Gödel Platform

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ABSTRACT

This article shows, on a Boolean multivalued logic, a model of logic analysis named "Gödel platform" that evaluates consistency among any knowledges given arbitrarily by a user, recommends improving some knowledges for maintaining consistency, and calculates arbitrary inferences. This article shows also several typical examples that simulate behavior of Gödel platform.

Keywords: Boolean Multivalued Logic, Logic Analysis, Consistency, Inference, Learning.

1. INTRODUCTION

This article shows, on a Boolean multivalued logic, a model of logic analysis that evaluates consistency among any knowledges given arbitrarily by a user, recommends improving some knowledges for maintaining consistency, and calculates arbitrary inferences. In this article, we name the model a "Gödel platform" after Gödel's great works (cf. [1]).

Section 2 defines a Boolean multivalued logic. Section 3 specifies architecture of Gödel platform. Section 4 shows several typical examples that simulate behavior of Gödel platform.

2. BOOLEAN MULTIVALUED LOGIC

This section, in contrast to Boolean binary logics, defines newly a multivalued logic such that the set of logic formulas forms a Boolean algebra. We call it a Boolean multivalued logic, which appears first in [2] and corresponds to a bitwise-processable realization of "complementary fuzzy logic system" (cf. [3]).

Atoms and Logic Operations

Each *atom* takes a *truth value*, that is, a binary number $0.x^{(1)}x^{(2)}\ \cdots\ x^{(n)}{}_{\mathrm{b}}$ ($x^{(1)}\in\{0,1\}$, $x^{(2)}\in\{0,1\}$, $\cdots$, $x^{(n)}\in\{0,1\}$) in fixed-point notation, where n is a constant positive integer called *dimension*. *Logic formulas* are results of applying the following logic operations to atoms and to logic formulas themselves.

The *negation* $\neg x$ of a logic formula $x = 0.x^{(1)}x^{(2)}\cdots x^{(n)}{}_{\mathrm{b}}$ takes the truth value $0.z^{(1)}z^{(2)}\ \cdots\ z^{(n)}{}_{\mathrm{b}}$ consisting of the Boolean binary negation $z^{(i)} = \neg x^{(i)}$ of $x^{(i)}$ for $i = 1, 2, \cdots, n$. For example, the negation of $x = 0.0000\ 0000\ 0000\ 1111_{\mathrm{b}} \approx 0.00023$ is $\neg x = 0.1111\ 1111\ 1111\ 0000_{\mathrm{b}} \approx 0.99977$.

The *conjunction* $x \wedge y$ of logic formulas $x = 0.x^{(1)}x^{(2)}\cdots x^{(n)}{}_{\mathrm{b}}$ and $y = 0.y^{(1)}y^{(2)}\ \cdots\ y^{(n)}{}_{\mathrm{b}}$ takes the truth value $0.z^{(1)}\ z^{(2)}\ \cdots\ z^{(n)}{}_{\mathrm{b}}$ consisting of the Boolean binary conjunction $z^{(i)} = x^{(i)} \wedge y^{(i)}$ of $x^{(i)}$ and $y^{(i)}$ for $i = 1, 2, \cdots, n$. For example, the conjunction of $x = 0.0000\ 0000\ 0000\ 1111_{\mathrm{b}} \approx 0.00022$ and $y = 0.1001\ 1001\ 1001\ 1001_{\mathrm{b}} \approx 0.59999$ is $x \wedge y = 0.0000\ 0000\ 0000\ 1001_{\mathrm{b}} \approx 0.00014$.

Similarly, the *disjunction* $x \vee y$ of x and y takes the truth value $0.z^{(1)}\ z^{(2)}\ \cdots\ z^{(n)}{}_{\mathrm{b}}$ consisting of $z^{(i)} = x^{(i)} \vee y^{(i)}$. The *implication* $x \rightarrow y$ from x to y takes the truth value $0.z^{(1)}z^{(2)}\ \cdots\ z^{(n)}{}_{\mathrm{b}}$ consisting of $z^{(i)} = x^{(i)} \rightarrow y^{(i)}$. The *equivalence* $x \leftrightarrow y$ between x and y takes the truth value $0.z^{(1)}z^{(2)}\ \cdots\ z^{(n)}{}_{\mathrm{b}}$ consisting of $z^{(i)} = x^{(i)} \leftrightarrow y^{(i)}$.

