Modeling education and science as evolutionary systems

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ABSTRACT

General mathematical theory of evolutionary system developed earlier is implemented to the education and science and their interaction.

Keywords: Evolutionary system; Mathematical model; Optimization problem; Education; Science

1. INTRODUCTION

All the features of evolutionary systems (ES) in the general theory [1], [6], [7], [8], [9], including work places (WP) as the main elements of ES, existence of the subsystem A for realization of the internal functions of the system development and the subsystem B for realization of the external functions, allocation of the system's resources between its internal and external functions, the out of date or obsolete WP, the inflow of resources from the outside, and existence of the moment of ES origin or ES prehistory, are also suitable for the case of education and science.

An essential difference between education and science and any artificial ES, creating by the human beings and functioning with their participation, consists of their WP and their products.

The educational product is the number of specialists of determined quality, and the index of efficiency of WP is the number of new specialists of determined quality per unit of WP in the subsystem B per unit time. The quality of specialists can be designated by different rules. To avoid the question: "And who are the judges?" different tests are used and their results are calculated with the help of computers.

The subsystem B in education as ES is a set of WP, the product of which is the number of the specialists. The subsystem A in education as ES is a set of WP, the products of which are new, more effective WP in both A and B subsystems of education. Creation of these WP also means training of the respective, more effective specialists in the field of education. Any science-methodical centers in the field of education, of public educational office, and administration of any educational center will belong to the subsystem A if their functions are the creation of new WP. Actually, any educator belongs to A while he/she is increasing his/her own qualification.

Thus, the primary indication of whether WP belongs to the subsystem A or B is not place, time, or person, but rather the kind of labor function fulfilled.

The science product is a new, more effective technology. Since new technology results in higher-level values of indices of WP efficiency, we can assume that the science external products are these indices of WP efficiency for different ES. The internal product of the subsystem A in science as

ES is a new technology, which is new WP efficiency indices in A that further create new technology, including creation or restoration of A itself.

The external product of the subsystem B in science is a new technology for other branches of human activities. So, we can conclude that indices of WP efficiency in the subsystem A of science and the main products of WP are actually the same. Any scientific center, any branch of science, and science as a whole can be considered as ES. At all events, the subsystem A of these ES is the set of WP. The product of this set of WP is new, more effective WP, which create new technology for themselves and for external ES. Any scientist, while his/her labor function is perfection of his/her own WP, belongs to the subsystem A of science as ES.

Thus, it is not difficult to see that the essence of development of science is the presence of the subsystem self-development or perfection.

2. MATHEMATICAL MODELS OF DEVELOPMENT

The base, minimal or simplest MM of development has the form

$$\begin{aligned}
&m(t) = \int_{a(t)}^{t} \alpha(t,s) y(s) ds, \\
&0 \le y \le 1,\ 0 \le a(t) \le t,\ \alpha \ge 0, \\
&c(t) = \int_{a(t)}^{t} \beta(t,s) z(s) ds, \\
&z(s) = 1 - y(s),\ \beta \ge 0, \\
&R(t) = \int_{a(t)}^{t} m(s) ds, M(t) = \int_{0}^{t} m(s) ds, \\
&G(t) = M(t) - R(t), \\
&f(t) = m(t) + c(t),\ t \ge t^*. \qquad (1)
\end{aligned}$$

where $m(t)$ is the rate of creation of the first kind new generalized product (resource) quantity at the time instant t, which provides the fulfillment of the internal functions of ES, that is, restoration of itself and creation of the second kind product; $y(t)m(t)$ is a share of $m(t)$ for fulfillment of internal functions in the subsystem A of restoration and perfection of the system as a whole; $\alpha(t,s)$ is the efficiency index for functioning of the subsystem A along the channel $\alpha(t,s)y(s)m(s) - -m(t)$, i.e., the number of units of $m(t)$ created in the unit of time starting from the instant t per one unit of $y(s)m(s)$; $a(t)$ is a special temporal bound: the new product creating before $a(t)$ is never used at the instant t, but created after $a(t)$ is used entirely; $c(t)$ is the rate of creation of the second kind new generalized product quantity at the instant t, which provides the realization of the external functions of ES; $[1 - y(s)]m(s)$ and $\beta(t,s)$ are similar to ym and α respectively but for the subsystem B of creation of the second kind product; $R(t)$ is the total quantity of the first kind product functioning at the instant t; $M(t)$ is the total quantity of the first kind product to be created during the time $[t, 0]$; $G(t)$ is the total quantity of the obsolete product at the instant t; $f(t)$ is the rate of the resource inflow from the outside ($m(t)$ and $c(t)$ are measured in the units of $f(t)$); t^* is the starting point for modeling; $[0, t^*]$ is the prehistory of ES, for which all the functions are given (their values will be noted by the same symbols but with the sign "*", e g, $m(t) = m^*(t)$, $t \in [0, t^*]$).

