Detection of Locally Stationary Region for Universal GMM and its Application in denoising X-ray CT Images

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Abstract—An adaptive Wiener filter (AWF) for denoising X-ray CT image has been proposed based on the universal Gaussian mixture distribution model (UNI-GMM). The universal model can be estimated by an assumption that the GMM is stationary. In the previous UNI-GMM-AWF method, a fixed observation block size of UNI-GMM has been adopted, assuming smaller block size makes the block more stationary, but the small block tend to suffer observation error due to image noise. Thus in the previous method, the observation region size was not small enough to satisfy the stationary assumption. Inversely the observation region size is not large enough for precise model detection and high denoising ability in stationary region. To overcome the problems, variable observation block sizes of the UNI-GMMs are adopted in this paper. Actually, in the new UNI-GMM-AWF method, two sizes of the UNI-GMMs are applied for each observation region and the most stationary UNI-GMM for each observation region is selected according to the normalized likelihood function, related to the Akaike’s information criteria (AIC). Moreover, the new UNI-GMM which has a observation region with hole in its central region is applied to detect a small point shape structure like a small vessel or a bronchiole. Then the new UNI-GMM using observation region with hole is also selected for each observation block based on the AIC. Simulation results show that the proposed method performs better than median filter as a standard method in terms of the denoising and point like shadow preservation ability. Furthermore a simulation result shows that the new UNI-GMM-AWF is more flexible than the previous UNI-GMM-AWF method in terms of the applicability of fitting the stationary model.

I. INTRODUCTION

Reduction of the patient dose unfortunately degrades the quality of medical X-ray CT Images, because the signal to noise ratio (SNR) on projection data, called sinogram, decrease. In medical examination, the slice thicknesses adjust the resolution of target organs. The thinner slice X-ray CT images provides the higher resolutonal interpretations of small objects, decreasing partial volume effect, ex. peripheral blood vessels. In spite of the advantage of the thin slice CT scan, it degrades reconstructed images by the noise appeared on the sinogram. Thus the denoising medical X-ray CT images contributes to not only patient dose reduction but also image quality improvement.

The noise in the X-ray CT image appears with the fluctuation of incident X-ray photon and the linear attenuation coefficients of the objects. Using incomplete filtered back projection (FBP) method to such fluctuated and finite resolution projection data generates visible striped pattern. In the X-ray CT images reconstructed by such incomplete FBP method, noise images take a variety of forms by superposition of the striped patterns [2], [3], [4]. Thus the noise on the X-ray CT images are non-stationary and non-Gaussian, due to the variation of amplitude which varies with linear attenuation coefficients at non-stationary objects.

In the case of white noise removal for ordinary images, an adaptive Wiener filter based on a universal Gaussian mixture distribution model (UNI-GMM) has been proposed as a minimum mean square error (MMSE) filter [5] which is known to be global optimum filter including non-linear filters. In this method, an image is divided into small blocks and each block is classified into one of the Gaussian stationary process in the UNI-GMM, assuming that the smaller block is more stationary.

In the previous UNI-GMM-AWF method [6], the size of observation region of UNI-GMM has been fixed for simplicity. Thus the method failed into two problems. First, the observation region size is not large enough for precise model detection and high denoising ability in stationary region. Second, the observation region size is not small enough for detecting non-stationary region, e.g. cross-section of bronchioles and vessels. In this paper, a new UNI-GMM-AWF for denoising X-ray CT images is proposed to improve the fitness of locally stationary CT GMM assumption by applying a set of UNI-GMM in various observation region size. In the proposed method, each image block signal is restored using a Wiener filter on the most stationary UNI-GMM, selected from the set of UNI-GMMs. As a parameter to detect the most stationary UNI-GMM, proposed method introduces likelihood function normalized by the size of observation region, which is related to the Akaike’s information criteria (AIC). Simulation result shows that the proposed model performs better than the conventional model. It also shows that the normalized likelihood criteria can be
flexibly applicable to the variably shaped observation detection such as block with hole.

II. PRINCIPLE

In this section, the principle of UNI-GMM-AWF method is reviewed, in the case of white noise removal on ordinary image.

A. UNI-GMM

Fig.1 illustrates the UNI-GMM. In this figure, \( x \) denotes the vector of local image signal which has probability density function (PDF) \( p(x) \), \( s_k \) denotes the \( k \)'th Gaussian stationary process which has PDF \( p(x|s_k) \) and \( P(s_k) \) denotes a priori probability or mixture weight of \( s_k \).

