Comparison of Extended Fuzzy Logic Models of A-IFS and HLS: Detailed Analysis of Inclusion in the A-IFS of the Data Sets for Implication Operations

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ABSTRACT

After the proposal of Zadeh's "Fuzzy Set Theory", several kinds of extended fuzzy set/logic models have been proposed. Some extended models treat any multi-dimensional fuzzy logic system. Typically, the models assume a pair of (*t, f*) for the truth-value of an ambiguous proposition **A**, while $t,\ f \in [0,1]$. The *t* means truthfulness and *f* means the falsity of the proposition **A**. Both Atanassov's "Intuitionistic Fuzzy Set" (A-IFS) model and Oda's Hyper Logic Space (HLS) model treat the two-dimensional logic space, but there are some differences regarding (1) the domain areas of definition and (2) the formulas of negation operation. For comparison of the two models, the authors investigated the following two conditions. Condition 1: Both propositions **A**=(*ta, fa*) and **B**=(*tb, fb*) are in the A-IFS area ($t+f\leq 1$), and both results of the implication operations by the two models return to the A-IFS area. Here, the HLS model can use both the A-IFS area and the contradiction area ($t+f>1$) only as a calculating space. Condition 2: Though both propositions **A** and **B** are in the A-IFS area, the results of the implication operations by the HLS model are no longer in the A-IFS area but in the contradiction area. For the purpose of comparing the results, the following two methods are applied. Method 1: The resultant points (*t, f*) calculated by the implication are converted to the corresponding Interval Valued sets [$t,\ 1\text{-}f$] in one dimensional numerical truth-value space **V**. Method 2: As the contradicted area data is impossible to convert to the Interval Valued set, the resultant points from both models are integrated using one of Oda's formulas named $\boldsymbol{I_4}$. Since $\boldsymbol{I_4}$ is symmetrical to both data areas, it is proper to apply. Analyzing the results of both conditions, we concluded that the HLS model, especially the usage of the contradiction area, is useful for implication operation not only for treating the HLS data but also for treating the A-IFS data.

Keywords: Fuzzy Logic, Extended Fuzzy Logic, Intuitionistic Fuzzy Set, Hyper Logic Space, Fuzzy-set Concurrent Rating Method

1. INTRODUCTION

In 1965, L. A. Zadeh proposed the "Fuzzy Set Theory" for mathematically modeling a kind of ambiguous concept. After the proposal, many investigators including Zadeh himself followed and developed the theory from various points of view. In the early stage of this development, the Numerical Truth-value model was proposed. The Numerical Truth-value model assumes a real number value $\boldsymbol{t}$, ranging from 0 to 1, as a truth-value of a fuzzy proposition. Nowadays, the model is recognized to be representative of Fuzzy Logic (FL) models. Several kinds of extended Fuzzy Set/Logic models have since been developed. For example, the Interval Valued Fuzzy Set (IVF) model (See Fig. 1) is a well-known extended model. According to D. Dubois & H. Prade [1], the IVS includes most of the extended Fuzzy Set/Logic models.

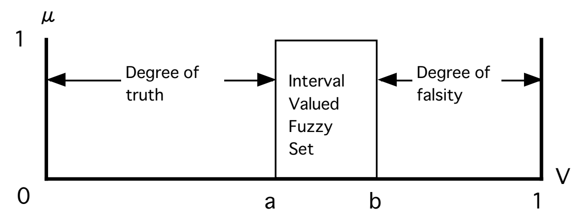

Figure1: Interval Valued Fuzzy Set (IVS)

In Bulgaria, K. Atanassov [2] proposed the Intuitionistic Fuzzy Set (IFS) model. IFS assumes not only the degree of membership function $\boldsymbol{\mu}(x)$ but also the degree of non-membership function $\boldsymbol{\nu}(x)$ of an ambiguous set. In Japan, the Hyper Logic Space (HLS) model was proposed by T. Oda [3]. The HLS model extended the numerical truth-value of the Fuzzy Logic in order to define the special indexes of the newly developed psychological measurement method namely Fuzzy-set Concurrent Rating (FCR) method. As HLS closely resembles IFS, the advantage of HLS over IFS has not been so clear, though HLS can also treat contradictory data sets while the IFS can not.

