perform optimization in stage two. Again, a multi-variable quadratic function is matched using the stress data from FE analyses. The fitted model is then tested for its adequacy, and the testing reveals a satisfactory outcome for both 1 mm and 1.5 mm compression cases: adjusted R^2 values of 0.966 and 0.919 for the 1 mm case and the 1.5 mm case, respectively. Within 10 iterations, both cases are converged. The optimum model for the 1 mm case, situating at w_1=0.516 mm, w_2=1.196 mm, w_3=1.192 mm and w_4=0.502 mm, yields σ_{max}=511.156 MPa, shown in Fig. 8. Similarly, the optimum model for the 1.5 mm case, positioning at w_1=0.728 mm, w_2=0.500 mm, w_3=0.937 mm and w_4=0.815 mm, generates σ_{max}=628.926 MPa, shown in Fig. 9. These optimized responses represent significant improvements over those of the initial models (σ_{max}=685.524 MPa and 741.917 MPa). Notice that only one of the design parameters attains its optimal value at the boundary for the 1.5 mm case.

Contact Normal Force vs. Displacement

Beside the stresses in the contact spring, the contact normal force is another important factor dictating the performance of a spring. Figures 10 and 11 show the contact normal forces experienced by the optimum designs of the spring after stage one and stage two optimizations, respectively. When the contact head is compressed downward by approximately 1 mm, the force reaches the maximum at 663.83 mN for the stage-one optimum model, and at 718.50 mN for the stage-two optimum model. A further examination reveals that the spring suffers no plastic strain and undergoes no plastic deformation. If the contact pad is moved back to the initial position, the spring will recover completely to its original shape. When the compressing distance of the spring is extended to 1.5 mm, plastic deformations are clearly visible. Figures 12 and 13 display the force vs. displacement curves as the contact pad travels downward and back. The maximum forces are observed at 1459.29 mN for the stage-one optimum model and at 909.44 mN for the stage-two optimum model, both occurring near the 1.2 mm mark of distance traveled by the contact pad. When the contact pad further moves down, the yielding strength of the spring is exceeded and the normal force drops. As the contact pad continues to press downward to the farthest distance mark of 1.575 mm and then returns to its initial position, the spring endures a maximum plastic strain at the location coincident to that of the maximum stress. The plastic strains lead to a permanent deformation of -0.41 mm (downward) for the stage-one optimum model and -0.43 mm (downward) for the stage-two optimum model, at the tip of the contact head after the contact pad disengages the spring. These irreversible deformations may be significant enough to cause a reliability problem. One possibility to resolve the dilemma is to conduct a new optimization practice that seeks to minimize both the maximum stress and the maximum equivalent plastic strain simultaneously.

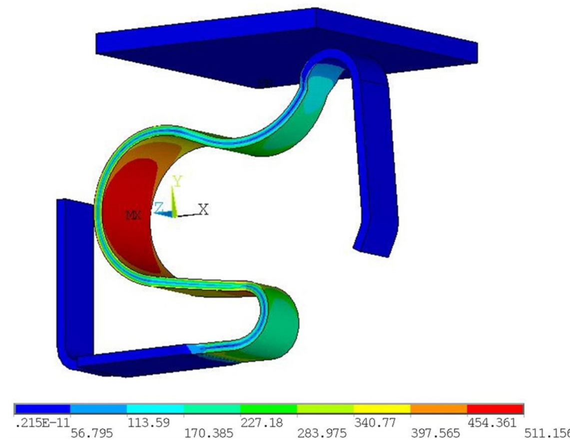

Fig. 8. Stress distribution of the optimum model for the 1 mm compression case after <u>stage two</u> optimization

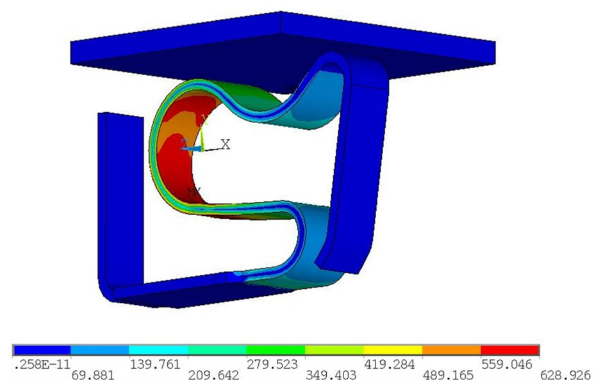

Fig. 9. Stress distribution of the optimum model for the 1.5 mm compression case after <u>stage two</u> optimization

5. CONCLUSIONS

Optimum designs of a contact spring used in an electrical connector have been achieved using a sequential metamodeling approach, with the objective function of the optimization problem being defined as the maximum von Mises stress in the contact spring. In order to ease the computational burden of the optimization process, the procedure was split into two stages, each with four design variables. This two-stepped scheme utilized the FCCD experimental design concept, performed non-linear contact finite element analysis on every design point, built response surface models with regression analysis, and used the quadratic programming technique to optimize the approximated models. Two cases were studied, one with the contact head of the spring pressed downward 1 mm, the other 1.5 mm. Both cases showed that the responses of the optimum models represented significant improvements over those of the initial models. However, simulation results revealed that significant permanent deformations were observed for the 1.5 mm case.

