TAKING INTO ACCOUNT PARAMETRIC UNCERTAINTIES IN ANTICIPATIVE ENERGY MANAGEMENT FOR DWELLINGS

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***Abstract*—Energy management in dwellings is addressed in this paper. The energetic impact of dwellings in the current energetic context is first depicted. The formulation of the global energy management problem of dwellings is defined as an optimization problem based on a Mixed Integer Linear Programming (MILP) algorithm. It aims at adjusting the energy consumption to both the energy cost and the inhabitant's comfort. The available flexibilities, provided by domestic appliances, are associated to time windows or heating storage abilities. The energy consumption in houses is very dependent to uncertain data such as weather forecasts and inhabitants' activities. The paper focuses on the taking into account parametric uncertainties in anticipative energy management for dwellings. Robust linear programming is implemented in order to provide the robust energy allocation. Application example is given.**

***Keywords*-Building, Energy management, Robust optimization, Uncertainties, Mixed integer linear programming.**

I. INTRODUCTION

A home automation system basically consists of household appliances linked via a communication network allowing interactions for control purposes [1]. Thanks to this network, a load management mechanism can be carried out: this is a functionality of the so-called smart home. Load management allows inhabitants, in this paper, to adjust power consumption according to expected comfort and energy price variation. For instance, during the consumption peak periods when power plants rejecting higher quantities of CO_2 are used and when energy price is high, it could be possible to decide to delay some consumption activities by reducing some heater set points. Load management is all the more interesting that the availability and price of the energy vary. It is very complex to manage by users in a dynamic pricing context.

A building energy management system consists in two aspects: the load management and the local energy production management. [2] have proposed optimal control strategies for HVAC (Home Ventilation and Air Conditioning) systems taking into account the natural thermal storage capacity of buildings that shift the HVAC consumption from peak period to off-peak period. [2] has shown that this control strategy can save up to 10% of the electricity cost of a building. The control temperature in buildings and more generally HVAC is widely studied in automatic control. Nowadays, a lot of studies promote the Model Predictive Control (MPC) for HVAC systems [3]. MPC consists in tracking a reference trajectory. This trajectory is the predicted thermal information for example. The predictive approach proposed in this paper could provide such trajectories for the MPC strategies. However, these approaches do not take into account the energy resource constraints, which generally depend on the autonomy needs of off-grid systems [4] or on the total power production limits of the suppliers in grid connected systems.

The household global load management problem is a larger problem than HVAC control. It can be formulated as an assignment problem in which energy is a resource shared by appliances, and tasks are energy consumption of appliances. A discrete optimization method of shortest path is proposed in [5] to deal with the prediction of optimal indoor temperature.

The generic home energy management problem as a scheduling problem is proposed in [6]. The available electric power at each time is a cumulative resource shared by the appliances. The tasks are the activities requested by the inhabitant that consume the supplied power in given time windows. A mathematical formulation of this problem is proposed that can be written as a Mixed Integer Linear Program. A lot of data are assumed in this formulation such as weather forecasts and inhabitants' requests. An optimal energy planning is proposed over a given planning horizon, typically one day.

In this paper, parametric uncertainties on the data are taken into account in order to propose robust energy planning in which a performance is guaranteed over the expected data. The associated robust formulation which is proposed to deal with is adapted from the Bertsimas and Sim formulation proposed in [7]. Parametric uncertainties, such as weather forecasts, are addressed through this formulation. This formulations lead to the definition of the Robust Home Energy Scheduling Problem.

The deterministic Home Energy Scheduling Problem is defined in section 2. The uncertainties that the home energy management system has to face and the robust optimization of parametric uncertainties as well as an application example are performed in section 3.

II. Anticipative Energy Management Problem

The Home Energy Scheduling Problem (HESP) takes as input a set of activities called *services*. One can distinguish the *provider services* that produce the energy resource and the *consumer services* having to be processed using the energy resource. The *consumer services* take place of the activities required by the inhabitant. The energy is a cumulative resource which availability and price vary from time to time. The optimization system aims at planning the services by finding the best compromise between the cost of the consumed energy (to be minimized) and the inhabitant's satisfaction (to be maximized) under constraints of power supply and inhabitant's requests. The HESP is modeled through a MILP formulation.