Among extended Fuzzy Set/Logic theories, K. Atanassov's IFS theory may be the most widely well-known model. (It is now also called the A-IFS, to distinguish it from another model with the same name proposed by G. Takeuchi and S. Chitani [4]. Takeuchi and Chitani's model has been called T-IFS.)

Apart from the European academic background, other models have been proposed in Japan, which assumed that *t* (truthfulness) and *f* (falsity) of a proposition are mutually independent as A-IFS assumed. One of the early models is Mukaidono and Kikuchi's "Between Fuzzy Logic" (BFL) model [5], in which they extended "Interval Valued Fuzzy Logic" by permitting the lower limit *a* to exceed the upper limit *b* of the interval value [*a, b*]. (See Fig. 2)

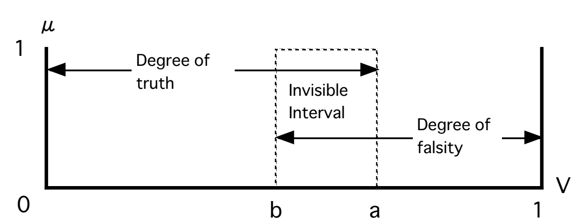

Figure 2: Between Fuzzy Logic (the case of invisible interval)

Against this background, the authors proposed the HLS model in which ***t*** and ***f*** are defined as perfectly independent, while A-IFS and other models have been assumed limited independencies of $t+f \leq 1$.
This study includes two analyses. Analysis 1 treats the condition that both results of the implication operations, derived by the models, of two points **A** and **B** in the A-IFS area return to the A-IFS area. In Analysis 2, another condition is investigated.
Throughout this paper, the effort of analysis is concentrated on uncovering the differences of the implication operation formulas between ¬**A**∨**B** by HLS and ~**A**∨**B** by IFS.
This study is intended to show some advantages of the HLS model over the IFS model under the restriction of applying only A-IFS data sets.
Since the resultant point of the implication ¬**A**∨**B** can appear in the contradiction area, while the resultant point of ~**A**∨**B** never spills out from the A-IFS area, the two models cannot be compared directly. So, the results are indirectly compared by calculating their integrated values. For the integration algorithm, the authors adopted the Combined Scoring Method 1 (***I_4***) developed by T.Oda.

2. FCR-METHOD ,HLS MODEL AND OTHER CONCEPTS

In this chapter, the compendium of the HLS model, the FCR-method, and related concepts used in the latter part of this paper are introduced.

2.1 Hyper Logic Space

Hyper Logic Space (HLS) is the two-dimension fuzzy logic space **T** × **F.** Here, **T**=[0,1] means truthfulness while **F**=[0,1] means falsity. HLS has five special points in the coordinates:
0=(0,0) , **T**=(1,0) , **F**=(0,1) , **1**=(1,1) and **C**=(0.5,0.5).
The meanings of the points are as follows.

- **0**: empty point (Perfectly irrelevant)
- **T**: true point ("Truth" in Classical Logic.)
- **F**: false point ("False" in Classical Logic.)
- **1**: contradicting point (Perfectly contradicting)
- **C**: center

The points on the line t+f=1 compose the numerical truth-value space **V** of FL. The three points F, C and **T** appear on the line. The other points in HLS have not been treated in traditional FL. The area composed of the points t+f>1 is named "contradiction area", while the area composed of the points t+f<1 is named "irrelevance area". A-IFS has been treats only the area defined by t+f ≦ 1.
The logical operations of HLS are discussed in Chapter 3.

