Examining the Validity of Credit Ratings Assigned to Credit Derivatives

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ABSTRACT

This paper examines the validity of ratings assigned by rating agencies on structured products: ABS and ABS CDO. The rating agencies have been criticized for assigning AAA ratings to the structured products created from mortgages. The ratings might give a false sense of confidence to investors with which a vast market was created and then collapsed. Using a new loss function to compute the attachment points, we show that the previously considered reasonable ratings may need more scrutiny.

Keywords: Rating agency, Asset-backed security (ABS), Collateral debt obligation (CDO), Loss function.

1. INTRODUCTION

The rating agencies have been criticized for assigning AAA ratings to the structured products created from mortgages. The ratings might give a false sense of confidence to investors with which a vast market was created and then collapsed. There have been various attempts to check whether the assigned ratings to structured products are reasonable. For example, Fender, et al. (2008) concludes that investors who narrowly focus on ratings can seriously misjudge the value-at-risk of collateral debt obligations (CDOs). Benmelech and Dlugosz (2009) examine the rating practices of rating companies.

Hull and White (2010) examines the risk in the tranches of asset-backed securities (ABSs) and ABS CDOs using the criteria of the rating agencies. The rating agencies test the should-be attachment point for an AAA-rated tranche if it has the same probability of losses as a AAA-rated corporate bond. The test involves:

1. The expected default rate (EDR) for the mortgages portfolio.

2. A correlation model that is needed to link the EDR to a probability distribution for the actual default rate.

3. A specified ratio of the expected loss given default (ELGD) to the initial mortgage principal.

Hull and White empirically show that some BBB tranches of ABS cannot be considered BBB bonds for the purposes of subsequent ABS CDO securitizations. Therefore, investors may be lulled into a false sense of confidence. This conforms to the market observation that investors suffer tremendous losses after the outbreak of subprime crisis in July 2007.

However, Hull and White also conclude that the AAA ratings assigned by rating agencies are not totally unreasonable. This conclusion appears contrary to the extreme default rates observed in the financial crisis period, namely, the widespread default of AAA rated securities.

In this paper, we use the loss functions proposed by Lee, et al. (2004), and Kuo and Lee (2007), for computing the minimum attachment point in order to better assess the validity of credit ratings assigned to credit derivatives. The main difference between Hull and White model and our approach is the assumption of default correlation. In Hull and White model, the probability of default is assumed to follow a cumulative normal distribution function. Therefore the realized default rate, conditional on a common macro factor, is a specific factor copula. The disadvantage or this model is that the specific copula does not fit the real situation precisely. In our approach, default loss is described by a Poisson distribution, which has the potential to describe the mortgage default loss more accurately without using an ad-hoc copula function.

The next section describes the structure design of ABS CDO tranches. The third section derives the loss function for calculating the attachment points of ABS CDO tranches. Finally, a simple conclusion follows.

2. METHODOLOGY

ABS CDO is created from a portfolio of ABS tranches. Hence, there are two levels in ABS CDO structure. In the first level, ABS consists of several mortgages. The underlying mortgage collateral was allocated to senior tranches rated AAA, mezzanine tranches, and subordinated tranches that were either unrated or rated BB. The rules for allocating cash flows from mortgages to tranches were defined by what was known as a waterfall.

In the second level of securitization, ABS CDOs were formed by creating tranches from tranches. Two types were common: a High Grade ABS CDO, created from AAA, AA, and A tranches of ABSs, and a Mezz ABS CDO, created from the BBB tranches of ABSs. The structure is illustrated in Figure 1 and Figure 2.

