Conservativeness Judgement Of The Controller For Systems With Time-varying Delay

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ABSTRACT

The conservatism of asymptotic stability conditions is considered in terms of linear matrix inequalities for time-varying delay systems. The conservative index is defined to evaluate the conservativeness for both delay-dependent and delay-independent stability conditions. The general results on H_∞ performance analysis are presented based on descriptor system approach. The optimization approach is given to obtain the upper delay and rational performances for the state-feedback controller of time-delay systems. Experimental results verify the effectiveness of the new method.

Keywords: time-varying delay systems, conservative index, descriptor system

1. INTRODUCTION

Time-delay existing in many control systems is often a source if instability. For the stability analysis of time-delay systems, there are mainly two type of stability conditions proposed: delay-dependent and delay-independent stability criteria[1-3]. The delay-dependent conditions are generally less conservative than delay-independent ones which do not include any information on the size of delays. The choice of an appropriate Lyapunov-Krasovskii functional is crucial for obtaining a solution to various H_∞ control problems[4-7]. In the state-feedback H_∞ controller design, special forms of Lyapunov-Krasovkii functionals lead to simpler delay-independent and delay-dependent linear matrix inequalities(LMIs)[8-12]. Concerning the H_∞ control problem for some time-delay systems, only a delay-independent state-feedback solution has been achieved[13]. In order to reduce the conservatism of these stability conditions, the descriptor system approach has been proposed in [1-2,5]. The improved solutions are delay-dependent, however delay-independent results can be obtained, for certain values of the design parameters in [1]. We integrate the delay-dependent conditions with time-independent ones by descriptor system approach. A conservativeness index is introduced to evaluate the conservatism for the state-feedback controller of time-varying delay systems. The general asymptotic stability results are presented on H_∞ performance analysis. The optimization problem is given to compute the upper value of delay and minimum disturbance attenuation performance for system with various size of delay. At last, the given numeral experiments show that the new method is effective.

Notation: Throughout the paper the superscript "T" stands for matrix transposition, R^n denotes the n dimensional Euclidean space with vector norm $|\cdot|$. The space of functions that are square integrable $[0\,\infty)$ is denoted by $L_2[0,\infty)$. For real symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). I is an identity matrix with appropriate dimension. In symmetric block matrices or long matrix expressions, an asterisk (*) is used to represent a term that is induced by Symmetry. Matrices, if not explicitly stated, are assumed to have compatible dimension.

2. ELEMENTARY KNOWLEDGE

We consider the following time-delay system[2]:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=0}^{2} A_i x(t-\tau_i(t)) + B_1\omega(t) + B_2 u(t)\\ x(t) &= \phi(t),\ \forall t\in[-h,0]\\ z(t) &= Cx(t) + D_1\omega(t) + D_2 u(t)\end{aligned} \tag{1}$$

Where $x(t)\in R^n$ is the state, $\tau_0 \equiv 0, \phi(t)$ is the initial condition, the scalar $h>0$ is an upper bound on the time delays $\tau_i(t), i=1,2$, and $A_i, i=0,1,2, B_1, B_2, C, D_1, D_2$ are known real constant matrices. $\omega(t)\in R^q$ is the noise signal which is assumed to be in $L_2[0,\infty)$; $z(t)\in R^p$ is the output. However, the results in this paper can be extended to the case of multiple delays.

As in [3], we consider two different cases for time-varying delays:

(1) $\tau_i(t)$ are differentiable functions, satisfying for all $t\geq 0$:

$$0\leq \tau_i(t)\leq h_i,\quad \dot{\tau}_i(t)\leq d_i<1,\quad i=1,2. \tag{2}$$

(2) $\tau_i(t)$ are continuous functions, satisfying for all $t\geq 0, 0\leq \tau_i(t)\leq h_i, i=1,2$.