2.2 FCR-method

In 1993, T. Oda proposed a new technique to identify the fuzzy membership function of a concept by using three monopole scales. Then, the idea was developed to the Fuzzy-set Concurrent Rating method (FCR method) as a new technique for general psychological measurement [10]. The FCR method is designed to measure a subject's opinions or attitudes more naturally than traditional bi-polar rating scale methods. Since then, various new concepts and related algorithms have been developed [3,6-12].
The two-item type FCR method uses two independent rating scales for each question. (See Fig. 3)
The scales are used to measure positive and negative responses respectively. Thus, one question obtains a pair of responses and is represented as (*p,n*). By combining the pair, an integrated value is calculated. The integrated value is an alternative to a rated score value on a traditional bi-polar rating scale.

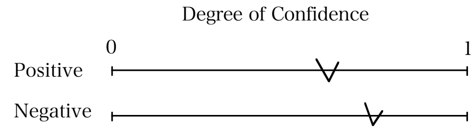

Figure 3: FCR-scale (two-item type)

Furthermore, possible contradictions in the responses or possible irrelevancies to the question can also be observed from the pair. In the FCR-method, the irrelevancy-contradiction index is very important for analyzing actual data. So, various kinds of formulas have been developed. But, the detailed explanations of the indexes are omitted here. Nowadays, $C_3=p+n-1$ is generally used as the irrelevancy-contradiction index in the FCR-method because of its simplicity and linearity. ($-C_3$ is identical to the "hesitation margin" introduced by P. Merin in her extended fuzzy logic/set model "Medative Fuzzy Logic" [13].) Furthermore, as a psychological measurement system, special instructions, that both scales should be independently rated, are needed when it is practically applied.

2.3 Integrated Value of the FCR-method

Assume the positive scale value ***p*** and negative scale value ***n*** of the FCR-scale are directly assigned to truthful ***t*** and falsity ***f*** respectively in HLS.
Fig.4 is illustrating the fundamental integration algorithms of the FCR-method, I_1, I_2 and I_3 has been proposed. In this figure, they are explained by using the projection lines pass through the observed point A.

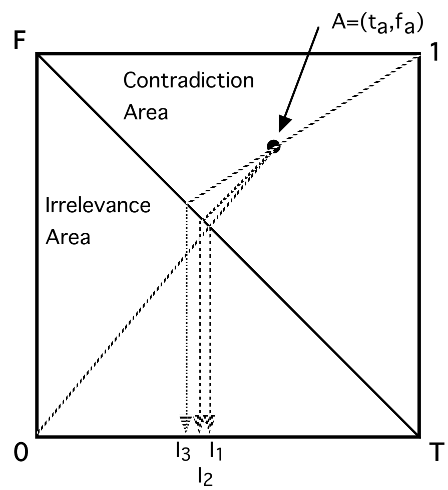

Figure 4: Fundamental integrated values illustrated in the HLS

Simple scoring method (I_1): By assigning the score value ***1*** for ***t*** while **0** for ***f***, the weighted average score for the pair (*t, f*) is calculated by the following formula.

$$I_1 = \frac{t}{t+f} \quad if\ t+f \neq 0 \quad otherwise\ I_1 = 0.5 \qquad \textbf{(1)}$$

Reverse item averaging method (I_2): Since the degree of falsity ***n*** can coincide with the degree of truthful ***t*** by negation, one of the basic integration formulas is defined by averaging ***t*** and ***1-n***.
In other word, the negation of the falsity (***1-f***) can be considered to be the alterative of the truthful ***t.***

$$I_2 = \frac{t+1-f}{2} \qquad \textbf{(2)}$$

Inverse scoring method (I_3): By assuming that (1-t) can be the alternative of n while (1-f) can be the alternative of t, the weighted average of the pair ((1-f), (1-t)) when the scores 1 and 0 are assumed respectively.

$$I_3 = \frac{1-f}{2-t-f} \quad if \ t+f \neq 2 \quad otherwise \ I_3 = 0.5 \tag{3}$$

By combining these integrated values, various combined scoring method were proposed and named I_4 to I_{11}.(See Table 1) [12]