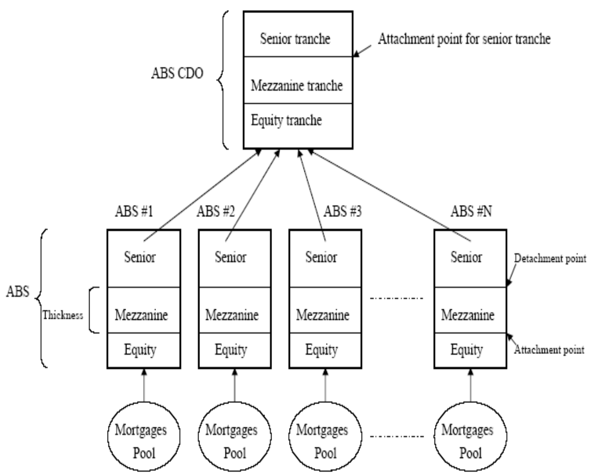

Figure 1: The structure of High Grade ABS CDO, created from senior tranches of ABSs.

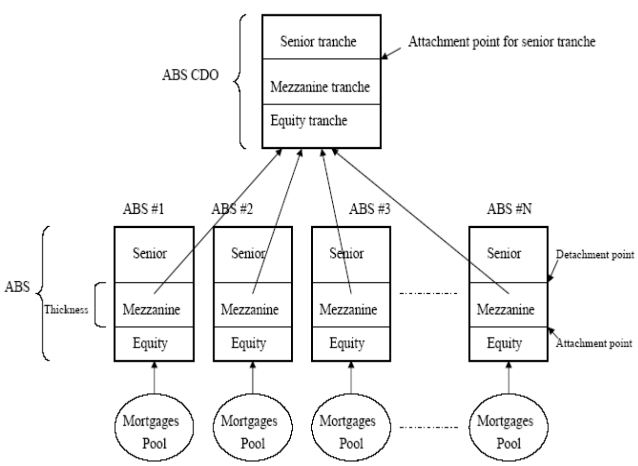

Figure 2: The structure of Mezz ABS CDO, created from mezzanine tranches of ABSs.

Each tranche in an ABS or an ABS CDO has its own attachment point (AP) and detachment point (DP). The AP is the lower bound of the loss covered by a tranche. The upper bound is called the DP. The range between the two points is called the thickness of the tranche. The tranche of an ABS begins to suffer loss when the cumulative losses of the reference entity exceed the AP. The tranche experiences total loss when the cumulative losses reach the DP. The loss of ABS tranches will pass through to the ABS CDO.

Similarly, tranches of ABS CDO suffer loss when the cumulative losses of underlying ABS tranches exceed the AP. The ABS CDO tranche experiences total loss when the cumulative losses reach the DP. In other words, the tranche is defined by establishing the AP and DP.

3. THE LOSS FUNCTION

To calculate the minimum attachment point for AAA tranche of ABS CDO formed from various ABS tranches, we use multi-pool correlation loss function. To justify the AAA rating assigned to tranches of ABS CDO, we have to compare attachment points for AAA tranche of ABS CDO with that for ABS tranches of the same rating. The latter attachment point can be calculated using a single pool correlation loss function.

First, we derive a single pool correlation loss function. Assume mortgages in the pool to have equal principals and to have same probability of default. Also denote ABS CDO loss function as

$$L = \rho L_C + (1-\rho) L_i \qquad (1)$$

where L_C follows a Poisson distribution with hazard rate parameter λ_C, and L_i, a factor specific to mortgage *i*, also follows a Poisson distribution with hazard rate arameter λ_i. The parameter ρ is the correlation between the losses of any two mortgages.

In this way, L_C is related to ratings of mortgages. The purpose of L_i is to decide the attachment point for a specific rating ABS tranche. The loss function is the sum of two Poisson distributions, and would be a random variable independent of the Poisson distribution. By using moment generating function (MGF), we can derive that expected value E(L) of the loss function L as

$$E(L) = E(\rho L_C) + E((1-\rho) L_i) = \rho\lambda_C + (1-\rho)\lambda_i \qquad (2)$$

Hence,

$$\lambda_i = (E(L) - \rho\lambda_C)/(1-\rho) \qquad (3)$$

It should be noted that E(L) divided by the tranche principal is equivalent to the expected default rate (EDR). This is the expected proportion of the mortgages in the portfolio that will default.