Based on the above definition of logic operations, each logic formula takes a binary number $0.x^{(1)}x^{(2)}\ \cdots\ x^{(n)}{}_{\mathrm{b}}$, that is, one of $0, 2^{-n}, 2{\cdot}2^{-n}, 3{\cdot}2^{-n}, \cdots, 1{-}2{\cdot}2^{-n}, 1{-}2^{-n}$ that are reals extracted from $[0, 1)$ with interval 2^{-n}. Here, the maximum $1_n = 1 - 2^{-n}$ of truth values approaches 1 by increasing n, e. g., $1_{16} = 0.9999847412109375$.

Boolean Properties

Since each of Boolean binary conjunction and disjunction satisfies a commutative law, each of multivalued conjunction $\wedge$ and disjunction $\vee$ satisfies also a *commutative law*. Since each of Boolean binary conjunction and disjunction satisfies an associative law, each of multivalued conjunction $\wedge$ and disjunction $\vee$ satisfies also an *associative law*. Since Boolean binary conjunction and disjunction satisfy absorption laws, multivalued conjunction $\wedge$ and disjunction $\vee$ satisfy also *absorption laws*. Thus, the universal set of logic formulas forms a *lattice* with meet $\wedge$ and join $\vee$.

Further, since $x \wedge \neg x = 0.00 \cdots\ 0_{\mathrm{b}} = 0$ and $x \vee \neg x = 0.11 \cdots\ 1_{\mathrm{b}} = 1_n$ for an arbitrary logic formula x and its complement $\neg x$, the universal set of logic formulas forms a *complemented lattice* with identity elements 0 and 1_n.

Since Boolean binary conjunction and disjunction satisfy distributive laws, multivalued conjunction $\wedge$ and disjunction $\vee$ satisfy also *distributive laws*. Thus, the universal set of logic formulas forms a *distributive lattice*.

Since the universal set of logic formulas is both a complemented lattice and a distributive lattice, it forms a *Boolean algebra*.

Bayesian Conditional

For efficient handling of Bayesian theories (cf. [4]), this article defines an arithmetic operation called *conditional* from a logic formula x to another logic formula y in the following way:

$$y \mid x \ = \ \frac{x \wedge y}{x} \,. \tag{1}$$

3. ARCHITECTURE OF GÖDEL PLATFORM

Gödel platform basesed on a Boolean multivalued logic consists of four modes bridged by a "Home" (cf. Fig. 1).

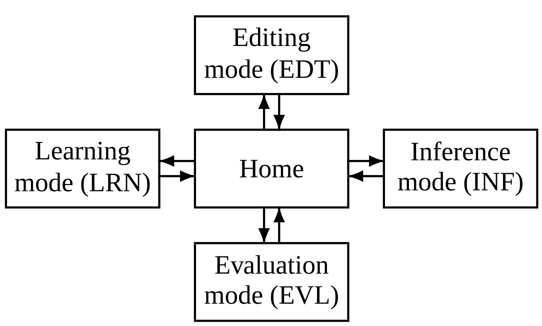

Fig. 1. Architecture of Gödel platform.

Specification of Editing Mode

This article defines a *knowledge* $x \equiv r$ as a constraint such that: the left-hand side x is a logic formula; the right-hand side r is a desired truth value in $[0,1]$; both sides are connected by "$\equiv$" meaning that x should take r.

For example, we consider the following set of eight knowledges consisting of seven atoms, that is, *falcon*, *pigeon*, *penguin*, *bird*, *fly*, *reflex*, and *bicycle*:

$$\textit{bird} \mid \textit{falcon} \equiv 1 \,, \tag{2}$$
$$\textit{fly} \mid \textit{falcon} \equiv 0.9 \,, \tag{3}$$
$$\textit{bird} \mid \textit{pigeon} \equiv 1 \,, \tag{4}$$
$$\textit{fly} \mid \textit{pigeon} \equiv 0.9 \,, \tag{5}$$
$$\textit{reflex} \mid \textit{fly} \equiv 0.75 \,, \tag{6}$$
$$\textit{bird} \mid \textit{penguin} \equiv 1 \,, \tag{7}$$
$$\textit{fly} \mid \textit{penguin} \equiv 0 \,, \tag{8}$$
$$\textit{bicycle} \mid \textit{falcon} \equiv 0 \,. \tag{9}$$