Now set $\dot{x}(t)=y(t)$, then as in [2], the delay system (1) can be transformed into an equivalent descriptor form

$$\begin{aligned}&\dot{x}(t)=y(t)\\ &0=-y(t)+\{\sum_{i=0}^{2}A_i\}x(t)-\sum_{i=1}^{2}A_i\int_{t-\tau_i(t)}^{t}y(s)ds+B_1\omega(t)+B_2u(t)\end{aligned} \tag{3}$$

or

$$E\dot{\bar{x}}(t)=\begin{bmatrix}0 & I\\ \sum_{i=0}^{2}A_i & -I\end{bmatrix}\bar{x}(t)-\sum_{i=1}^{2}\begin{bmatrix}0\\ A_i\end{bmatrix}\int_{t-\tau_i(t)}^{t}y(s)ds+\begin{bmatrix}0\\ B_1\end{bmatrix}\omega(t)+\begin{bmatrix}0\\ B_2\end{bmatrix}u(t) \tag{4}$$

$$z(t)=\begin{bmatrix}C & 0\end{bmatrix}\bar{x}(t)+D_1\omega(t)+D_2u(t) \tag{5}$$

Where

$$\bar{x}(t)=\begin{bmatrix}x(t)\\ y(t)\end{bmatrix},\quad E=\begin{bmatrix}I & 0\\ 0 & 0\end{bmatrix}.$$

The following lemma is delay-dependent result on H_∞ performance analysis as theorem 5 in [2].

Lemma 1 Suppose $\gamma>0$ is a given scalar. Then, under zero input $u(t)=0$, for all $\tau_i(t), i=1,2$, satisfying (2), the time-delay system (4),(5) is asymptotically stable and satisfies

$$\|z\|_2<\gamma\|\omega\|_2 \tag{6}$$

Under zero initial condition for all non-zero $\omega\in L_2[0,\infty)$ if there exist matrices $P_1>0, P_2, P_3, R_i>0, S_i$, and $Y_i, i=1,2$, such that the following LMI holds:

$$\begin{bmatrix}\Omega & P^T\begin{bmatrix}0\\ A_1\end{bmatrix}-Y_1^T & P^T\begin{bmatrix}0\\ A_2\end{bmatrix}-Y_2^T & -h_1Y_1^T & -h_2Y_2^T & P^T\begin{bmatrix}0\\ B_1\end{bmatrix} & \begin{bmatrix}C^T\\ 0\end{bmatrix}\\ * & -(1-d_1)S_1 & 0 & 0 & 0 & 0 & 0\\ * & * & -(1-d_2)S_2 & 0 & 0 & 0 & 0\\ * & * & * & -h_1R_1 & 0 & 0 & 0\\ * & * & * & * & -h_2R_2 & 0 & 0\\ * & * & * & * & * & -\gamma^2 I & D_1^T\\ * & * & * & * & * & * & -I\end{bmatrix}<0 \tag{7}$$

where
$$\Omega = P^T \begin{bmatrix} 0 & I \\ A_0 & -I \end{bmatrix} + \begin{bmatrix} 0 & I \\ A_0 & -I \end{bmatrix}^T P + \begin{bmatrix} \sum_{i=1}^{2} S_i & 0 \\ 0 & \sum_{i=1}^{2} h_i R_i \end{bmatrix} + \sum_{i=1}^{2} \begin{bmatrix} Y_i \\ 0 \end{bmatrix} + \sum_{i=1}^{2} \begin{bmatrix} Y_i \\ 0 \end{bmatrix}^T \quad (8)$$

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix} \quad (9)$$

There is the proof in [2].

A Lyapunov functional candidate is

$$V(t) = V_1(t) + V_2(t) + V_3(t)$$

Where

$$V_1(t) = \bar{x}(t)^T EP\bar{x}(t)$$

$$V_2(t) = \sum_{i=1}^{2} \int_{-h_i}^{0} \int_{t+\theta}^{t} y(\alpha)^T R_i y(\alpha) d\alpha d\theta$$

$$V_3(t) = \sum_{i=1}^{2} \int_{t-\tau_i(t)}^{t} x(\alpha)^T S_i x(\alpha) d\alpha$$

Similarly, for all constants $\tau_i(t), i = 1, 2$, the delay-dependent result on H_∞ performance analysis was given by Theorem 6 in [2], corresponding to (7) with $S_i = 0, i = 1, 2$.

3. CONSERVATISM JUDGEMENT OF TIME-DELAY SYSTEM

The conditions of delay-dependent stability criteria take into account of the size of delays. The delay-dependent approach often has a limit of upper delay the time-delay system. The delay-independent approach is developed to stabilize the systems with long delay.

Set $R_i = 0, i = 1, 2$, the condition given by Lemma is transformed into the delay-dependent result on H_∞ performance analysis. It is known that the delay-dependent stability conditions are generally less conservative than delay-independent ones which do not include any information on the size of delays. So, we define an index to evaluate the conservativeness of stability conditions[1].