6. ACKNOWLEDGEMENT

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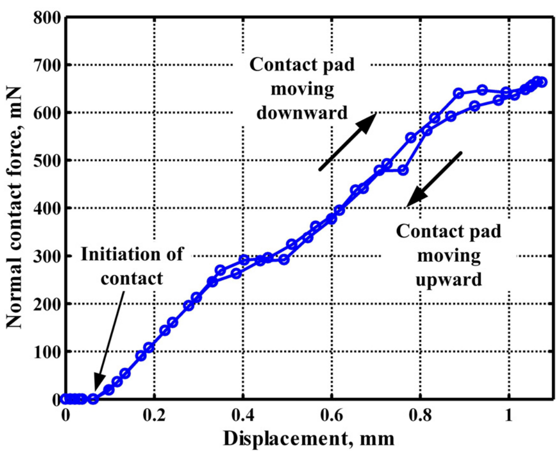

Fig. 10. Contact normal force experienced by the optimum model for the 1 mm case after stage one optimization

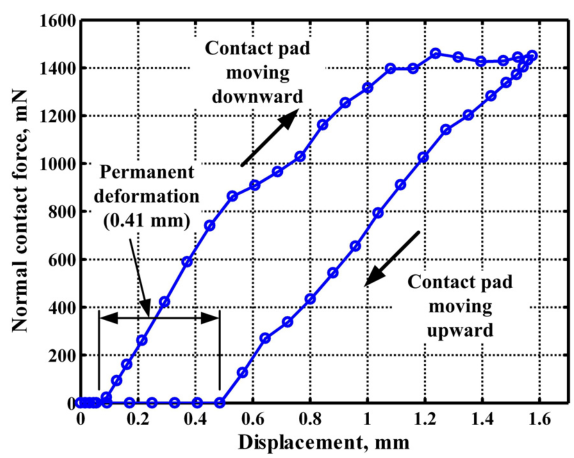

Fig. 12. Contact normal force experienced by the optimum model for the 1.5 mm case after stage one optimization

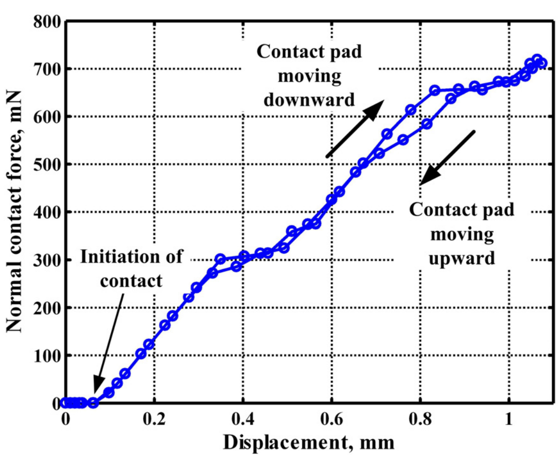

Fig. 11. Contact normal force experienced by the optimum model for the 1 mm case after stage two optimization

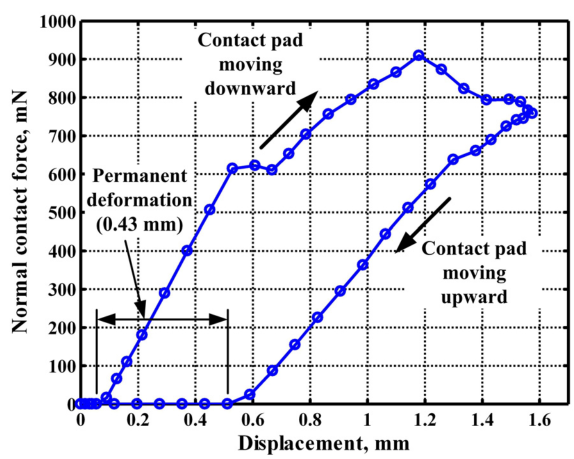

Fig. 13. Contact normal force experienced by the optimum model for the 1.5 mm case after stage two optimization