Semantic interpretation of (2)–(9) is as follows. Knowledge (2) means that if one is a falcon, it is absolutely a bird, where adverb "absolutely" means that the truth value is 1. Knowledge (3) means that if one is a falcon, it nearly flies, where adverb "nearly" means that the truth value is 0.9. (An injured falcon may not fly.) Knowledge (4) means that if one is a pigeon, it is absolutely a bird. Knowledge (5) means that if one is a pigeon, it nearly flies. Knowledge (6) means that if one flies, it possibly has good reflexes, where adverb "possibly" means that the truth value is 0.75. Knowledge (7) means that if one is a penguin, it is absolutely a bird. Knowledge (8) means that if one is a penguin, it never flies, where adverb "never" means that the truth value is 0. Knowledge (9) means that if one is a falcon, it never rides a bicycle.

Thus, "Editing mode (EDT)" is a mode in which a user can input and update knowledges.

Specification of Learning Mode

Temporarily supposing a set of truth values of atoms, the truth value of an arbitrary logic formula can be calculated, and hence the truth value of x for each knowledge $x \equiv r$ can be calculated. The gap is the absolute value $|x - r|$ of the subtraction of the desired truth value r from the calculated truth value of x, which should be as small as possible. *Learning* is the procedure of searching for the truth values of atoms so as to minimize the gaps concerning all knowledges. It is a kind of mathematical programming problem, to which algorithms of linear programming [5] (that is, more precisely, algorithms of integer programming) or other methods are applicable.

The following shows the truth values of atoms obtained by a trial-and-error method that tries to minimize locally the deviation of gaps concerning all knowledges (2)–(9):

$$\textit{falcon} = 0.0101\ 1000\ 1100\ 0111_{\mathrm{b}} \,, \tag{10}$$
$$\textit{pigeon} = 0.0101\ 0111\ 1111\ 1111_{\mathrm{b}} \,, \tag{11}$$
$$\textit{penguin} = 0.0000\ 1010\ 1010\ 0000_{\mathrm{b}} \,, \tag{12}$$
$$\textit{bird} = 0.0101\ 1111\ 1111\ 1011_{\mathrm{b}} \,, \tag{13}$$
$$\textit{fly} = 0.1111\ 0000\ 0000\ 0000_{\mathrm{b}} \,, \tag{14}$$
$$\textit{reflex} = 0.1011\ 0010\ 1001\ 1001_{\mathrm{b}} \,, \tag{15}$$
$$\textit{bicycle} = 0.1010\ 0010\ 0011\ 1011_{\mathrm{b}} \,. \tag{16}$$

In this case, the calculated truth values of logic formulas (2)–(9) are as follows, where the slant numerals denote the desired truth values:

$$\begin{aligned}
\textit{bird} \mid \textit{falcon} &\approx 0.99982 = \mathit{1} - 0.00018\,, && (17)\\
\textit{fly} \mid \textit{falcon} &\approx 0.90114 = \mathit{0.9} + 0.00114\,, && (18)\\
\textit{bird} \mid \textit{pigeon} &\approx 0.99982 = \mathit{1} - 0.00018\,, && (19)\\
\textit{fly} \mid \textit{pigeon} &\approx 0.90913 = \mathit{0.9} + 0.00913\,, && (20)\\
\textit{reflex} \mid \textit{fly} &\approx 0.73334 = \mathit{0.75} - 0.01666\,, && (21)\\
\textit{bird} \mid \textit{penguin} &= 1 = \mathit{1} - 0\,, && (22)\\
\textit{fly} \mid \textit{penguin} &= 0 = \mathit{0} + 0\,, && (23)\\
\textit{bicycle} \mid \textit{falcon} &\approx 0.00014 = \mathit{0} + 0.00014\,. && (24)
\end{aligned}$$

Here, the deviation of gaps is around 0.00673 and the maximum gap is around $|-0.01666|$ caused by the 5th knowledge (21).

Thus, "Learning mode (LRN)" is a mode that optimizes the truth values of atoms.

Specification of Evaluation Mode
"Evaluation mode (EVL)" is a mode that, given atoms with some truth values, simply calculates the deviation of gaps and the maximum gap without optimizing the truth values of atoms.

Note that this mode is a part of "Learning mode".

Specification of Inference Mode
After the learning, that is, after optimizing the truth values of atoms, we can calculate the truth value of an arbitrary logic formula. *Inferences* are procedures of simply calculating the truth values of logic formulas after the learning. (Note that inferences on many logic systems, i. e. Prolog, need complex procedures.)