Table 1. Combined Integration algorithms

Name of the algorithm	Definition	Eq.
Combined scoring method 1	***I4*** = ***I1*** *if t + f ≤ 1 otherwise* ***I4***=***I3***	(4)
Combined scoring method 2	***I5*** = ***I3*** *if t + f ≤ 1 otherwise* ***I5***=***I1***	(5)
Combined scoring method 3	***I6*** = (***I1*** + ***I3***) / 2	(6)
Combined scoring method 4	***I7*** = $\sqrt{I_1 + I_3}$	(7)
Combined scoring method 5	***I8*** = 2/(1/***I1*** + 1/***I3***)	(8)
Combined scoring method 6	***I9*** = (***I1***+***I2*** + ***I3***) / 3	(9)
Combined scoring method 7	***I10*** = $\sqrt[3]{I_1 \cdot I_2 \cdot I_3}$	(10)
Combined scoring method 8	***I11*** = 3/(1/***I1***+1/***I2*** + 1/***I3***)	(11)

2.4 Converting principle from a point to an interval

In this investigation, the well-known converting principle is introduced. According to the principle, one point of **A**=(*ta*,*fa*), which is in the two-dimensional fuzzy logic space **T** × **F**, which is identical to the HLS introduced here, is converted into the closed interval [*ta, 1-fa*] in the one dimensional numerical truth-value space **V**.

3. COMPARING HLS MODEL AND IFS MODEL

3.1 Common assumptions and definitions

Both models are closely resembles excepting the data area and the negation operations.

Assume both **A** and **B** are fuzzy propositions. The pairs of truth-value and false-value are referred to:

A = (*ta, fa*), **B** = (*tb, fb*) while *ta, tb, fa, fb* ∈ [0,1] (12)

The logical OR (∨) and the logical AND (∧) operations are just the same as follows.

$$A \vee B = (\max(t_a, t_b), \min(f_a, f_b)) \tag{13}$$

$$A \wedge B = (\min(t_a, t_b), \max(f_a, f_b)) \tag{14}$$

Since the fuzzy set operations are defined by the logical operations of the elements, the operations can be compared.

3.2 Differences of data area

Any data pair (***t, f***) of the A-IFS has the constraint of t + f ≦ 1 by the definition of the model. Meanwhile, t and f are completely independent with each other in HLS model.

3.3 Differences of negation operation

A negation operation in the HLS model is a natural extension of Zadeh's negation, and is so-called external negation of a proposition.

¬ **A** = (*1-ta ,1-fa*) (15)

Meanwhile, a negation operation in the A-IFS model is so-called internal negation; referred to exchange ***t*** and ***f.***

~ **A** = (*fa, ta*) (16)

For distinguishing the negation operations of each model, the other symbol ~ is used for the negation operation of A-IFS.

The result of negation of A-IFS by (3) never protrudes out of the A-IFS areas.

3.4 Differences of implication operator

As a common definition for the implication operation (**A** → **B**) for both models, the "the negation of **A** or **B**" is adopted. But the definition of the negation operations are different depending the models, the algorithms are different.

The operations of each model are distinguished by the symbols of negation operation.

Hereafter, the hyper-logical space (i.e. [0,1]×[0,1] fuzzy logical space) is used for mathematical explanations and graphical presentations of both models.

The implication operations for HLS and A-IFS are as follows respectively.

¬**A**∨**B**={*max(1-ta, tb), min(1-fa, fb)*} (17)

∼**A**∨**B**={*max(fa, tb), min(ta, fb)*} (18)

4. ANALYSIS 1:
The case in which the result of HLS return to the IFS area.

s

In the HLS model, not only the data in the A-IFS area (t+f≦1), but also the data in the contradiction area (t + f> 1) can be treated while the A-IFS model can only treat the data in the irrelevance area (t+f<1) and the data in the numerical truth-value area (t+f=1). The authors compare the results obtained by each model to prove the superiority of the HLS model when the data A and B are both in the A-IFS area, because the A-IFS model is impossible to treat if a data is in the contradiction area. The constraints for A and B assumed here are as follows.

ta+fa≦1, tb+fb≦1 (19)

In this chapter, first, under the conditions of (6), the conditions are clarified in which the result of implication operation ¬ **A** ∨ **B** by the HLS model returns into the A-IFS area. Then, the properties of both models' implication operators are compared.

