Low-frequency magnetostrictive inertial actuator

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ABSTRACT
The present paper presents the design of an innovative low-frequency magnetostriuctive inertial actuator that is able to multiply by almost a factor of 10 the amplitude of striction-elongation amplitude of the magnetostrictive bar, thus leading to an increase in the generated force amplitude, and to obtain a working frequency from 30Hz up, i.e. well below the working frequencies of traditional magnetostrictive inertial actuators. Moreover, the design has been optimized through an analytical model and a finite element model taking into account all design parameters. The optimised low-frequency magnetostriuctive inertial actuator has then be produced and its frequency response compared to that of a traditional magnetostrictive actuator made up of the same components (except for the supporting structure).

1. INTRODUCTION
Magnetostrictive inertial actuators are able to effectively generate an actuation force only above their first eigenfrequency that is determined by the inertial mass and the stiffness along the actuation direction ([1]). In traditional magnetostrictive actuators the inertial mass is directly connected to the actuator foot through the bar of magnetostrictive material (plus a rigid spring that is required for applying the necessary mechanical pre-load to the magnetostrictive bar, [2]). Thus, the stiffness along the actuation direction is strictly related to Young’s modulus of the magnetostrictive material. To obtain low-frequency actuators one could therefore either increase the inertial mass (e.g. by adding more weight on top of the actuator) or decrease the cross-section of the magnetostrictive bar (thus reducing its axial stiffness). There is however a limit to the increase in inertial mass and/or decrease in the bar’s cross-section imposed by Euler’s critical load (buckling of the magnetostrictive bar) and by the frailty of the actuator (magnetostrictive materials usually have a fragile behaviour, [3]). This leads to traditional magnetostrictive actuators that have their first eigenfrequency well above 100Hz ([4]).

To obtain a low-frequency magnetostrictive inertial actuator (with the first eigenfrequency below 100Hz) it is necessary to re-design the actuator in order to decouple the stiffness along the actuation direction from Young’s modulus of the magnetostrictive material. In the present paper an innovative design of the magnetostrictive actuator is proposed that uses the same components of traditional magnetostrictive actuators but with a different layout: the magnetostrictive bar is mounted orthogonally with respect to the direction of actuation and is connected to the inertial mass through a special deformable structure having low stiffness along the direction of actuation. This innovative configuration allows to improve two performances:

- the up and down motion of the inertial mass is not equal to the amplitude of striction-elongation (about 1% of the length of the magnetostrictive bar) but can be multiplied by almost a factor of 10 thus leading to an increase in the force amplitude that the inertial actuator is able to generate (at equal inertial mass);
- the eigenfrequency of the actuator is determined by the deformability of the supporting structure and not to the axial stiffness of the magnetostrictive bar thus allowing to reduce the working frequency from about 30Hz on (i.e. significantly extending the bandwidth of the inertial actuator towards low frequencies).

Moreover, the proposed configuration allows to greatly reduce the overall height of the actuator thus allowing to mount it even in narrow cavities.

The design of the low-frequency magnetostrictive inertial actuator has been optimized both through analytical models and through finite element models in order to achieve the best compromise between force amplitude, bandwidth, stresses in the supporting structure, height and complexity of the assembly/final cost of the device. The comparison between the developed models (simple analytical and finite element ones) shows that the analytical model is able to correctly estimate the motion amplification of the innovative device while the finite element model is required for correctly assessing to increase in bandwidth and for determining the stresses the supporting structure will be subjected to. Optimization parameters are all the design parameters of the supporting structure, i.e. its lengths, widths, radii, points of connection to the inertial mass and to the magnetostrictive bar, etc. Also the material of the supporting structure has been considered in this optimization phase although, at the end, the most bounding requirement was that of the cost of the material of the supporting structure.
Once optimized, the low-frequency magnetostrictive inertial actuator has been produced (the supporting structure has been obtained through water-jet technology, and its performances compared to those of a traditional magnetostrictive inertial actuator made up of the same components (except for the supporting structure) and having the same inertial mass ([5]). This comparison has shown to correctness of the design approach and that, through the addition of just the deformable structure, performance indexes are greatly improved.