An example based on atoms (10)–(16) is

$$\begin{aligned}
&\textit{fly} \mid \textit{bicycle}\\
&= 0.1111\,0000\,0000\,0000_{\mathrm{b}} \mid 0.1010\,0010\,0011\,1011_{\mathrm{b}}\\
&= \frac{0.1010\,0010\,0011\,1011_{\mathrm{b}} \wedge 0.1111\,0000\,0000\,0000_{\mathrm{b}}}{0.1010\,0010\,0011\,1011_{\mathrm{b}}}\\
&= \frac{0.1010\,0000\,0000\,0000_{\mathrm{b}}}{0.1010\,0010\,0011\,1011_{\mathrm{b}}} \approx 0.98625\,, \qquad (25)
\end{aligned}$$

which means that if one rides a bicycle, it almost flies. (However, we may feel somewhat strangeness in common sense. Indeed, the knowledge currently given concerning the bicycle is too insufficient, that is, only (9) meaning that if one is a falcon, it never rides a bicycle.)

Another example is

$$\begin{aligned}
&\neg \textit{bird} \wedge \textit{reflex}\\
&= \neg 0.0101\,1111\,1111\,1011_{\mathrm{b}} \wedge 0.1011\,0010\,1001\,1001_{\mathrm{b}}\\
&= 0.1010\,0000\,0000\,0100_{\mathrm{b}} \wedge 0.1011\,0010\,1001\,1001_{\mathrm{b}}\\
&= 0.1010\,0000\,0000\,0000_{\mathrm{b}}\,, \qquad (26)
\end{aligned}$$

hence

$$\begin{aligned}
&\textit{bicycle} \mid (\neg \textit{bird} \wedge \textit{reflex})\\
&= 0.1010\,0010\,0011\,1011_{\mathrm{b}} \mid 0.1010\,0000\,0000\,0000_{\mathrm{b}}\\
&= \frac{0.1010\,0000\,0000\,0000_{\mathrm{b}} \wedge 0.1010\,0010\,0011\,1011_{\mathrm{b}}}{0.1010\,0000\,0000\,0000_{\mathrm{b}}}\\
&= \frac{0.1010\,0000\,0000\,0000_{\mathrm{b}}}{0.1010\,0000\,0000\,0000_{\mathrm{b}}} = 1\,, \qquad (27)
\end{aligned}$$

which means that if one is no bird and has good reflexes, it absolutely rides a bicycle. (We may feel little strangeness in metacognition of "one" as a human.)

Thus, "Inference mode (INF)" is a mode that calculates instantaneously the truth value of a logic formula given arbitrarily by a user.

4. SIMULATION OF GÖDEL PLATFORM

This section shows several typical examples that simulate behavior of Gödel platform.

In Case of Consistent Knowledges
As mentioned in the subsection "Specification of Learning Mode" in Section 3, if knowledges (2)–(9) are given, a result of learning is as follows: The optimized truth values of atoms are (10)–(16), for which we have (17)–(24), where the deviation of gaps is around 0.00673 and the maximum gap is around $|-0.01666|$ caused by the 5th knowledge (21). Thus, both the deviation and the maximum are quite small, which means that the knowledges are consistent.

An example of inference based on atoms (10)–(16) is (25), which means that if one rides a bicycle, it almost flies. Another example is (27), which means that if one is no bird and has good reflexes, it absolutely rides a bicycle.

Scenario A: A scenario simulating the conversation between a computer and a user that reflects the above situation is as follows. (Such a conversation system is partially realized in Japanese language at present.)

> Input knowledges in succession.
EDT: Input, please.
> If one is a falcon, it is absolutely a bird. Continue.
> If one is a falcon, it nearly flies. Continue.
> If one is a pegion, it is absolutely a bird. Continue.
> If one is a pegion, it nearly flies. Continue.
> If one flies, it possibly has good reflexes. Continue.
> If one is a penguin, it is absolutely a bird. Continue.
> If one is a penguin, it never flies. Continue.
> If one is a falcon, it never rides a bicycle.
EDT: Accepted eight knowledges.
> Display all knowledges.
EDT:
1: If one is a falcon, it is a bird with truth value 1.
2: If one is a falcon, it flies with truth value 0.9.
3: If one is a pegion, it is a bird with truth value 1.