4.1 Analysis of the result of inclusion operation by HLS model

The inclusion formula (Eq. 17) by the HLS model can be classified into four cases by its parameter values.

Case 1 : (1-ta≧tb) & (1-fa<fb),

¬A∨B={1-ta, 1-fa} (20)

----- coincident with ¬A

Case 2 : (1-ta≧tb) & (1-fa≧fb),

¬A∨B={1-ta, fb} (21)

Case 3 : (1-ta<tb) & (1-fa<fb),

¬A∨B={tb, 1-fa} (22)

Case 4 : (1-ta<tb) & (1-fa≧fb),

¬A∨B={tb, fb} (23)

----- coincident with B

(1) About the Case 1: Since the result of this case does not match with prerequisite condition (6), it is not treated in this chapter.

(2) About the Case 2: The restriction *(1-ta≧tb)&(1-fa≧fb)* can be transformed to *(ta≦1-tb)&(fa≦1-fb)*. (24)

The pentagonal area with hatching in Fig. 5 is showing the possible existence zone of the point **A** in which the point satisfying the restriction. In this figure, the hatched area means the relative location of **A** if the location of the point **B** was given.

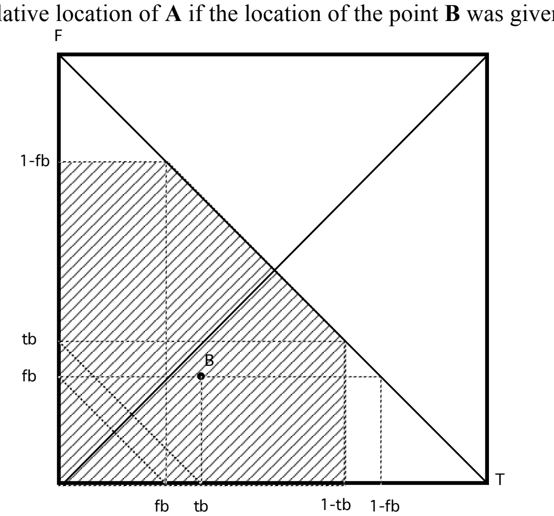

Figure 5: The zone in which point A satisfies the restriction of the Case 1

In Case 2, the condition in which the result of ¬**A**∨**B** returns to the A-IFS domain is $1 - ta + fb \leqq 1$. It means $ta \geqq fb$. Then, the result is shown as a trapezoidal area with hatching in the Fig. 6, as a relative location of **A** when **B** was given.

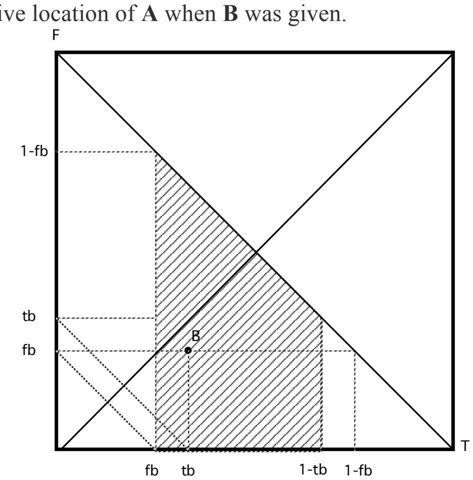

Figure 6: The zone that the point A satisfies the restriction of Case 2

(3) About Case 3: The condition (1-ta<tb)&(1-fa<fb) can be transformed to (ta>1-tb)&(fa>1-fb).

It is clear that the area which fulfilling the condition does not exist.

Proof:

From the condition,

(ta>1-tb)&(fa>1-fb) (25)

Then,

ta+fa>(1-tb)+(1-fb)

ta+fa>2-(tb+fb) (26)

By the way, since **B** is a point in the A-IFS area,

tb+fb≦1 (27)

therefore *ta+fa>1*. (28)

This inequality is contradicting to the assumption that "**A** is existing in the A-IFS area".

Q.E.D.