It is obvious that all the relations (1) are faithful representations by definition. In a general case, the indices α and β depend on m, c, a, y, R, M, G, and f.

Thus, (1) consists of 7 equalities and 7 inequalities connecting 14 values, namely: m, c, α, y, β, $1-y$, a, R, M, G, t, t^*, f, 0, all of which are nonnegative. Usually, α, β, y, f, and/or R are given, and the others are to be found.

As can be seen, even in the simplified formulation, MM (1) is the system of nonlinear functional relations, in which along with the nonlinear integral equation of the unusual form (the lower bound $a(t)$ can be unknown function) we have the system of functional inequalities. Note that we have a particular case here where the intensities of ES functioning are

$$\begin{aligned}
&\lambda(t,s) = \mu(t,s) = 0,\ 0 < s < a(t); \\
&\lambda(t,s) = \mu(t,s) = 1,\ a(t) < s < t. \qquad (2)
\end{aligned}$$

It is not hard to introduce different generalizations of MM (1).

The n-product MM, $n > 2$, can be formally written in the same form (1), where m, a, and c are the vector functions, and α, y, β, and z are the respective matrices (where the inequalities for the vectors and matrices are the same inequality for their appropriate components).

The continuous MM can be described in the same form considering t and s as many-dimensional variables and examining the appropriate integrals as multivariate ones.

The stochastic MM can be obtained by considering α, β, and f as functions of a random factor ω.

The discrete MM can be represented in the same form if the integrals in (1) are understood in the sense of Stieltjes.

One of the important typical optimization problems for ES is maximization of the functional

$$I(y) = \int_{t^*}^{t} c(t)dt =$$
$$\int_{t^*}^{t} \left(\int_{a(t)}^{t} \beta(t,s)[1-y(s)]m(s)ds \right) dt, \quad (3)$$

over y with regard to MM (1). The first essential result on the properties of solutions of the problem (3) has been seen in [4],[5].

The result consisted qualitatively in that for "small" $T - t^*$ the desired $y(t)$ is minimally possible, but for "large" $T - t^*$ the desired $y(t)$ may differ from the minimally possible on the larger initial part of the segment $[t^*, T]$. Only on the smaller final part of $[t^*; T]$ the desired $y(t)$ is minimally possible.

The notions "small" and "large" depend on the values of the functions α and β; namely, the greater the functions in question, the closer to t^* is the boundary between "small" and "large" segments.

The result has obtained, in sequel, an important qualitative general interpretation:

The record of an external function for any evolutionary system (ES) can be obtained only under the conditions of its sufficiently comfortable guarantee, that is, under the significant fraction of resources sent to internal needs of ES.

We dwell on the comparisons between MM in question and the classical MM. Everybody is familiar with the approach of the so-called "black box", when only input $X = (x_1, ..., x_n)$ and output $Y = (y_1, ..., y_m)$ of a dynamic system are given. We have, in the linear approximation,

$$Y(t) = \int_{t-T}^{t} K(t,s)X(s)ds, \quad (4)$$

Here, T is the upper bound for all the transient end times, $K(t,u)$ is the matrix of the pulse transition functions $k_{ij}(t,u)$ that are the response functions of the system for $x_j(s) = \delta(u-s)$, $t > u > t-T$, where δ is Dirac δ-function, and all the other $x_k(s) = 0$. A nonlinear dynamic system can be also represented as (4), but then K depends on X.

The MM (1) deals with the so-called "gray box" when the structure of a dynamic system is partly revealed. Indeed, it is possible to say that in (1) the matrix K in the case of the subsystem A has been factored into two parts α and λ (in our particular case, $\lambda = 1$, $a(t) \le s \le t$; $\lambda = 0$, $0 \le s \le a(t)$) such that each of them has its own applied sense.

In addition, several outputs of the system in (1) have served as its inputs. At last, due to the functions $c(t)$, $G(t)$, and $f(t)$ we deal with the so-called non-autonomous or open dynamic system.