When an image is divided into small blocks, it is known that the non-stationary statistics of images decreases. This property of ordinary images makes the stationary UNI-GMM profitable \([5]\), \([7]\). In order to estimate the stationary model, the UNI-GMM employs discrete cosine transform (2-D DCT) AC coefficients as characteristics vector and it is assumed that their covariance matrix is diagonal. Under these assumptions, the PDF of 2-D DCT vector \( \nu \) belongs to \( s_k \) is modeled as follows,

\[
p(\nu|s_k) = N[\nu : 0, \Lambda_k]; \quad k = 1, 2, \ldots, K,
\]

where \( N[\nu : \mu, \Sigma] \) denotes the value of Gaussian PDF whose mean vector \( \mu \) and covariance matrix \( \Sigma \) evaluated at \( \mu \). Note that the mean vector is assumed to be zero. In the case of white noise removal for ordinary images, a local image vector \( y \) in observed image is modeled as original image vector \( x \) plus stationary Gaussian noise vector \( n \).

\[
y = x + n
\]

(2)

Because \( x \) and \( n \) are uncorrelated, PDF of 2-D DCT AC vector \( \zeta \) of \( y \) can be modeled as follows,

\[
p(\zeta|s_k) = N[\zeta : 0, \Lambda_k + I_{n^2}]; \quad k = 1, 2, \ldots, K,
\]

where \( \sigma_n^2 \) denotes noise variance and \( I \) denote identity matrix.

B. UNI-GMM-AWF

It is known that the MMSE estimate \( \hat{x}_{\text{MMSE}} \) that minimize mean square error \( E[\|x - \hat{x}\|^2] \) is reduced to the Wiener estimate \( \hat{x}_{\text{WF}} \) in Gaussian process. Thus using the finite UNI-GMM, illustrated in Fig.1, \( \hat{x}_{\text{MMSE}} \) can be estimated by \( \hat{x}_{\text{WF}} \) for each Gaussian process \( s_k \) \([5]\). Because the UNI-GMM models the statistics of local image blocks as Gaussian stationary processes, \( \hat{x}_{\text{WF}} \) can be estimated using finite impulse response filter whose support region \( S \) is illustrated in Fig.2.

In Fig.2, \( B \) and \( OB \) denote block and its observation block, and \( N \) and \( M \) denote sizes of \( B \) and \( OB \), respectively. It is shown that the OB covers all support regions for pixels in \( B \) to observe sufficient statistics for \( B \).

The UNI-GMM-AWF coefficients vector \( a_k \) for each class \( s_k \) is estimated under the constraint that the sum of all coefficients is 1, in order to preserve local average of image as follows,

\[
a_k = C^{-1}_k c_k - \frac{C^{-1}_k 1^T C^{-1}_k}{1^T C_k^{-1} 1} (1^T C_k^{-1} c_k - 1),
\]

(4)

where \( .^T \) denotes transpose of \( . \), \( -1 \) denotes inverse of \( . \), \( C \) denotes the covariance matrix of the vector \( y_S \) on the support region \( S \) in observed image, \( c \) denotes the cross covariance vector between original image signal \( x \) and its corresponding \( y_S \), and \( I \) denotes the vector whose all elements are 1.

The restored signal \( \hat{x} \) is estimated by convolving the UNI-GMM-AWF coefficient \( a_k \) with the filter support vector in the observation image \( x_S \) as follows,

\[
\hat{x} = a_k^T x_S
\]

(5)

C. Issues in previous UNI-GMM-AWF method

The previous UNI-GMM-AWF method had two issues as follows.

i The observation region size is not large enough for precise model detection and high denoising ability in stationary region.

ii The observation region size is not small enough for detecting non-stationary region.

D. AIC and \( \overline{\text{AIC}} \)

The classification of the observation block is based on the theorem "the larger \( \overline{\text{AIC}} \) of the OB, the more stationary the OB is". Generally AIC is obtained as follows,

\[
AIC = -2 \ln L + 2m.
\]

(6)


\[ \text{AIC} = 2 \ln \frac{L}{m}, \quad (7) \]

where

\[ \ln L = -\frac{m}{2} \ln |\Sigma| - \frac{1}{2} y^T \Sigma^{-1} y. \quad (8) \]

In the equation (8), \( y \), \( \Sigma \) and \( |\Sigma| \) denote the DCT AC vector of the \( \text{OB} \), covariance matrix of \( y \) and the determinant of \( \Sigma \) respectively. Then \( m \) denotes dimension of vector \( y \) and is equivalent to \( M^2 - 1 \).