The *consumer services* are the activities required by the inhabitants that are energy consuming. Some of them are the tasks to be planned under the constraint of power supply service. Several types of consumer services can be defined in the optimization problem according to the type of involved control. One can distinguish the *permanent services* to be controlled all over the planning horizon (see section II-A) and the *timed services* to be scheduled in a time window according to the inhabitant's requests (see section II-B). The *unsupervised service* is associated to the set of activities that cannot be planned because they are totally driven by the inhabitants. Another segmentation is proposed in [8] based on the number of settings of the appliances and their automation level.

Let $H = \{0, \ldots, T-1\}$ be the planning horizon composed of T time periods of length Δ. The time period Δ is a data given by the variation of the resource. At every time period k the amount of energy allocated to each service has to be decided. The energy cost and the resource availability are assumed to be constant over a time period Δ. [6] and [9] propose a formulation of the energy management problem in which the execution of the services are assumed to be synchronized to the time period.

A. Permanent services

The permanent services depict services that are continuously delivered and controlled all over the planning horizon. Typically the room heating and refrigerating services are permanent services. Let us assume $SRV(i)$ such a permanent service characterized by the following data :

- $P(i)$ the required power in execution
- $T_{opt}(i,t)$, $T_{\min}(i,t)$ and $T_{\max}(i,t)$ respectively the optimal, the minimum and maximum satisfying controlled parameters at time t

For example the inhabitant requires a temperature in his room in the satisfying interval [18°C, 20°C]. The optimization problem aims at setting the best temperature at each time to minimize the energy cost and maximize the inhabitant's satisfaction. The thermal ability storage of the building is then used to reach this optimum. In HESP only one type of permanent service is addressed. A permanent service is assumed to control a physical variable such as temperature. The temperature set point $T_{in}(i,k)$ is the decision variable associated to the permanent service i. The set point modulation corresponds to a variable amount of energy $E(i,k)$ allocated to the permanent service. $T_{in}(i,k), E(i,k)$ are the decision variables of this optimization problem.

The permanent services that are modeled by a first order dynamic are addressed in HESP. The discrete time model of a room heating service $SRV(i)$ is as follows:

$$\begin{aligned} T_{in}(i,k+1) = \quad & e^{\frac{-\Delta}{\tau(i)}} T_{in}(i,k) + (1 - e^{\frac{-\Delta}{\tau(i)}}) T_{out}(i,k) \\ & + G(i)(1 - e^{\frac{-\Delta}{\tau(i)}}) E(i,k) \\ & + G_s(i)(1 - e^{\frac{-\Delta}{\tau(i)}}) \phi_s(i,k) \end{aligned} \tag{1}$$

$$T_{min}(i,k) \leq T_{in}(i,k) \leq T_{max}(i,k) \tag{2}$$

with the following data parameters:

- T_{in}, T_{out} the indoor and outdoor temperatures
- ϕ_s the equivalent electric power generated by the solar radiation
- G, G_s the gains of the first order dynamic from the heating power and the solar radiance respectively
- τ the constant time of the first order dynamic

According to the typical models for the thermal comfort proposed in [10], the *dissatisfaction index* for the permanent service $SRV(i)$ at each time period k of a given type of thermal space is computed as follows :

$$D(i,k) = \begin{cases} \frac{T_{opt}(i,k) - T_{in}(i,k)}{T_{opt}(i,k) - T_{\min}(i,k)} & \text{if} \quad T_{in}(i,k) \leq T_{opt}(i,k) \\ \frac{T_{in}(i,k) - T_{opt}(i,k)}{T_{\max}(i,k) - T_{opt}(i,k)} & \text{if} \quad T_{in}(i,k) > T_{opt}(i,k) \end{cases} \tag{3}$$

The global dissatisfaction $D(i)$ associated to $SRV(i)$ can then be defined as follows:

$$D(i) = \sum_{k=0}^{T-1} D(i,k) \tag{4}$$

B. Timed services

A timed service depicts an activity that is required at some time and which execution has a given duration lower than the planning horizon. The timed services are associated with appliances which number of settings is not too large, typically few times per day. The set of timed services is a set of tasks to be scheduled with time windows constraints (ready times and due dates) and resource sharing constraints. Preemption is not available. Typically cooking a meat with an oven, washing dishes are examples of timed services. A timed service $SRV(i)$ is characterized by the following input data:

- $P(i)$ the required power in execution
- $f_{opt}(i)$, $f_{\min}(i)$ and $f_{\max}(i)$ the requested, earliest and latest allowed ending times respectively

- $d(i)$ the execution time

Timed services such as washing are expected by the inhabitant to be finished at a given preferred time denoted $f_{opt}(i)$. The earliest and latest requested ending times are given by the inhabitant for the programmable services or predicted from the user's behavior for the not programmable services. The ending time $f(i)$ is the decision variable associated to a timed service.