2. DESIGN

As already observed, magnetostrictive inertial actuators are not able to generate a significant force below their first (axial) eigenfrequency ([1]) and this eigenfrequency is well above 100Hz due to the fact that their stiffness is associated to the magnetostrictive bar:

\[
\omega_0 = \sqrt{\frac{k_s}{m}} = \sqrt{\frac{A}{s^H L m}}
\]

where \(m\) being the inertial mass fixed on top of the magnetostrictive bar and \(k_s\) being its axial stiffness equal to \(A/s^H L\) (where \(A = 9 mm^2\) is the cross-section area of the magnetostrictive bar, \(s^H = 3.3 \times 10^{-11} m^2/N\) its mechanical compliance when a constant magnetic field is applied and \(L\) its length). Figure 1 shows the eigenfrequency of the magnetostrictive inertial actuator evaluated through equation 1 as a function of the suspended mass and the length of the magnetostrictive bar. It can be seen that, to obtain a magnetostrictive inertial actuator working at low frequencies, a very slender bar with high suspended mass is necessary. However, such configuration is impossible to obtain practically due to the reaching of Euler’s critical load (buckling of the magnetostrictive bar).

In order to develop a low-frequency magnetostrictive inertial actuator, it is therefore necessary to design a mechanism that is able to decouple the stiffness of the actuator from the stiffness of the axial magnetostrictive bar and, if possible, to amplify the deformations of the bar that are usually about 1% of the bar length itself. The power balance of such decoupled magnetostrictive inertial actuator sounds:

\[
m \ddot{y} \cdot \dot{y} + k_s x \cdot \dot{x} = 0
\]

where \(x\) is the elongation of the terfenol-D bar and \(y\) is the displacement of the inertial mass. The transmission ratio is equal to the ratio between the output speed, i.e. \(\dot{y}\), and the input speed, i.e. \(\dot{x}\):

\[
\tau = \frac{\dot{y}}{\dot{x}}
\]

Substituting the definition of \(\tau\) in equation 2 we get:

\[
m x \cdot \dot{x}^2 + k x \cdot \dot{x} = 0
\]

and thus the eigenfrequency of the system becomes:

\[
\omega = \sqrt{\frac{k}{m} \tau^{-2}} = \frac{\omega_0}{\tau}
\]

It is therefore clear that, by conveniently designing the support structure of the magnetostrictive bar, the eigenfrequency of the actuator can be reduced by a factor equal to the transmission ratio \(\tau\).

The proposed mechanism exploits a double motion amplification as schematically shown in figure 2: both the lever mechanism and elastic deformation of the arc/bow cooperate to amplify the deformation of the magnetostrictive bar.

![Figure 1: Eigenfrequency of a magnetostrictive inertial actuator as a function of the suspended mass \(m\) and bar length \(L\) (A=9mm²).](image1.png)

![Figure 2: Kinematic scheme of the motion amplification mechanism.](image2.png)
3. Optimization

In order to design a low-frequency magnetostrictive inertial actuator that is able to maximise the transmission ratio $\tau$ as well as the force transmitted to the inertial mass keeping the stresses of the supporting structure as low as possible, thus guaranteeing an infinite fatigue life, several different configurations were taken into account. Figure 3 shows the main configurations considered.

![Main configurations considered for the low-frequency magnetostrictive inertial actuator](image)

Figure 3: Main configurations considered for the low-frequency magnetostrictive inertial actuator

For each considered configuration, the lever effect, the generated force, the eigenfrequency and the maximum stresses in the supporting structure have been determined. From a preliminary analysis, it has been found that the most promising, as well as the easiest one to produce, configuration is the last one shown in figure 3. The parameter optimization procedure has therefore been applied to this last configuration.