### E. Measures in new UNI-GMM-AWF method

To overcome these issues mentioned above, in the new UNI-GMM-AWF method, the fitness of locally stationary GMM assumption is improved by applying a set of UNI-GMMs in various observation block. Thus the new UNI-GMM-AWF method is able to classify \( B \) in various observation block. Thus the new UNI-GMM-AWF assumption is improved by applying a set of UNI-GMMs.

- **i** In order to detect the stationary \( \text{OB} \), two sizes of the UNI-GMM which is small size (8*8 pixels) and large size (16*16 pixels) are prepared. Then the AICs of the OBs are compared to each other. If the AIC of the large sized \( \text{OB} \) is larger than the small sized \( \text{OB} \), then the block \( B \) is in stationary local region, that is large \( \text{OB} \) is more profitable.
- **ii** Simultaneously, in the UNI-GMM-AWF, a modified observation block \( \text{OB}_{\text{hole}} \) showed in fig.2 is prepared in order to detect point-like shadow. \( \text{OB}_{\text{hole}} \) has a hole of 2*2 pixels at the central region of the \( \text{OB} \). In the case of existing the point-like shadow, like a small vessel or a bronchiole, at the block \( B \), the stationary GMM cannot detect it because small point tends to detect as white noise. If the AIC is larger than AIC without hole, this block is detected as non-stationary block and filtered with AWF.

### F. Procedure of the new UNI-GMM-AWF with \( \text{AIC} \)

Brief overview of the proposed method as follows.

- **I** Prepare two UNI-GMMs with respect to the size of \( \text{OB} \). e.g. UNI-GMM_{small}: \( M^2 = 8 \times 8 \), UNI-GMM_{large}: \( M^2 = 16 \times 16 \), are prepared.

- **II** Evaluate four AICs for each \( \text{OB} \).

- **III** Estimate the class \( s_k \) of \( B \) based on each UNI-GMM by maximum a posteriori probability (MAP) for each \( \text{OB} \) and evaluate four AICs for each \( \text{OB} \).

- **IV** Denoise according to four \( \text{OB} \)s GMM as follows.

  - if AIC_{large} > AIC_{small},
  - if AIC_{large} of \( \text{OB} \) > AIC_{large} of \( \text{OB}_{\text{hole}} \),
    Adopt UNI-GMM_{large} based on \( \text{OB} \).
  - else
    Adapt UNI-GMM_{large} based on \( \text{OB}_{\text{hole}} \).
  - else
    if AIC_{small} of \( \text{OB} \) > AIC_{small} of \( \text{OB}_{\text{hole}} \),
    Adopt UNI-GMM_{small} based on \( \text{OB} \).
  - else
    Adopt UNI-GMM_{small} based on \( \text{OB}_{\text{hole}} \).

### G. Corrected AIC

In practice it is difficult to recognize whether the point like signal detected by proposed method is significant signal or noise. To compensate this error, AIC is compensated by adding a correction \( \alpha \). In this paper \( \alpha \) is determined experimentally showed in fig.4 described as follows.

### III. SIMULATION(RESTORATION OF PHANTOM IMAGE)

#### A. preparation of the image set for training

The original image set \( \mathcal{O} \) and the observed image set \( \mathcal{D}_d \) for training are prepared. 149 chest phantom images are obtained for \( \mathcal{O} \) and \( \mathcal{D}_{40} \) mAs using 190 milliampere second (mAs) and 40 mAs respectively by scanning chest phantom (N1) developed by Kyotokagaku Co. Ltd. using X-ray CT (Asteion multi™) developed by Toshiba medical systems and imaging condition is listed in Table I.

#### B. Experimental conditions

The observed chest phantom image for restoration is prepared to scan using 40 mAs. This observed image is not included in \( \mathcal{D}_{40} \) mAs prepared beforehand for training. Using this observed image, we compare the new UNI-GMM-AWF with median filter and previous UNI-GMM-AWF. The images restored by each method are compared by Signal to Noise Ratio Improvement (ISNR) and horizontal profile in restored images. ISNR is defined as follows,

\[ \text{ISNR} = 10 \log_{10} \frac{\sum(y - x)^2}{\sum(\tilde{x} - x)^2}, \quad (9) \]

where \( y \), \( \tilde{x} \) and \( x \) denote the pixels in the observed image, the restored image and the original (\( d_0 = 190 \) mAs) image, respectively, and \( \sum \) is taken on all pixels. Horizontal profile is measured on the white line segment showed in fig.3(c).
Fig. 3. white box area in the observed phantom image fig.3(b) and profile of white dashed line in fig.3(c) are applied to evaluate ability of filters.