If we consider the case, when in (4) $X = f$ and $Y = (c, g)$, $g(t) = G'(t) = m[a(t)]a'(t)$ (supposing that derivative $a'(t)$ of $a(t)$ exists), then for determination of the respective K, we need to solve rather complicated system of nonlinear equations. As a result, K will be a rather sophisticated nonlinear vector-functional of f. The essential differences are that all the values in (1) are nonnegative by definition and a diminution of the output values is regulated not by a sign, but, for example, by the rate of $a(t)$ growth.

Detailed comparison of MM (1)-types and some integral dynamic MM is published in [7], [8], [9], and [10].

3. GENERALIZED STRUCTURE OF TWO ES INTERACTIONS

Of course, we can consider both kinds of ES as one ES, but this is not the case when they considered in more details [1], [2], and [3].

The main functions of ES1 are the creation of new WP for itself and for ES2. So, external products of ES1 are the new additional WP for ES2, including a new "personnel-ware" for ES2 as a result of training persons with new professions for ES2. In response, ES2 gives to ES1 a certain part of its external products.

This part is an important additional control function, of which the best value can be found, for example, from analysis of joined maximal efficiency of both ES1 and ES2.

In the case of the interaction between two independent countries as ES1 and ES2, determined parts of all kinds of products of ES1, including new technologies (which are products of its subsystem A1) and new goods and services (which are products of its subsystem B1), are subject to interchange with determined parts of all kinds of products of ES2 and vice versa. These parts are the important additional control functions. MM of ES allow us to select the appropriate values, in a certain sense, of these parts. One of the criteria of optimization is the idea of coordinated maximization of profits for both ES1 and ES2.

The classical example of interactions between two populations, specifically between beasts or birds of prey (ES1) and their victims (ES2), was investigated by V. Volterra [11]. Here the subsystems A1 and A2 are subsystems of reproduction of plunderers and victims, respectively. The subsystem B1 consumes the maximal possible part of the victims, and the subsystem B2 minimizes that part.

So, we have the mini-max problem under the condition of finite external resources. The solution of this problem gives determined values to the distribution in each of ES: between A1 and B1 in ES1 and between A2 and B2 in ES2.

We consider in the next section in more details interaction between education and science.

4. APPLICATION TO THE CASE STUDY

4.1 Modeling education as ES

Any educational units, from the separate school to education in the country to the education in the world, can be studied with the help of the same models (1)-types. Difference will be in the functions of α, β, and the prehistory on $[0, t^*]$. The bigger unit, the more stable α and β. In any case, for numerical realization of the problem considered, the problem of approximate identification of α and β has to be solved. However, since the MM of (1)-types are exact by definition, qualitative results can be achieved using only certain qualitative properties of α and β.

The usual problems of the study are 'if..., then...', and the optimization problems by various criterions.

According to the general law of optimization, mentioned above, by the criterion maximization of external products (the numbers of required specialists), the results optimization can be achieved only under the distribution the most part of the resources given to the subsystem A, which is the subsystem of WP for perfection of teachers' process, i.,d., perfection of WP in the subsystem B. These results depend on the time cycle. At the end of the each time cycle, most part of the resources has to be directed to the subsystem B. The identification and numerical solution of the respective optimization problems can give more exact value of the notation of 'the end of the each time cycle'.

The certain additional results of educational process of modeling can be seen in the book [1].

4.2 Modeling science as ES

All, what were asserted in 4.1, in principle, are also valid in the case of science. For maximization of new-kind of technologies, going to various other ES, in particular, to education, most part of the resources given to science has to be directed to the subsystem A of WP, the main function of which is the perfection of WP in the subsystem B. However, at the end of planning period, most part of the resources has to be directed to the subsystem B in science.

There is the so-called 'horizon' of planning $[t^*, T^*]$, so that for $[t^*, T]$, $T > T^*$, the initial part of $[t^*, T]$, on which the control y is not minimally possible, remains the same (This information was given to one of the authors of this paper by Professor Yu. P. Yatsenko) .

4.3 Modeling interaction between education and science

This modeling assumes rather big units of education and science as ES.

It is obvious that a certain part of the science products in its subsystem B, which are new more effective technologies for WP in both subsystems A and B of education, will be directed to education, and a certain part of the education products in its subsystem B, which are new specialists necessary for science, will be directed to science. However, the optimal values of those parts as well as the respective criterion of optimization are open problems.

In our opinion, the criterion of optimization can be the so-called principle of concerted optimum, for example, maximization of the sum of the profits for both ES, education and science.

The qualitative and numerical results in this field remain still open.

5. CONCLUSION

We considered above the certain problems and results of investigation of education and science as ES.

It is clear that the respective approach can be applied for study many other ES, and hence it is the right approach for study of various interdisciplinary problems.

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