At first, a simple kinematic model has been adopted for the optimization procedure to limit the parameter space taking into account just the lever effect. Then, a detailed finite element model has been adopted in order to maximise not only the lever effect but also the generated force as well as to minimize the eigenfrequency and the maximum stresses in the supporting structure.

**Analytical Model**

In figure 4 the undeformed kinematic model adopted is shown while in figure 5 the deformed kinematic model adopted is displayed.

![Undeformed configuration of the analytical kinematic model adopted](image)

Figure 4: Undeformed configuration of the analytical kinematic model adopted

![Deformed configuration of the analytical kinematic model adopted](image)

Figure 5: Deformed configuration of the analytical kinematic model adopted

- the maximum dimensions of the support structure ($H$ and $m$) are fixed,
- the point of application of the displacement imposed by the magnetostrictive bar ($l$) is fixed,
- the amplitude of half the elongation of the magnetostrictive bar ($x$) is imposed,
- the length of the arc of the crossbow remains constant ($Ro = R'\alpha'$)

The equations that govern the kinematics of the analytical model are the following:

$$\alpha = a \sin \left( \frac{m}{R} \right)$$

$$L = H - R \left( 1 - \cos \alpha \right)$$

$$\beta = a \sin \left( \frac{x}{l} \right)$$

$$n = L \sin (\beta)$$

$$Ro = R'\alpha'$$

$$m + n = R' \sin \alpha'$$

$$e = R' \left( 1 - \cos \alpha' \right)$$

$$y = H - L \cos (\beta) - e$$

The reference configuration of the supporting structure is characterized by a height $H = 15mm$, by a width $2m = 30mm$ and by a point of application of the displacement imposed by the magnetostrictive bar $l = 3mm$ from the basis.

The model results are shown in figure 7. As can be clearly seen, the crossbow lever effect (equal to the transmission ratio $\tau$) significantly varies as a function of the crossbow curvature radius and reaches a maximum value of 7.4 for a curvature radius equal to 39.2mm.

Also the effect of other parameters has on the lever effect has been investigated. Figure 7 shows the influence of the width $m$ of the crossbow: at increasing width, higher lever effects are reached but for greater crossbow curvature radii (with $2m = 40mm$ the lever effect becomes equal to 8.3 for a curvature radius equal to 60.9mm).
Reducing the distance between the hinge and the point of application of the imposed displacement $l$ (figure 9) a significant increase in the lever effect for slightly greater curvature radii is obtained (with $l = 2\text{mm}$ the lever effect reaches a value of 9.5 for a curvature radius equal to 52.2mm). Figure 8, instead, shows how the increase in the height $H$ of the crossbow determines a significant increase in the lever effect (with only a slight increase in the crossbow curvature radii): for $H = 20\text{mm}$ the lever ratio becomes 9.2 while the curvature radius becomes equal to 53.0mm (for a distance between the hinge and the point of application of the imposed displacement $l = 4\text{mm}$).

**Finite Element Model**

The optimization of the geometry of the supproting structure as thus been carried out using a detailed finite element model that allows to predict not only the lever effect but also the reaction forces, the eigenfrequency of the structure as well as the stresses inside the structure. The developed finite element model of the structure has the following characteristics:

- approx. 7000 linear 8-node brick elements (characteristics dimensions of the mesh equal to 0.5mm),
- approx. 25000 degrees of freedom,
- lower surface bounded to the ground,
- inertial mass equal to 0.3$\text{kg}$,
- imposed deformation fo the magnetostrictive bar and
- linear elastic material for the supproting structure ($\rho = 7800\text{kg/m}^3$, $E = 210000\text{MPa}$, $\nu = 0.3$).