Now, choose a new $V_2(t)$:

$$V_2(t) = \sum_{i=1}^{2} \int_{-h_i}^{0} \int_{t+\theta}^{t} y(\alpha)^T (1-\varepsilon) R_i y(\alpha) d\alpha d\theta$$

Where the conservativeness index $\varepsilon \in [0,1]$. For $\varepsilon = 0$, the stability condition is derived as the delay-dependent result in Lemma 1; for $\varepsilon = 0$, the stability condition is derived as the delay-independent result.

Then, we have the following stability condition on H_∞ performance analysis.

Theorem 1 Suppose $\gamma > 0$ is a given scalar, $\varepsilon \in [0,1]$. Then, under zero input $u(t) = 0$, for all $\tau_i(t), i = 1, 2$, satisfying (2), the time-delay system (4),(5) is asymptotically stable and satisfies (6), Under zero initial condition for all non-zero $\omega \in L_2[0, \infty)$ if there exist matrices $P_1 > 0, P_2, P_3, R_i > 0, S_i$, and $Y_i, i = 1, 2$, such that the following LMI holds:

$$\begin{bmatrix} \Omega_1 & P^T \begin{bmatrix} 0 \\ A_1 \end{bmatrix} - Y_1^T & P^T \begin{bmatrix} 0 \\ A_2 \end{bmatrix} - Y_2^T & -h_1 Y_1^T & -h_2 Y_2^T & P^T \begin{bmatrix} 0 \\ B_1 \end{bmatrix} & \begin{bmatrix} C^T \\ 0 \end{bmatrix} \\ * & -(1-d_1)S_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & -(1-d_2)S_2 & 0 & 0 & 0 & 0 \\ * & * & * & -h_1(1-\varepsilon)R_1 & 0 & 0 & 0 \\ * & * & * & * & -h_2(1-\varepsilon)R_2 & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & D_1^T \\ * & * & * & * & * & * & -I \end{bmatrix} < 0 \quad (10)$$

where $\Omega_1 = P^T \begin{bmatrix} 0 & I \\ A_0 & -I \end{bmatrix} + \begin{bmatrix} 0 & I \\ A_0 & -I \end{bmatrix}^T P + \begin{bmatrix} \sum_{i=1}^{2} S_i & 0 \\ 0 & \sum_{i=1}^{2} h_i(1-\varepsilon)R_i \end{bmatrix} + \sum_{i=1}^{2} \begin{bmatrix} Y_i \\ 0 \end{bmatrix} + \sum_{i=1}^{2} \begin{bmatrix} Y_i \\ 0 \end{bmatrix}^T$, P is given in (9).

We consider the state-feedback control law:

$$u(t) = Kx(t) \tag{11}$$

Substituting (9) into (4),(5), wo obtain the structure of (4), (5) with

$$\overline{A}_0 = A_0 + B_2 K, \quad \overline{C} = C + D_2 K,$$

$A_i, i = 1, 2, B_1, C, D_1$ are the same. It is obvious that the stability condition of the state-feedback control system on H_∞ performance analysis is generated for Theorem.

Theorem 2 Suppose $\gamma > 0$ is a given scalar, $\varepsilon \in [0,1]$. For all $\tau_i(t), i = 1, 2$, satisfying (2), the time-delay system (4),(5) with control law (11) is asymptotically stable and satisfies (6), Under zero initial condition for all non-zero $\omega \in L_2[0,\infty)$ if there exist matrices $P_1 > 0, P_2, P_3, R_i > 0, S_i$, and $Y_i, i = 1, 2$, such that the following inequality holds:

$$\begin{bmatrix} \Omega_2 & P^T\begin{bmatrix} 0 \\ A_1 \end{bmatrix} - Y_1^T & P^T\begin{bmatrix} 0 \\ A_2 \end{bmatrix} - Y_2^T & -h_1 Y_1^T & -h_2 Y_2^T & P^T\begin{bmatrix} 0 \\ B_1 \end{bmatrix} & \begin{bmatrix} \overline{C}^T \\ 0 \end{bmatrix} \\ * & -(1-d_1)S_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & -(1-d_2)S_2 & 0 & 0 & 0 & 0 \\ * & * & * & -h_1(1-\varepsilon)R_1 & 0 & 0 & 0 \\ * & * & * & * & -h_2(1-\varepsilon)R_2 & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & D_1^T \\ * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{12}$$