The service quality achievement depends on the amount of time it is shifted from this preferred value. Then the following *dissatisfaction index* $D(i)$ can be computed for a timed service.

$$D(i) = \begin{cases} \frac{f(i)-f_{opt}(i)}{f_{\max}(i)-f_{opt}(i)} & \text{if} \quad f(i) > f_{opt}(i) \\ \frac{f_{opt}(i)-f(i)}{f_{opt}(i)-f_{\min}(i)} & \text{if} \quad f(i) \leq f_{opt}(i) \end{cases} \tag{5}$$

C. Unsupervised service

All energy consuming activities in housing cannot be taken into account as services to be scheduled. Lighting is one of the best examples of unsupervised service. Indeed put the light on is totally dependent on the inhabitant's presence in a room, a parameter that is neither controllable nor predictable at the planning level. Then all activities that cannot be controlled and/or individually predicted have no interest to be planned to optimality. Those activities are merged together into one unsupervised service. The unsupervised service is defined by the energy $P_u(k)$ consumed at period k given as a data for the optimization problem. No decision variable is associated to the unsupervised activities in the scheduling problem.

D. Power supply

Let us denote $SRV(i)$ a power supply activity. The power supply is the cumulative resource and represents the available power $P(i,k)$ over the planning horizon and the associated cost $C(i,k)$ at every time period for a given production mean. In the problem HESP, the available power can be consumed by the services or not. Then a power supply service $SRV(i)$ is associated to the following constraint:

$$E(i,k) \leq P(i,k)\Delta \quad \forall k \in \{0, ..., T-1\} \tag{6}$$

where $E(i,k)$ the provided energy during the time window $]k\Delta, (k+1)\Delta]$. This constraint aims at translating the available power into a maximal amount of energy per time period.

E. Energy balance

A constraint modeling the production/consumption balance has to be added. This constraint can be written as follows:

$$\sum_{j \in \mathcal{I}_{\mathcal{S}}} E(j,k) = \sum_{i \in \mathcal{I}_{\mathcal{C}}} E(i,k) + P_u(k)\Delta \quad \forall k \in \{0, \ldots, T-1\} \tag{7}$$

where $\mathcal{I}_{\mathcal{S}}$ and $\mathcal{I}_{\mathcal{C}}$ are respectively the set of indexes of the power supply services and the consumer services.

F. Objective function

Depending on the inhabitants' requests, a compromise between the cost and the comfort has to be exhibited. This is generally the case when the energy cost changes. The highest cost corresponds to the peak consumption periods. An aggregation approach has been implemented to exhibit such a compromise. The corresponding objective function to be minimized is depicted by equation (8):

$$\sigma = \sum_{j \in \mathcal{I}_{\mathcal{S}}} \sum_{k=0}^{T-1} C(j,k)E(j,k) + \frac{\beta}{\sum_{i \in \mathcal{I}_{\mathcal{C}}} \alpha(i)} \sum_{i \in \mathcal{I}_{\mathcal{C}}} \alpha(i)D(i) \tag{8}$$

The parameters $\alpha(i)$ depict the priorities between the consumer services and the parameter β depicts the relative importance given by the user to the energy cost and the comfort.

The optimization problem is defined by equations (1) to (8). The linearization of this formulation is not addressed in this paper. It can be performed using integer variables. In the following of the paper the linear equation (1) will be used to illustrate the robust parametric optimization.

III. Robust energy management problem

In this section, parametric uncertainties are taken into account in the optimization problem. The robust formulation introduced by Bertsimas and Sim [7] is depicted and then adapted to the home energy management problem. Uncertain parameters in the permanent services are addressed with this robust approach.

A. Uncertainties analysis in the home energy system

The home energy management problem has been modeled with a lot of parameters that are uncertain data. This is generally the case in modeling. This is particularly true in the energy management problem because of the high sensitivity of the decision (the energy allocation) to the data. Generally speaking, every forecasted data involved into a model is not exactly known.