(4) About Case 4: From the condition *(ta<tb)&(1-fa≧fb)*, obtain

(ta>1-tb)&(fa≦1-fb). (29)

As the result of the implication operation is just the same to the point **B**, it is always in the A-IFS area. In Fig. 7, the hatched triangle area is illustrating the possible existing zone of **A** satisfying the result of Case 4 as a relative location if **B** was given.

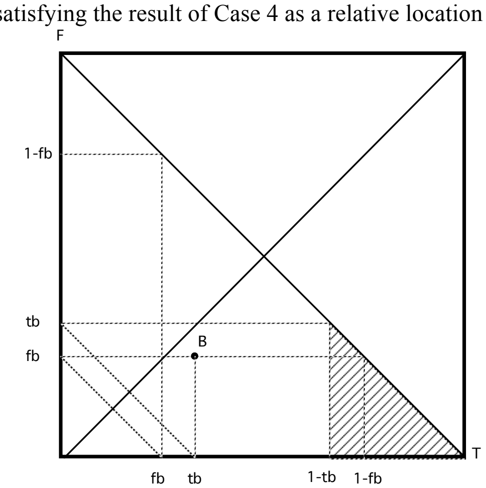

Figure 7: The zone that the point A satisfies the restriction of Case 4

Both ¬ **A** ∨ **B** and **B** fit into the IFS area without problems.

(5) Summarize the four cases: From Case 2,

1-fb≦ta <1-tb therefore

¬**A**∨**B**=*(1-ta, fb)* (30)

From Case 4, *ta≧1-tb* therefore

¬**A**∨**B**=*(tb, fb)* (31)

In these cases, any result of the implication operation **A** → **B** returns into the A-IFS area.

In summary, the results of implication operation by the HLS model are shown as below.

$$\neg A \vee B = \begin{cases} (1-t_a, f_a) & if\ f_b \le t_a < 1-t_b \\ (1-t_b, f_b) & if\ t_a \ge 1-t_b \end{cases} \tag{32}$$

4.2 Analysis of the results of implication operation by A-IFS model

The result of implication operation by A-IFS model is expressed as below.

$$\sim A \vee B = \{\max(f_a, t_b), \min(t_a, f_b)\} \tag{33}$$

Eq. (33) can be divided into four cases by the conditions.

Case i :

$$if\ f_a \ge t_b\ and\ t_a \ge f_b\ then \sim A \vee B = (f_a, f_b) \tag{34}$$

Case ii :

$$if\ f_a \ge t_b\ and\ t_a < f_b\ then \sim A \vee B = (f_a, t_a) \tag{35}$$

Case iii :

$$if\ f_a < t_b\ and\ t_a \ge f_b\ then \sim A \vee B = (t_b, f_b) \tag{36}$$

Case iv :

$$if\ f_a < t_b\ and\ t_a < f_b\ then \sim A \vee B = (t_b, t_a) \quad (37)$$

These four cases are obtained through analyzing the relative position of the point **A** to the point ~ **B**. (See Fig. 8)
In these, only Case iii is special, because the result of the implication operation is just overlap with **B**.

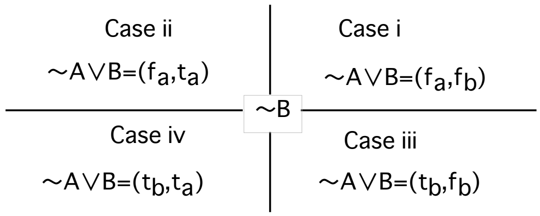

Figure 8: The four cases for analyzing the results of implication operation by the A-IFS model. The figure is showing the relative geometrical position of the point **A** to the point ~**B** for each case.

4.3 Evaluation of the result of implication operations of each model

In summary, the results of implication operations of both IFS and HLS models are expressed together in one figure.