4: If one is a pegion, it flies with truth value 0.9.
5: If one flies, it has good reflexes with truth value 0.75.
6: If one is a penguin, it is a bird with truth value 1.
7: If one is a penguin, it flies with truth value 0.
8: If one is a falcon, it rides a bicycle with truth value 0.
> Learn.
LRN: Learning..........completed.
> Evaluate.
EVL: The knowledges are consistent.
> Detail.
EVL: The deviation of gaps is around 0.00673. The maximum gap is around $|-0.01666|$ caused by the 5th knowledge. Both the deviation and the maximum are quite small.
> Infer that if one rides a bicycle, it flies.
INF: If one rides a bicycle, it flies with truth value around 0.98625.
> Infer that if one is no bird and has good reflexes, it rides a bicycle.
INF: If one is no bird and has good reflexes, it rides a bicycle with truth value 1.

In Case of Inconsistent Knowledges

Let us suppose that a new knowledge concerning the bicycle is given:

$$\textit{bicycle} \mid \textit{reflex} \;\equiv\; 0.75\,, \tag{28}$$

which means that if one has good reflexes, it possibly rides a bicycle. If it is added to the current knowledges (2)–(9), a result of learning is as follows:

$$\begin{aligned}
\textit{falcon} &= 0.0100\,0111\,0110\,0010_{\mathrm{b}}\,, & (29)\\
\textit{pigeon} &= 0.1100\,1011\,0101\,1101_{\mathrm{b}}\,, & (30)\\
\textit{penguin} &= 0.0010\,0110\,0111\,1010_{\mathrm{b}}\,, & (31)\\
\textit{bird} &= 0.1110\,1111\,1110\,1011_{\mathrm{b}}\,, & (32)\\
\textit{fly} &= 0.1101\,0000\,1010\,1001_{\mathrm{b}}\,, & (33)\\
\textit{reflex} &= 0.1001\,1111\,1111\,1111_{\mathrm{b}}\,, & (34)\\
\textit{bicycle} &= 0.1000\,1000\,0000\,0000_{\mathrm{b}}\,, & (35)
\end{aligned}$$

for which we have

$$\begin{aligned}
\textit{bird} \mid \textit{falcon} &= 1 = \mathit{1} - 0\,, & (36)\\
\textit{fly} \mid \textit{falcon} &\approx 0.89833 = \mathit{0.9} - 0.00167\,, & (37)\\
\textit{bird} \mid \textit{pigeon} &\approx 0.99962 = \mathit{1} - 0.00038\,, & (38)\\
\textit{fly} \mid \textit{pigeon} &\approx 0.94429 = \mathit{0.9} + 0.04429\,, & (39)\\
\textit{reflex} \mid \textit{fly} &\approx 0.69328 = \mathit{0.75} - 0.05672\,, & (40)\\
\textit{bird} \mid \textit{penguin} &\approx 0.99838 = \mathit{1} - 0.00162\,, & (41)\\
\textit{fly} \mid \textit{penguin} &\approx 0.00406 = \mathit{0} + 0.00406\,, & (42)\\
\textit{bicycle} \mid \textit{falcon} &= 0 = \mathit{0} + 0\,, & (43)\\
\textit{bicycle} \mid \textit{reflex} &\approx 0.85002 = \mathit{0.75} + 0.10002\,, & (44)
\end{aligned}$$

where the deviation of gaps is around

$$0.04110 \tag{45}$$

and the maximum gap is around

$$|+0.10002| \tag{46}$$

caused by the 9th knowledge (44). Thus, the maximum is slightly large, which means that *the knowledges include some inconsistency.*

An example of inference based on atoms (29)–(35) is

$$\textit{fly} \mid \textit{bicycle} \;=\; 0.94118\,, \tag{47}$$

which means that if one rides a bicycle, it almost flies. (Note that addition of the new knowledge (28) has not effected reduction of strangeness in comparison with (25).)

Scenario B (followed Scenario A): A scenario simulating the conversation between a computer and a user that reflects the above situation is as follows.