Sources of uncertainties in the problem of energy management are very numerous. In this paper uncertainties related to the uncertain character of data is only studied. The uncertain weather forecast is a typical example of such uncertainties. The weather forecast is responsible for two main parameters that play a key role in the dynamic equation of the permanent services (i.e. the outdoor temperature and the solar radiation). These data define the required amount of energy to be consumed to satisfy the requested indoor temperature. In order to take into account such uncertainties a parametric model is assumed and implemented into the linear equations. This formulation can be used for every uncertain parameter a_i. In the assumed model of uncertainty the parameter a_i is associated with a random variable $\tilde{a}_i$ that takes value according to a symmetric distribution in an interval centered around the nominal or expected value. The robust linear optimization will be depicted in this section to face these uncertainties.

B. The robust formulation of Bertsimas and Sim

A linear formulation is proposed in [7] to face parametric uncertainties under a convex model of data uncertainty. Let us assume the ith constraint of the deterministic problem, i.e. $a_i x \leq b_i$. The formulation of the robust optimization of parametric uncertainties is firstly explained in the basic case of independent parameters. It is then extended to the case of correlated parameters.

1) Independent parameters: Let J_i be the set of coefficients a_{ij}, $j \in J_i$ that are subject to parameter uncertainty. These parameters are assumed to be independently uncertain. Then the parameter uncertainty is defined as a random variable $\widetilde{a}_{ij}$, $j \in J_i$ that takes value according to a symmetric distribution with mean equal to the nominal value a_{ij} in the interval $[a_{ij} - \widehat{a}_{ij}, a_{ij} + \widehat{a}_{ij}]$. For each constraint i a parameter Γ_i is introduced. It takes values in the interval $[0, |J_i|]$. The parameter Γ_i allows adjusting the robustness of the proposed method against the required level of conservatism of the solution. The coefficients of the ith constraint are allowed to change up to $\widehat{a}_{it}$ and one coefficient a_{it} changes up to $(\Gamma_i - \lfloor \Gamma_i \rfloor)\widehat{a}_{it}$. This robust formulation can be written as follows:

$$\begin{array}{ll} \text{maximize} & \mathbf{cx} \\ \text{subject to} & \sum_j a_{ij}x_j + \beta_i(\mathbf{x}, \Gamma_i) \leq b_i \quad \forall i \\ & -y_j \leq x_j \leq y_j \quad \forall j \\ & \mathbf{y} \geq 0 \end{array} \tag{9}$$

with $\beta_i(\mathbf{x}, \Gamma_i)$ the protection function of the ith constraint that is written as follows:

$$\beta_i(\mathbf{x}, \Gamma_i) = \max_{S_i \cup t_i | S_i \subseteq J_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i} \left\{ \sum_{j \in S_i} \widehat{a}_{ij} y_j + (\Gamma_i - \lfloor \Gamma_i \rfloor) \widehat{a}_{it_i} y_{t_i} \right\} \tag{10}$$

This definition of the robust optimization can be derived into a linear formulation as follows:

$$\begin{array}{ll} \text{maximize} & \mathbf{cx} \\ \text{subject to} & \sum_j a_{ij}x_j + z_i\Gamma_i + \sum_{j \subseteq J_i} p_{ij} \leq b_i \quad \forall i \\ & z_i + p_{ij} \geq \widehat{a}_{ij} y_j \quad \forall i, j \subseteq J_i \\ & -y_j \leq x_j \leq y_j \quad \forall j \\ & p_{ij}, y_j, z_i \geq 0 \quad \forall i, j \subseteq J_i \end{array} \tag{11}$$

Thanks to this formulation the probability that the ith constraint is violated can be computed. Let us denote x_j^* an optimal solution of the robust problem 11. Then it yields:

$$\Pr\left(\sum_j \tilde{a}_{ij} x_j^* \geq b_i\right) \leq \frac{1}{2^n} \left\{ (1-\mu) \binom{n}{\lfloor \nu \rfloor} + \sum_{l = \lfloor \nu \rfloor + 1}^{n} \binom{n}{\lfloor l \rfloor} \right\} \tag{12}$$

with $n = |J_i|$, $\nu = \frac{\Gamma_i + n}{2}$ and $\mu = \nu - \lfloor \nu \rfloor$.