where $\Omega_2 = P^T \begin{bmatrix} 0 & I \\ \overline{A}_0 & -I \end{bmatrix} + \begin{bmatrix} 0 & I \\ A_0 & -I \end{bmatrix}^T P + \begin{bmatrix} \sum_{i=1}^{2} S_i & 0 \\ 0 & \sum_{i=1}^{2} h_i(1-\varepsilon)R_i \end{bmatrix} + \sum_{i=1}^{2} \begin{bmatrix} Y_i \\ 0 \end{bmatrix} + \sum_{i=1}^{2} \begin{bmatrix} Y_i \\ 0 \end{bmatrix}^T$, P is given in (9).

4. CONSERVATIVENESS EVALUATION AND EXPERIMENTAL RESULTS

The inequality (12) in Theorem is affine in the system matrices and feedback gain *K*. The solver of Matlab is not directly applied to solve the non-linear matrix inequality. We decompose the condition (12) as the followings (13.1)+(13.2).

$$\begin{bmatrix} \Omega_1 & P^T\begin{bmatrix} 0 \\ A_1 \end{bmatrix} - Y_1^T & P^T\begin{bmatrix} 0 \\ A_2 \end{bmatrix} - Y_2^T & -h_1 Y_1^T & -h_2 Y_2^T & P^T\begin{bmatrix} 0 \\ B_1 \end{bmatrix} & \begin{bmatrix} \overline{C}^T \\ 0 \end{bmatrix} \\ * & -(1-d_1)S_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & -(1-d_2)S_2 & 0 & 0 & 0 & 0 \\ * & * & * & -h_1(1-\varepsilon)R_1 & 0 & 0 & 0 \\ * & * & * & * & -h_2(1-\varepsilon)R_2 & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & D_1^T \\ * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{13.1}$$

$$\Omega_3 = P^T \begin{bmatrix} 0 & 0 \\ B_2 K & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ B_2 K & 0 \end{bmatrix}^T P < 0 \tag{13.2}$$

The LMI (13.1) is solved by the solver of Matlab under the constraint (13.2). For the prescribed scalar $\gamma > 0$, and a conservativeness index $\varepsilon \in [0,1]$, we applied the Theorem 2 to obtain the maximum value of the delay *h*.

For convenient computation, the optimization problem is formulated as

$$\begin{aligned} &\min_{\gamma,\varepsilon\in[0,1]} 1/h \\ &s.t.\ (13.1), (13.2) \\ &\quad P > 0 \\ &\quad R > 0 \end{aligned} \tag{14}$$

Example 1. We consider the following system[1]:

$$\begin{aligned} \dot{x}(t) &= A_0 x(t) + A_1 x(t-\tau(t)) + B_1\omega + B_2 u(t) \\ z(t) &= C_0 x(t) + D_1\omega + D_2 u(t) \end{aligned} \tag{15}$$

where $D_1 = 0$, $A_0 = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$, $A_1 = \begin{bmatrix} 0 & 0.9 \\ -1.3 & -1.9 \end{bmatrix}$,

$B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C_0 = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $D_2 = 0.1$,

$0 \le \tau(t) \le h, \quad \dot{\tau}(t) \le d = 0.5$.

The LMI of Theorem 2 can be used to find the maximum value of *h* for which a state-feedback controller stabilizes the system. For $\varepsilon = 0.31$, we obtained that $h = 1$, and a minimum value of $\gamma = 4.12$ with a corresponding gain $K = [-10\ 0]$. For another $\varepsilon = 0.1$, a value of $h = 0.9$, $\gamma = 2.1$ was achieved. For $\varepsilon = 0.95$, an upper delay $h = 0.9$, and a minimum value of $\gamma = 1.86$ were achieved. We will have better results in cost of conservatism raising.

5. CONCLUSION

In this paper, we utilize the descriptor system approach for time-varying systems. The conservatism of asymptotic stability condition is analysed on both delay-dependent and delay-independent methods. The delay-dependent stability condition is less conservative than delay-independent stability condition. We introduce the conservativeness index to denote the conservatism between delay-dependent and delay-independent condition. The general results with the conservativeness index on H_∞ performance analysis are presented for time-delay systems. This method links delay-dependent condition with delay-independent condition. So, it is convenient to compute the maximum value of delay and optimal performances for time-varying delay systems, or systems with various size of delay. The experimental results of numeral example show that the new method is effective.

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