> Add a new knowledge.
EDT: Input, please.
> If one has good reflexes, it possibly rides a bicycle.
EDT: Accepted nine knowledges.
> Display all knowledges.
EDT:
1: If one is a falcon, it is a bird with truth value 1.
2: If one is a falcon, it flies with truth value 0.9.
3: If one is a pegion, it is a bird with truth value 1.
4: If one is a pegion, it flies with truth value 0.9.
5: If one flies, it has good reflexes with truth value 0.75.
6: If one is a penguin, it is a bird with truth value 1.
7: If one is a penguin, it flies with truth value 0.
8: If one is a falcon, it rides a bicycle with truth value 0.
9: If one has good reflexes, it rides a bicycle with truth value 0.75.
> Learn.
LRN: Learning..........completed.
> Evaluate.
EVL: The knowledges are slightly inconsistent.
> Detail.
EVL: The deviation of gaps is around 0.04110. The maximum gap is around $|+0.10002|$ caused by the 9th knowledge. The maximum exceeds a warning threshold 0.1.
> Infer that if one rides a bicycle, it flies.
INF: If one rides a bicycle, it flies with truth value around 0.94118. *Note that this inference is unreliable because the knowledges include some inconsistency.*

A Possibility of Improving Consistency

In the current knowledges (2)–(9) and (28), let us suppose that the 9th knowledge (28) corresponding to the cause (44) of the maximum gap is replaced with

$$\textit{bicycle} \mid \textit{reflex} \;\equiv\; 0.85\,, \tag{48}$$

which means that if one has good reflexes, it rides a bicycle with truth value 0.85. (*Note that a computer always can recommend improving the knowledge that corresponds to the cause of maximum gap.*) Then, for the current atoms (29)–(35), we have

$$\begin{aligned} \textit{bird} \mid \textit{falcon} \;&=\; 1 \;=\; \mathit{1} - 0\,, & (49)\\ \textit{fly} \mid \textit{falcon} \;&\approx\; 0.89833 \;=\; \mathit{0.9} - 0.00167\,, & (50)\\ \textit{bird} \mid \textit{pigeon} \;&\approx\; 0.99962 \;=\; \mathit{1} - 0.00038\,, & (51)\\ \textit{fly} \mid \textit{pigeon} \;&\approx\; 0.94429 \;=\; \mathit{0.9} + 0.04429\,, & (52)\\ \textit{reflex} \mid \textit{fly} \;&\approx\; 0.69328 \;=\; \mathit{0.75} - 0.05672\,, & (53)\\ \textit{bird} \mid \textit{penguin} \;&\approx\; 0.99838 \;=\; \mathit{1} - 0.00162\,, & (54)\\ \textit{fly} \mid \textit{penguin} \;&\approx\; 0.00406 \;=\; \mathit{0} + 0.00406\,, & (55)\\ \textit{bicycle} \mid \textit{falcon} \;&=\; 0 \;=\; \mathit{0} + 0\,, & (56)\\ \textit{bicycle} \mid \textit{reflex} \;&\approx\; 0.85002 \;=\; \mathit{0.85} + 0.00002\,, & (57) \end{aligned}$$

where the deviation of gaps is around 0.02404 and the maximum gap is around $|-0.05672|$ caused by the 5th knowledge (53). Thus, the deviation and the maximum decrease to around fifty percent of (45) and (46), respectively.

Scenario C (followed Scenario B): A scenario simulating the conversation between a computer and a user that reflects the above situation is as follows.

> Recommend.
EVL: The 9th knowledge should be replaced with: If one has good reflexes, it rides a bicycle with truth value 0.85.
> Accept the recommendation.
EDT: Accepted nine knowledges.
> Display all knowledges.
EDT:
1: If one is a falcon, it is a bird with truth value 1.
2: If one is a falcon, it flies with truth value 0.9.
3: If one is a pegion, it is a bird with truth value 1.
4: If one is a pegion, it flies with truth value 0.9.
5: If one flies, it has good reflexes with truth value 0.75.
6: If one is a penguin, it is a bird with truth value 1.
7: If one is a penguin, it flies with truth value 0.
8: If one is a falcon, it rides a bicycle with truth value 0.
9: If one has good reflexes, it rides a bicycle with truth value 0.85.
> Evaluate.
EVL: The knowledges are consistent.
> Detail.
EVL: The deviation of gaps is around 0.02404. The maximum gap is around $|-0.05672|$ caused by the 5th knowledge. Both the deviation and the maximum decreased to around fifty percent.