From this principle for robust optimization one can extend the formulation to take into account correlated parameters.

2) Correlated parameters: Let us now assume the correlated data a_{ij}. The variations of all a_{ij} are due to $|K_i|$ sources of data uncertainty. Then the following model is assumed:

$$\tilde{a}_{ij} = a_{ij} + \sum_{k \in K_i} \tilde{\eta}_{ik} g_{kj} \tag{13}$$

with $\tilde{\eta}_{ik}$ independent and symmetrically distributed random variables in [-1, 1] and g_{kj} the range of variation of the parameter a_{ij} related to the source of uncertainty k. From this model of uncertainty a linear formulation of the robust problem can be written as follow:

$$\begin{array}{ll} \text{maximize} & \mathbf{cx} \\ \text{subject to} & \sum_j a_{ij}x_j + z_i\Gamma_i + \sum_{k \subseteq K_i} p_{ik} \leq b_i \quad \forall i \\ & z_i + p_{ij} \geq y_{ik} \quad \forall i, k \subseteq K_i \\ & -y_{ik} \leq \sum_{j \subseteq J_i} g_{kj} x_j \leq y_{ik} \quad \forall i, k \subseteq K_i \\ & p_{ik}, y_{ik} \geq 0 \quad \forall i, k \subseteq K_i \\ & z_i \geq 0 \quad \forall i \end{array} \tag{14}$$

C. Robust optimization for parametric uncertainties

In this section robust optimization for parametric uncertainties is illustrated. Every parameter that is involved in the constraints of the HESP can be assumed as an uncertain parameter. Weather forecast are taken as example because they are the most illustrative and the most sensitive to energy allocation. The outdoor temperature T_{out} and the solar radiation ϕ_S are assumed to be known through expected values ($\bar{T}_{out}$, $\bar{\phi}_S$) and precision levels such that $T_{out} \in \left[\bar{T}_{out} - \widehat{T}_{out}, \bar{T}_{out} + \widehat{T}_{out}\right]$ and $\phi_S \in \left[\bar{\phi}_S - \widehat{\phi}_S, \bar{\phi}_S + \widehat{\phi}_S\right]$.

One can introduce the protection function in equation (1) using the formulation (11) applied to the right-hand-side coefficients:

$$\begin{aligned} T_{in}(i,k+1) - e^{\frac{-\Delta}{\tau(i)}} T_{in}(i,k) - G(i)(1 - e^{\frac{-\Delta}{\tau(i)}}) E(i,k) \\ -(1 - e^{\frac{-\Delta}{\tau(i)}}) T_{out}(i,k) x_1(i,k) \\ -G_s(i)(1 - e^{\frac{-\Delta}{\tau(i)}}) \phi_s(i,k) x_2(i,k) + \beta(i,k) = 0 \\ x_1(i,k) = x_2(i,k) = 1 \end{aligned} \tag{15}$$

1) Independent parameters: Let us assume that the variations of T_{out} and ϕ_S are independent. With the two assumed uncertain parameters, the parameter $\Gamma(i,k)$ takes values in the interval $[0, 2]$. From the equation (10) the protection function can be written as follow:

- If $0 \leq \Gamma(i,k) \leq 1$:

$$\beta(i,k) = [\Gamma(i,k) - \lfloor \Gamma(i,k) \rfloor] \max\left\{ \widehat{T}_{out}(i,k) y_1(i,k); \widehat{\phi}_s(i,k) y_2(i,k) \right\} \tag{16}$$

- If $1 < \Gamma(i,k) \le 2$:

$$\begin{aligned} \beta(i,k) = &\quad \max\{a_1, a_2\}, \\ a_1 = &\quad \widehat{T}_{out}(i,k)y_1(i,k) \\ &\quad +[\Gamma(i,k) - \lfloor\Gamma(i,k)\rfloor]\widehat{\phi}_s(i,k)y_2(i,k) \\ a_2 = &\quad [\Gamma(i,k) - \lfloor\Gamma(i,k)\rfloor]\widehat{T}_{out}(i,k)y_1(i,k) \\ &\quad +\widehat{\phi}_s(i,k)y_2(i,k) \end{aligned} \tag{17}$$