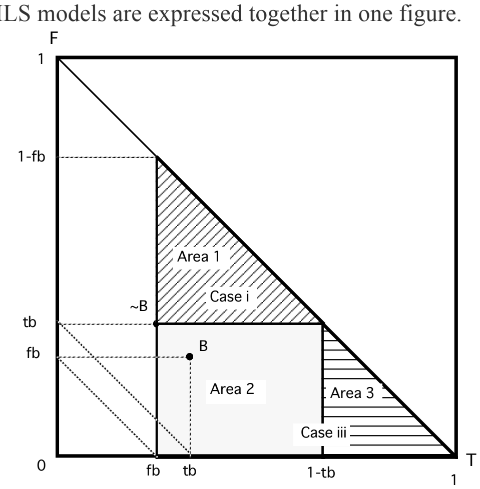

Figure 9: The results of implication operations by both IFS and HLS models

The areas to be analyzed are separated into 3 parts: i.e. Area 1 to Area 3.

Area 1: ¬**A**∨**B**=(*1-ta, fb*) -------- Interval = [1-ta,1-fb] (38)
~**A**∨**B**=(*fa, fb*) --------- Interval = [fa,1-fb] (39)
Area 2: ¬**A**∨**B**=(*1-ta, fb*) ---------Interval = [1-ta,1-fb] (40)
~**A**∨**B**=(*fa, fb*) ------------Interval = [fa,1-fb] (41)
Area 3: ¬**A**∨**B**=(*1-ta, fb*) ---------Interval = [1-ta,1-fb] (42)
~**A**∨**B**=(*tb, fb*) ------------Interval = [tb,1-fb] (43)

In the area 1, the right edges of the intervals have the same value *1-fb* for both models.
But, the left edges of the intervals have different values *1-ta* and *fa* respectively.
As the values *1-ta* and *fa* can vary under the restrictions *ta+fa≤1* and *fa≥tb*.
If and only if, the intervals are the same if *1-ta=fa*.
It is not known which model is better in this area.
In the area 2, the right edge of the interval is the same as bellow.
The left edge of ~ **A** ∨ **B** is a fixed value *tb*, ¬**A**∨**B** is *1-ta*, because Area 2 has a range of areas *fb ≦ ta ≦ 1-tb*, so transform *ta ≦ 1-tb*, obtain *tb-ta ≧ tb*. The interval of ¬ **A** ∨ **B** is narrow.
If *1-ta=tb*, then the range of the intervals are the same.
In the Area3, the results of the implication operations by each model are just the same point **B**.

5. ANALYSYS 2:
The case that the result of the implication operation by the HLS model does not return to the A-IFS area.

In this case, the results of the implication operation belong to different areas by models. If the result of the implication by the HLS model is in the contradiction area, it is out of the framework of the A-IFS model, because it is out of the data area of A-IFS. Commonly considering, it is impossible to compare them directly. On the other hand, from the view point of FCR-method or HLS model, it is not so difficult to compare, because the resultant pair (*t, f*) will be integrated when the result is used in some inference system of application, since the paired data is not so easy to understand or use directly.
So, in this section, the results of operations are compared in one-dimensional criteria by calculating the integrated values. As to calculate the integrated value of the resultant points, the "combined scoring method 1" (symbol I_4) developed for the FCR-method is applied (Refer to Eq. 4).

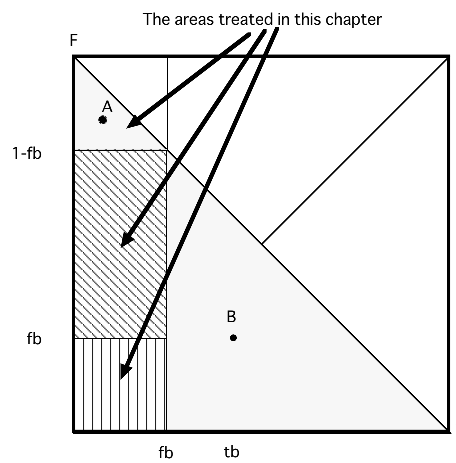

Figure 10: The areas where the results of the implication by HLS do not return to the A-IFS area.