Another Possibility of Improving Consistency

By replacing the 9th knowledge (28) with new (48), both the deviation and the maximum decreased. Though this is rational, *as another possibility we, enlightened by the computer's recommendation (48), may notice that* the relation among *reflex*, *bicycle* and also *bird* should be given explicitly. Thus, the following is an alternative approach to improving consistency instead of replacing simply the 9th knowledge (28) with (48).

In the current knowledges (2)–(9) and (28), let us suppose that the 9th knowledge (28) is replaced with

$$\textit{bicycle} \mid (\neg\, \textit{bird} \wedge \textit{reflex}) \;\equiv\; 0.75 \qquad (58)$$

in order to correct directly the result (27) of inference, which means that if one is no bird (that is, a human) and has good reflexes, it possibly rides a bicycle. (*Note that the computer cannot recommend automatically such radical improvement called an "abduction" in the field of artificial intelligence.*) Then, a result of learning is as follows:

$$\begin{aligned} \textit{falcon} \;&=\; 0.0010\;0000\;1010\;1110_{\mathrm{b}}\,, & (59)\\ \textit{pigeon} \;&=\; 0.1001\;0110\;1011\;0100_{\mathrm{b}}\,, & (60)\\ \textit{penguin} \;&=\; 0.0101\;1010\;1000\;1010_{\mathrm{b}}\,, & (61)\\ \textit{bird} \;&=\; 0.1111\;1110\;0000\;0000_{\mathrm{b}}\,, & (62)\\ \textit{fly} \;&=\; 0.1010\;0110\;0100\;0000_{\mathrm{b}}\,, & (63)\\ \textit{reflex} \;&=\; 0.1100\;0101\;1101\;1111_{\mathrm{b}}\,, & (64)\\ \textit{bicycle} \;&=\; 0.1101\;1111\;0111\;1111_{\mathrm{b}}\,, & (65) \end{aligned}$$

for which we have

$$\begin{aligned} \textit{bird} \mid \textit{falcon} \;&\approx\; 0.97920 \;=\; \mathit{1} - 0.02080\,, & (66)\\ \textit{fly} \mid \textit{falcon} \;&\approx\; 0.97920 \;=\; \mathit{0.9} + 0.07920\,, & (67)\\ \textit{bird} \mid \textit{pigeon} \;&\approx\; 0.99533 \;=\; \mathit{1} - 0.00467\,, & (68)\\ \textit{fly} \mid \textit{pigeon} \;&\approx\; 0.88916 \;=\; \mathit{0.9} - 0.01084\,, & (69)\\ \textit{reflex} \mid \textit{fly} \;&\approx\; 0.79549 \;=\; \mathit{0.75} + 0.04549\,, & (70)\\ \textit{bird} \mid \textit{penguin} \;&\approx\; 0.99405 \;=\; \mathit{1} - 0.00595\,, & (71)\\ \textit{fly} \mid \textit{penguin} \;&\approx\; 0.02209 \;=\; \mathit{0} + 0.02209\,, & (72)\\ \textit{bicycle} \mid \textit{falcon} \;&\approx\; 0.00549 \;=\; \mathit{0} + 0.00549\,, & (73)\\ \textit{bicycle} \mid (\neg\, \textit{bird} \wedge \textit{reflex}) & \\ &\approx\; 0.73277 \;=\; \mathit{0.75} - 0.01723\,, & (74) \end{aligned}$$

where the deviation of gaps is around

$$0.03294 \qquad (75)$$

and the maximum gap is around

$$|+0.07920| \qquad (76)$$

caused by the 2nd knowledge (67). Thus, both the deviation and the maximum are small, which means that the knowledges are consistent.

An example of inference based on atoms (59)–(65) is

$$\textit{fly} \mid \textit{bicycle} \;=\; 0.60068\,, \qquad (77)$$

which means that if one rides a bicycle, it perhaps, that is, in the sense of neutral, flies. (Note that the strangeness becomes little in comparison with (47), which is a secondary effect of (58).)

Scenario C' (instead of Scenario C followed Scenario B): A scenario instead of Scenario C simulating the conversation between a computer and a user that reflects the above situation is as follows.

> Recommend.