The equivalent linear program using (14) is as follow:

$$\begin{aligned} &T_{in}(i,k+1) - e^{\frac{-\Delta}{\tau(i)}}T_{in}(i,k) \\ &\quad -G(i)(1-e^{\frac{-\Delta}{\tau(i)}})E(i,k) \\ &\quad -(1-e^{\frac{-\Delta}{\tau(i)}})T_{out}(i,k)x_1(i,k) \\ &\quad -G_s(i)(1-e^{\frac{-\Delta}{\tau(i)}})\phi_s(i,k)x_2(i,k) \\ &\quad +z(i,k)\Gamma(i,k) + p_1(i,k) + p_2(i,k) = 0 \end{aligned} \tag{18}$$

$$z(i,k) + p_1(i,k) - y_1(i,k)\widehat{T}_{out}(i,k) \ge 0 \tag{19}$$

$$z(i,k) + p_2(i,k) - y_2(i,k)\widehat{\phi}_S(i,k) \ge 0 \tag{20}$$

$$-y_1(i,k) \le x_1(i,k) \le y_1(i,k) \tag{21}$$

$$-y_2(i,k) \le x_2(i,k) \le y_2(i,k) \tag{22}$$

$$x_1(i,k) = x_2(i,k) = 1 \tag{23}$$

$$p_1(i,k), y_1(i,k), p_2(i,k), y_2(i,k), z(i,k) \ge 0 \tag{24}$$

The violation probability can be calculated from equation (12) with $n = 2$. $\Gamma(i,k)$ is the attribute that has to be chosen to fix the required protection level.

Remark: The unsupervised consumed power $P_u(k)$ (7) could also be assumed as uncertain data and then studied in a robust formulation. However it is not realistic to assume that the variations of $P_u(k)$ satisfy an uniform distribution around a nominal value. A formulation derived from the definition of the service rate is better adapted. This kind of approach is not addressed in this paper.

2) Correlated parameters: Let us now assume the correlation between T_{out} and ϕ_S, that is the most realistic assumption. In this case, the parameter $\Gamma(i,k)$ takes values in the interval $[0,1]$. From the formulation (14), the robust formulation can be written as follow:

$$\begin{aligned} T_{in}(i,k+1) &\quad -e^{\frac{-\Delta}{\tau(i)}}T_{in}(i,k) \\ &\quad -G(i)(1-e^{\frac{-\Delta}{\tau(i)}})E(i,k) \\ &\quad -(1-e^{\frac{-\Delta}{\tau(i)}})T_{out}(i,k)x_1(i,k) \\ &\quad -G_s(i)(1-e^{\frac{-\Delta}{\tau(i)}})\phi_s(i,k)x_2(i,k) \\ &\quad +z(i,k)\Gamma(i,k) + p(i,k) = 0 \end{aligned} \tag{25}$$

$$z(i,k) + p(i,k) - y(i,k) \ge 0 \tag{26}$$

$$-y(i,k) \le \widehat{T}_{out}(i,k)x_1(i,k) + \widehat{\phi}_S(i,k)x_2(i,k) \tag{27}$$

$$\widehat{T}_{out}(i,k)x_1(i,k) + \widehat{\phi}_S(i,k)x_2(i,k) \le y(i,k) \tag{28}$$

$$x_1(i,k) = x_2(i,k) = 1 \tag{29}$$

$$p(i,k), y(i,k), z(i,k) \ge 0 \tag{30}$$

D. Application example

Let us consider a simple example of allocation plan computation for a housing for the next 24 hours. A 10 period planning horizon is assumed and four services have to be delivered: $SRV(1)$ is a grid power supplier, $SVR(2)$, $SVR(3)$ are respectively the room HVAC service of bedroom and living room, and $SVR(4)$ corresponds to a clothes washer. The forecasted outdoor temperature of this thermal zone as well as the forecasted solar radiation is given in the figure 1. The actual values are random variables supposed to be uniformly distributed around the forecasted value with a precision $\widehat{T}_{out} = 1$°C for the temperature and $\widehat{\phi}_S = 30$W for the solar radiation. These two uncertain parameters T_{out} and ϕ_S are assumed as independent parameters.