Figures 10, 11 and 12 show, respectively, the vertical displacement $y$ of the inertial mass (in mm) for an elongation of the magnetostrictive bar $x$ equal to 0.4mm, the inertia force generated by the magnetostrictive actuator (in N) for the same elongation of the magnetostrictive bar considered before and the first (vertical) eigenfrequency of the actuator (in Hz) as a function of the crossbow curvature radius and width. Comparing the vertical displacement $y$ obtained from the analytical model and the one determined through the finite element model, a good agreement can be found. Moreover, for a curvature radius of 50mm and a crossbow width of 0.3mm the maximum lever effect is reached. The force generated by the magnetostrictive actuator, instead, does not significantly vary in the parameter range considered (it varies from 370N a 420N) and increases at increasing crossbow width and curvature radius. Finally, the eigenfrequency decreases with the crossbow width and increases with the curvature radius. For the configuration that maximizes the lever ratio, the eigenfrequency is lower than 60Hz. It has therefore been decided to adopt the configuration that maximizes the lever ratio.
Figure 10: Vertical displacement $y$ of the inertial mass (in mm) for an elongation of the magnetostrictive bar $x$ equal to 0.4mm as a function of the curvature radius and crossbow width.

Figure 11: Inertia force generated by the magnetostrictive actuator (in N) for an elongation of the magnetostrictive bar $x$ equal to 0.4mm as a function of the curvature radius and crossbow width.

4. Production

Figure 13 shows the section of the 3D model of the low-frequency magnetostrictive inertial actuator prototype. The central part is the magnetostrictive bar and at its extremities two Neodymium-Iron-Boron magnet discs are placed. The supporting structure also supports the windings that generate the magnetic field inside the magnetostrictive material. Table 1 reports the most significant quantities of the designed low-frequency magnetostrictive inertial actuator.

Figure 14 shows a photograph of the produced prototype.

Table 1: Most significant quantities of the designed low-frequency magnetostrictive inertial actuator

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inertial mass</td>
<td>220 g</td>
</tr>
<tr>
<td>Material of supporting structure &amp; mass</td>
<td>steel</td>
</tr>
<tr>
<td>Material of magnetostrictive bar</td>
<td>Terfenol-D</td>
</tr>
<tr>
<td>Length of magnetostrictive bar</td>
<td>20 mm</td>
</tr>
<tr>
<td>Stiffness of magnetostrictive bar</td>
<td>$1.4 \times 10^7$ N/m</td>
</tr>
<tr>
<td>Number of windings</td>
<td>400</td>
</tr>
<tr>
<td>Total height</td>
<td>25 mm</td>
</tr>
<tr>
<td>External diameter</td>
<td>54 mm</td>
</tr>
</tbody>
</table>

5. Experimental Results

The test bench adopted for assessing the low-frequency magnetostrictive inertial actuator prototype characteristics...
is shown in figure 15.

Figures 14 and 15: Low-frequency magnetostrictive inertial actuator prototype

The inertial mass is free to vibrate and the actuator is driven through a time varying supply current $I$. Both random excitation and sweep sine excitation in the frequency range 1-2000 Hz were adopted. During the test both supply current and voltage are acquired as well as the base and the inertial mass accelerations (through piezolectric accelerometers) and the force transmitted to the ground (through a piezolectric load cell). Figure 16 shows the measured transfer function between supply current $I$ and force transmitted to the ground $F_T$. It can be seen that the first (vertical) eigenfrequency is equal to $\omega = 167$ Hz well below the eigenfrequency of the magnetostrictive inertial actuator without the designed supporting structure ($\omega_0 = 1260$ Hz). Moreover, the measured transmission ratio is equal to 7.5 quite close to the expected one.

Figure 16: Experimental transfer function $G = \frac{F_T}{I}$

6. Conclusions

In the present paper an innovative supporting structure for obtaining a low-frequency magnetostrictive inertial actuator has been designed and optimised through a simple kinematic analytical model and through a complex finite element model. Thus, a prototype has been produced and tested showing a good agreement with the simulated results.

References