EVL: The 9th knowledge should be replaced with: If one has good reflexes, it rides a bicycle with truth value 0.85.
> Replace the 9th knowledge.
EDT: Input, please.
> If one is no bird and has good reflexes, it possibly rides a bicycle.
EDT: Accepted nine knowledges.
> Display all knowledges.
EDT:
1: If one is a falcon, it is a bird with truth value 1.
2: If one is a falcon, it flies with truth value 0.9.
3: If one is a pegion, it is a bird with truth value 1.
4: If one is a pegion, it flies with truth value 0.9.
5: If one flies, it has good reflexes with truth value 0.75.
6: If one is a penguin, it is a bird with truth value 1.
7: If one is a penguin, it flies with truth value 0.
8: If one is a falcon, it rides a bicycle with truth value 0.
9: If one is no bird and has good reflexes, it rides a bicycle with truth value 0.75.
> Learn.
LRN: Learning..........completed.
> Evaluate.
EVL: The knowledges are consistent.
> Detail.
EVL: The deviation of gaps is around 0.03294. The maximum gap is around $|+0.07920|$ caused by the 2nd knowledge. Both the deviation and the maximum are small.
> Infer that if one rides a bicycle, it flies.
INF: If one rides a bicycle, it flies with truth value around 0.60068.

Improving Consistency
In the current knowledges (2)–(9) and (58), let us suppose that the 2nd knowledge (3) corresponding to the cause (67) of the maximum gap is replaced with

$$\mathit{fly} \mid \mathit{falcon} \;\equiv\; 1\,, \tag{78}$$

which means that if one is a falcon, it flies absolutely. Then, for the current atoms (59)–(65), we have

$$\begin{aligned}
\mathit{bird} \mid \mathit{falcon} &\approx 0.97920 = \mathit{1} - 0.02080\,, && (79)\\
\mathit{fly} \mid \mathit{falcon} &\approx 0.97920 = \mathit{1} - 0.02080\,, && (80)\\
\mathit{bird} \mid \mathit{pigeon} &\approx 0.99533 = \mathit{1} - 0.00467\,, && (81)\\
\mathit{fly} \mid \mathit{pigeon} &\approx 0.88916 = \mathit{0.9} - 0.01084\,, && (82)\\
\mathit{reflex} \mid \mathit{fly} &\approx 0.79549 = \mathit{0.75} + 0.04549\,, && (83)\\
\mathit{bird} \mid \mathit{penguin} &\approx 0.99405 = \mathit{1} - 0.00595\,, && (84)\\
\mathit{fly} \mid \mathit{penguin} &\approx 0.02209 = \mathit{0} + 0.02209\,, && (85)\\
\mathit{bicycle} \mid \mathit{falcon} &\approx 0.00549 = \mathit{0} + 0.00549\,, && (86)\\
\mathit{bicycle} \mid (\neg\,\mathit{bird} \wedge \mathit{reflex}) & \\
&\approx 0.73277 = \mathit{0.75} - 0.01723\,, && (87)
\end{aligned}$$

where the deviation of gaps is around 0.02088 and the maximum gap is around $|+0.04549|$ caused by the 5th knowledge (83). Thus, the deviation and the maximum decrease to around sixty percent of (75) and (76), respectively.

Scenario D (followed Scenario C'): A scenario simulating the conversation between a computer and a user that reflects the above situation is as follows.

> Recommend.
EVL: The 2nd knowledge should be replaced with: If one is a falcon, it flies with truth value 1.
> Accept the recommendation.
EDT: Accepted nine knowledges.
> Display all knowledges.
EDT:
1: If one is a falcon, it is a bird with truth value 1.
2: If one is a falcon, it flies with truth value 1.
3: If one is a pegion, it is a bird with truth value 1.
4: If one is a pegion, it flies with truth value 0.9.
5: If one flies, it has good reflexes with truth value 0.75.
6: If one is a penguin, it is a bird with truth value 1.
7: If one is a penguin, it flies with truth value 0.
8: If one is a falcon, it rides a bicycle with truth value 0.
9: If one is no bird and has good reflexes, it rides a bicycle with truth value 0.75.
> Evaluate.
EVL: The knowledges are consistent.
> Detail.
EVL: The deviation of gaps is around 0.02088. The maximum gap is around $|+0.04549|$ caused by the 5th knowledge. Both the deviation and the maximum decreased to around sixty percent.

5. CONCLUSION

This article showed a model of logic analysis on a Boolean multivalued logic, and named the model Gödel platform. For example, the model will be applicable directly to designing rule-bases for high-quality expert systems, or widely to managing logical consistency on various knowledge-processing systems.

A future subject is to find several approaches to automatic recommendation (including “abduction”) of improving knowledges for maintaining consistency.

6. REFERENCES

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