Secondly, we calculate the attachment points of ABS tranches given a specific (AAA or BBB) rating. In the previous loss function framework, the two-tailed 10% confidence level for CDO loss, conditional on L_C, is

$$\lambda_i + z_{0.1}\sqrt{\lambda_i} \qquad (4)$$

Assume the realized loss follow a normal distribution with mean λ_i and variance λ_i , where $z_{0.1}$ = 1.2816 is the 10th quantile of the Normal distribution. After the realized loss is calculated, we then know what the attachment point should be if it is to have the same expected loss of principal as a specifically rated corporate bond. In Hull and White (2010), they use one-factor Gaussian copula for calculating the attachment point. Their copula model has both a factor common to all mortgages, which is denoted by M, and a factor specific to mortgages i , which is denoted by Z_i . The factors M and Z_i are assumed to have independent standard normal distributions. Therefore, the attachment point is

$$(1-R)N\left[\frac{N^{-1}(EDR)-\sqrt{\rho}N^{-1}(PD)}{\sqrt{1-\rho}}\right] \qquad (5)$$

where *R* is the recovery rate, and *PD* is the probability of default for a specific rating bond.

To calculate the attachment point of ABS CDO tranche, multi-pool correlation model has to be used. To consider several pools simultaneously, we define a between-pool factor L_R, and within-pool factor L_W. The factor L_R affects probabilities of default for all mortgages, where as factor L_W affects expected losses of ABS CDO. Thus, ABS CDO loss function in a multi-pool case can be written as

$$L = \alpha\rho L_R + (1-\alpha)\rho L_W + (1-\rho)L_i \qquad (6)$$

The parameter ρ is the total within-pool correlation. The parameter α is the proportion of the default correlation that comes from a factor common to all pools. A multi-pool correlation model is useful when considering ABS CDOs. One of the potential advantages of ABS CDOs over ABSs is that investors benefit from both between-pool and within-pool diversification. Suppose that half the underlying pools of an ABS CDO consist of mortgages on homes in Pennsylvania and the other half consist of mortgages in California. If the mortgage default in California is not so correlated with that in Pennsylvania, investors receive a diversification benefit. The parameter α measures this benefit. If α is low, this extra diversification is valuable to investors, but if α is high, it has little value.

Finally, we calculate attachment points of ABS CDO tranches by using the expected loss criterion. Denoting $L(L_C)$ the loss on the mortgage portfolio for a particular value of L_C, the expected loss on the ABS CDO when the attachment point X for the senior tranche (with one hundred mortgages) is

$$\sum_{L_C^*}^{100}[L(L_C)-X]\theta(L_C) \qquad (7)$$

where L_C^* is the value of L_C. By adjusting factor L_R, we let the expected loss on the ABS CDO equals to a given expected loss, which is often calculated by rating agencies. After L_R is decided, the attachment point *X* is thus obtained.

By examining whether the attachment point of ABS tranches can withstand the total losses of the underlying mortgage pool, we can determine whether the assigned ratings are reasonable.[1] Given the probability of default for a specific rating bond, the attachment points of ABS are then calculated.

In addition to the probability of default, expected loss of a specifically rated bond is needed when calculating attachment points of ABS CDO tranches. The reason is risks of tranches in an ABS CDO are critically dependent on correlation between different asset pools. Therefore, valuing ABS CDO needs an additional parameter α, a between-pool correlation which describes correlation between underlying ABS tranches pools. It is for calculating this additional parameter that the expected loss is needed.

For rating, S&P/Fitch criterion ignores between-pool correlation and depends only on probability of default. Moody's criterion depends on both probability of default and expected loss for a specific rating bond. Various researches make different assumptions about the correlation structure. For example, Coval, et al. (2008) assumed that the asset pools underlying ABS CDOs have zero default correlation with each other.

By comparing attachment points of ABS CDO tranches with those of same rating bonds, one can determine whether the ratings assigned to ABS CDO tranches are reasonable. In the next section, we use a test to determine whether means of two attachment points are equal. If the null hypothesis is accepted, we can conclude that the ratings assigned to ABS CDO tranches are reasonable.

[1] For example, on page 62 of John and Hull (2010), with attachment points result from their Table 2, they calculate the corresponding losses and conclude that the AAA ratings were not totally unreasonable.