5.1 Comparing the integrated values of the results calculated by each model

For example, when **A** locates at the upper left of **B**, substitute the result of ¬ **A** ∨ **B** in the formula I_3, and substitute the result of ~ **A** ∨ **B** in the formula I_1, then analyze the magnitude relation of the integrated values. The authors illustrate the result of the analysis. (See Fig.11)

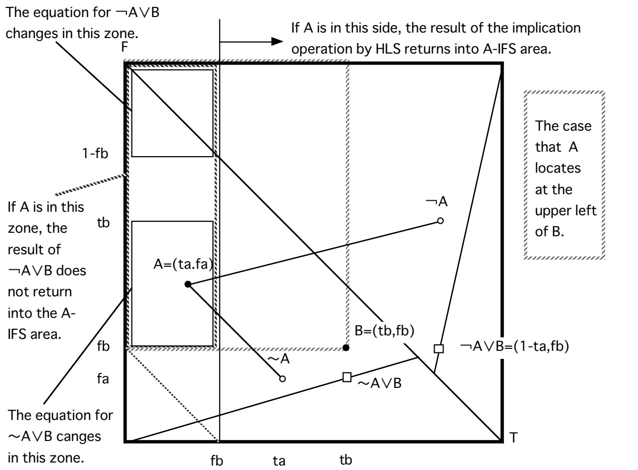

Figure 11: Summary of the results by both models
(The result by HLS does not return to the A-IFS area)

If the point **A** is in one of the three zones, the result of the implication operation does not return to the A-IFS area. The integrated value I_3 is equal to the value I_1 if the point **A** is in the top zone, while the values I_3 and I_1 are not consistent in the middle and the bottom zone. As illustrated in the Fig. 11, I_3 is greater than I_1. As seen as that, the integrated value of ¬ **A** ∨ **B** is greater than ∼**A** ∨ **B**, so the HLS is superior to the A-IFS in this case.

6. CONCLUSION

The HLS model can treat both contradiction area data and the A-IFS area while the A-IFS model precludes the contradiction area data by definition. The authors assumed that even if the original (observed) data sets are within the A-IFS area, if it is permitted to use the outside area of the definition of A-IFS, the result of the implication operation could be better than restricting the calculation space. Under such assumptions, this study analyzed the results of the implication operations by two extended fuzzy logic model, A-IFS and HLS, though both their data area is different. By classifying various cases, it is attempted to compare the results for testing the superiority of the models.

•For defining the implication equation **A** → **B**, adopt

$$(\text{Not } \mathbf{A}) \vee (\mathbf{B}) \; . \tag{44}$$

•For evaluating the result of implication, the same integration formula I_4 is applied.

(The I_4 can be used both the irrelevance area and the contradiction area by its symmetric feature. As it is a surjective function, it can be used for the inverse FCR-method proposed by E.Takahagi [9].)

Assuming that points **A** and **B** are in the A-IFS area, both of the resulting points appear inside and outside of A-IFS area were analyzed. It became clear that the result of HLS model is out of A-IFS area when a point **A** is at the left of a point **B**.

By setting three patterns for the results of implication are out of A-IFS area, followings became clear. In one pattern, the integrated value $I_3 = I_1$, in the other patterns, $I_3 > I_1$. By summarizing all of this investigation, it can be concluded that about the implications, the HLS model is showing superiority to the A-IFS model in most cases. But, in one case, Area 1 illustrated in the Fig. 9, A-IFS model can be superior to the HLS model depending on the values of the parameters of **A** and **B**.

According to the procedures of using the special integration algorithm, Analysis 2 could be seen as an attempt. But, at least Analysis 1 would be enough to demonstrate the usefulness of using the contradiction area for the logical calculation space.

In this paper, the only model compared to the A-IFS model is HLS. Regarding the definitions of the logical operations, logical OR and logical AND operations are the same in both models, but there can be better operations. For example, F. Smarandache [14] introduced the algebraic sum and algebraic product rules for the definitions as the logical OR and logical AND operations of his extended fuzzy/set logic model named "Newtorosophic Set/Logic". The definitions can be applied by using the method used in this paper. In the near future, such an investigation should be planned for exploring and constructing better fuzzy systems.

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