![Image](a) original image(190mAs)  (b) observed image(40mAs)  (c) enlarged image of the white box area in fig.3(b)

D. Denoising and point signal preservation ability of new UNI-GMM-AWF

Upper line in fig.5 shows the comparison of a restored phantom image among median filter, previous UNI-GMM-AWF and new UNI-GMM-AWF. Then for more detailed evaluation of denoising and point like shadow preservation ability, the lower line in fig.5 shows the profiles on the same line segment in fig.3(c). Evaluations of denoising and point like shadow preservation ability are described as follows:

- The comparison of the each profile shows more detail of the point like shadow preservation ability above. Then if taking notice to central peak on the profile, we see the intensity of the central peak of the new UNI-GMM-AWF is higher than the other methods. Furthermore we see that against the foot of the central peak of the new UNI-GMM-AWFs is sharp, the foot of the median filter is dull.
- Denoising ability between the previous UNI-GMM-AWF and the new UNI-GMM-AWF is almost same.
- Point like shadow preservation ability of the new UNI-GMM-AWF is higher than the previous UNI-GMM-AWF.

IV. RESTORATION OF CLINICAL THIN SLICE CT IMAGE

Fig.6 shows a thin slice chest CT image which is scanned by 2 mm slice thickness, 120 kV and 115 mAs. This thin slice image has a almost same variance of its noise as a 40 mAs phantom images. Thus the image is restored by UNI-GMM-AWFs designed using 40 mAs phantom images. Fig.7 shows restored images of fig.6 by median filter and UNI-GMM-AWFs. Result and discussion is described as follows in DISCUSSION.

V. DISCUSSION

In restoration of the chest phantom image, denoising ability with point like shadow preservation of the new UNI-GMM-AWFs is higher than median filter which is known as a standard denoising method with point like shadow preservation filter. New UNI-GMM-AWF works to leave a point

![Graph](ISNR vs. α (α is an additive correction for compensation of AIC))

C. ISNR vs. Correction of the AIC with α

As a result additive correction with α to the $\overline{\text{AIC}}_{\text{large}}$ does not work in any case. On the other hand additive correction with α to the $\overline{\text{AIC}}$ of OB$_{\text{hole}}$ which has "hole" in the central region of OB works functionally. These situations denote that the $\overline{\text{AIC}}_{\text{large}}$ is always larger than the $\overline{\text{AIC}}_{\text{small}}$ for each block. In other words, this means that large size (16*16 pixels) UNI-GMM is always more stationary than small size (8*8 pixels) UNI-GMM. Accordingly in this paper the correction with α to the $\overline{\text{AIC}}$ is just treated whether to adopt the model with "hole" or without.

Fig.4 shows that the highest ISNR is marked when the α is 0.15. Then we see that the new UNI-GMM-AWF(α is 0.15) marks the highest ISNR among the denoising methods showed in table II. Hereinafter we fix the α 0.15 in processing new UNI-GMM-AWF.
Fig. 5. upper line images: restored chest phantom images scanned with slice thickness 2 mm, dose 40 mAs using the median filter, the previous UNI-GMM-AWF and the new UNI-GMM-AWF. Each image is displayed with Window width 1600, Window level -500. $\alpha = 0.15$ in the new UNI-GMM-AWF. lower line graphs: profiles of CT numbers on the line segment pointed in fig.3(c) in each restored image.

Fig. 6. fig.6(a) shows an example of a clinical chest thin slice CT image. fig.6(b) shows a enlarged image of white box area in fig.6(a) like shadow which includes cross-section of a small vessel due to UNI-GMM with "hole". For this function new UNI-GMM-AWF is worked on effective restoration of thin slice chest CT images, because they include many axial cross-sections of small anatomical structure. In new UNI-GMM-AWF it can be clearly observed that the removal of noise and streaking artifacts at the dorsal region of the lung does not eliminate any lung nodule structures in the upper line of
fig. 7. If taking notice to central points in the lower line of
fig. 7 shows effectiveness of point signal preservation ability
in proposed method. However, although it is most important
whether new UNI-GMM-AWF recognize a point like shadow
as an anatomical object or as a noise, new UNI-GMM-AWF
can not recognize it. For this reason it remains a problem that
readers must adjust AIC with adding correction $\alpha$.

On the other hand, as a result small size UNI-GMM does not
work functionally because the large size UNI-GMM prepared
is always more stationary than the small size UNI-GMM. We
need to prepare the larger UNI-GMM for detection of larger
stationary region.

VI. CONCLUSION

Conclusion in this paper is described as follows,

- new UNI-GMM-AWF is effective to preserve a point
  like shadow with denoising of white noise and streaking
  artifact.
- Optimization of AIC with adding correction $\alpha$.
- Preparation of the larger UNI-GMM for detection of
  larger stationary region.

REFERENCES