4. TESTING RESULTS

In this section, we use statistics from Moody's for 1970-2007 concerning the cumulative five-year probability of default for AAA and BBB bonds. Probabilities of loss for AAA and BBB bonds are 0.1% and 1.8%, respectively. We assume the underlying ABS tranches are responsible for losses of 4-9 percent. Constant recovery rate 40% is given.

Table 1 shows results using Hull and White (2010) model for senior ABS CDO formed from AAA ABS rated tranche. When EDR = 10%, with probability of loss criteria, the CDO attachment will be 28.2% on average. With expected loss criteria used in multi-pool correlation model, the attachment is 35.4% on average. The two-tailed t test for equality of means is 1.224, which is not significant at the 5% level. It implies that the AAA ratings assigned to the senior tranche of ABS CDO is reasonable.

Table 1: Minimum Attachment Points Using Hull and White Model for High Grade ABS CDO senior Tranche

EDR = 10%

	α=0.05	α=0.5	α=0.95	Multi	Single
ρ= 0.05	28.67%	30.15%	35.75%	30.92%	16.34%
ρ= 0.1	31.61	33.31	36.44	33.66	22.45
ρ= 0.2	34.88	35.40	40.86	36.18	32.68
ρ= 0.3	40.13	40.84	42.14	40.99	41.30
			Average=	35.44	28.19

EDR = 5%

	α=0.05	α=0.5	α=0.95	Multi	Single
ρ= 0.05	21.90%	23.01%	28.55%	24.14%	9.83%
ρ= 0.1	22.87	24.81	29.05	25.36	14.45
ρ= 0.2	27.01	27.78	31.76	28.45	23.07
ρ= 0.3	31.07	32.19	34.22	32.41	31.36
			Average=	27.59	19.68

Notes:

1. Assuming constant recovery rate 40%, and using Gaussian copula.
2. Column "Multi" denotes average of attachment point using multi-pool correlation model given a specific ρ value.
3. Column "Single" denotes attachment point using single pool correlation model.
4. The two-tailed t test with $df = 3$ for equality of means for EDR = 10% and 5% are 1.224 and 1.549, respectively. Both t values are not significant at the 5% level.

Table 2 shows results using loss function model for senior ABS CDO formed from AAA ABS tranche. When EDR = 10%, with probability of loss criteria, the CDO attachment will be 33.9% on average. With expected loss criteria used in multi-pool correlation model, the attachment is 23.9% on average. The two-tailed t test for equality of means is -5.642, which is significant at the 5% level. The attachment point 23.9% for ABS tranche less than 33.9% for corporate bond implies that that the AAA ratings assigned to the senior tranche of ABS CDO is over-valued.

Table 2: Minimum Attachment Points Using Loss Function Model for High Grade ABS CDO senior Tranche

EDR = 10%

	α=0.05	α=0.5	α=0.95	Multi	Single
ρ= 0.05	0.48%	21.07%	45.12%	21.84%	31.25%
ρ= 0.1	0.50	21.86	46.71	22.65	32.28
ρ= 0.2	0.53	23.66	50.31	24.47	34.64
ρ= 0.3	0.56	25.11	53.21	26.73	37.57
			Average=	23.92	33.94

EDR = 5%

	α=0.05	α=0.5	α=0.95	Multi	Single
ρ= 0.05	0.33%	12.87%	28.85%	13.61%	20.79%
ρ= 0.1	0.34	13.40	29.88	14.13	21.43
ρ= 0.2	0.36	14.61	32.28	15.34	22.92
ρ= 0.3	0.39	22.01	35.19	18.00	24.75
			Average=	15.27	22.47

Notes: The two-tailed t test with $df = 3$ for equality of means for EDR = 10% and 5% are -5.642 and -5.467, respectively. Both t values are significant at the 5% level.

Table 3 shows results using Hull and White (2010) model for mezz ABS CDO formed from BBB rated corporate bond. When EDR = 10%, with probability of loss criteria, the CDO attachment will be 18.7% on average. With expected loss criteria used in multi-pool correlation model, the attachment is 31.3% on average. The two-tailed t test for equality of means is 3.842, which is significant at the 5% level. The attachment point 31.3% for ABS tranche more than 18.7% for corporate bond implies that that the BBB ratings assigned to the mezz tranche of ABS CDO is under-valued.

Table 3: Minimum Attachment Points Using Hull and White Model for Mezz ABS CDO senior Tranche

EDR = 10%

	α=0.05	α=0.5	α=0.95	Multi	Single
ρ= 0.05	21.50%	28.80%	34.89%	28.78%	12.13%
ρ= 0.1	28.11	29.28	35.11	30.55	15.43
ρ= 0.2	29.94	31.78	35.19	32.16	21.02
ρ= 0.3	32.33	33.21	36.96	33.82	26.21
			Average=	31.33	18.70

EDR = 5%

	α=0.05	α=0.5	α=0.95	Multi	Single
ρ= 0.05	13.08%	22.28%	27.79%	21.60%	6.83%
ρ= 0.1	21.31	22.46	27.86	23.50	9.02
ρ= 0.2	21.70	23.14	27.67	23.91	12.88
ρ= 0.3	23.00	24.46	27.22	24.78	16.59
			Average=	23.45	11.33

Notes: The two-tailed *t* test with *df* = 3 for equality of means for EDR = 10% and 5% are 3.842 and 5.371, respectively. Both *t* values are significant at the 5% level.

Table 4 shows results using losing function model for mezz ABS CDO formed from BBB rated corporate bond. When EDR = 10%, with probability of loss criteria, the CDO attachment will be 33.3% on average. With expected loss criteria used in multi-pool correlation model, the attachment is 23.3% on average. The two-tailed *t* test for equality of means is -7.007, which is significant at the 5% level. The attachment point 23.3% for ABS tranche less than 33.3% for corporate bond implies that that the BBB ratings assigned to the mezz tranche of ABS CDO is over-valued.

Table 4: Minimum Attachment Points Using Loss Function Model for Mezz ABS CDO senior Tranche

EDR = 10%

	α=0.05	α=0.5	α=0.95	Multi	Single
ρ= 0.05	0.48%	20.94%	44.87%	21.72%	31.09%
ρ= 0.1	0.49	21.60	46.18	22.38	31.95
ρ= 0.2	0.52	23.13	49.24	23.93	33.92
ρ= 0.3	0.54	24.52	52.03	25.33	36.38
			Average=	23.34	33.34

EDR = 5%

	α=0.05	α=0.5	α=0.95	Multi	Single
ρ= 0.05	0.32%	12.70%	28.51%	13.43%	31.09%
ρ= 0.1	0.33	13.04	29.18	13.77	31.95
ρ= 0.2	0.35	13.83	30.74	14.57	33.92
ρ= 0.3	0.36	14.82	32.69	15.55	36.38
			Average =	14.33	33.34

Notes: The two-tailed *t* test with *df* = 3 for equality of means for EDR = 10% and 5% are -7.007 and -15.008, respectively. Both *t* values are significant at the 1% level.

Our results also show the impact of EDR on attachment points. As might be expected, attachment points become lower because of lower probability of default.

5. CONCLUSION

Rating agencies have come under great scrutiny after the subprime crisis started in 2007. Investment banks created securities from underlying mortgages. Credit rating agencies assigned ratings on these instruments by calculating probability of loss. We evaluated whether ratings assigned to structured products by rating agencies were reasonable. We looked at both high grade and mezzanine ABS CDO.

Using Hull and White (2010) model, we confirm their finding that the AAA rating assigned to the senior tranches of ABS is not unreasonable, and this conclusion cannot be extended to the rating assigned to tranches created from mezzanine ABS. or ABS CDO. However, using our loss function model, we show that not only the rating assigned to tranches created from mezzanine ABS is unreasonable, the AAA rating assigned the senior tranches of ABS is also not reasonable. The reason is that the probability distribution of loss from a BBB tranche is quite different from the probability distribution of loss from a BBB bond. The same reasoning can also be applied to the AAA rating case.

6. REFERENCES

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