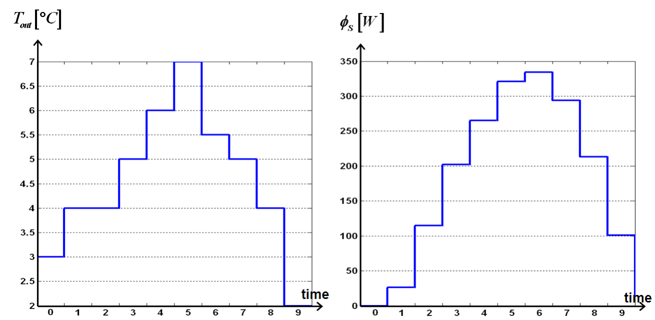

Figure 1. Forecasted values of the uncertain parameters

The optimization results of the heating service of bedroom and the clothes washer service are used to examine the quality of the robust solution. The heating bedroom $SRV(2)$ consumes $P(2) = 1000W$ in execution. The required minimum and maximum allowed temperatures equal respectively 22°C, 19°C and 25°C. The clothes washer is considered as a timed service $SVR(4)$. It just can be shifted providing that the following comfort constraints are satisfied: $f_{\min}(4) = 8$am, $f_{\max}(4) = 9$pm and $f_{opt}(4) = 5$pm. This service requires 1500W in execution.

The problem (18) is solved for various levels of $\Gamma(2,k)$. Figure 2 illustrates the energy reservation for the heating of bedroom at the 5th period $E^*(2,5)$ as a function of the protection level $\Gamma(2,5)$. The required energy increases from 0 to 113W with the increasing values of $\Gamma(2,5)$ to maintain the best thermal satisfaction in the bedroom at this period. The figure 3 shows the robust energy planning for the heating of bedroom service and the clothes washing service. As expected, more the protection level increases, more the energy reservation for the thermal service increases, so more the timed service is shifted from the requested time to satisfy the constrains of power supplier.

The violation probability of the constraint associated to the thermal model, from equation (12), is also given in the figure 4 as a decreasing function of $\Gamma(2,k)$. This kind of curve can be used to choose the value of $\Gamma(2,k)$ from a given risk to be covered or in other words from a given service rate. The so-called worst case is very easy to exhibit. A decisional strategy has to be added to choose a solution by capturing the best trade-off between risk and performance.

IV. Conclusion

Energy management in buildings is a very important problem due to the great impact of buildings in the current energetic context. In this paper, an optimization point of view for energy management in houses has been depicted. The energy consumption in buildings is very dependent to the value of the data of the optimization problem. The robust optimization approach has been introduced to take into account parametric uncertainties such as weather forecast. Robust energy allocations have been demonstrated. Some works have now to be developed to propose the way how to choose the protection level of parametric uncertainties with regards to the entire set of required services.

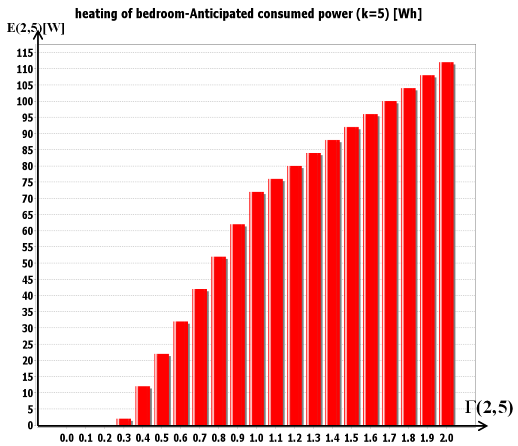

Figure 2. Energy reservation for the heating of bedroom service at the 5th period

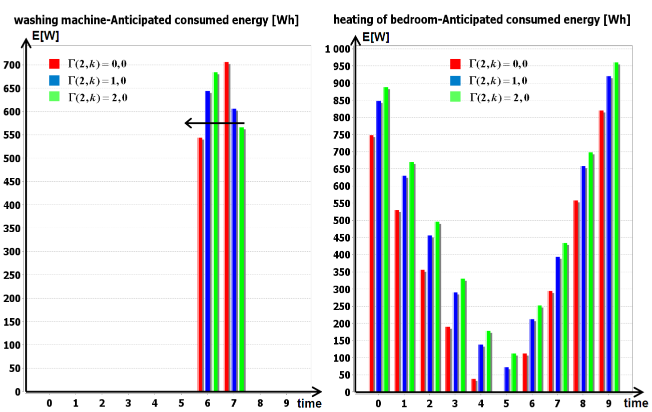

Figure 3. Robust energy planning for the heating of bedroom service and the clothes washer

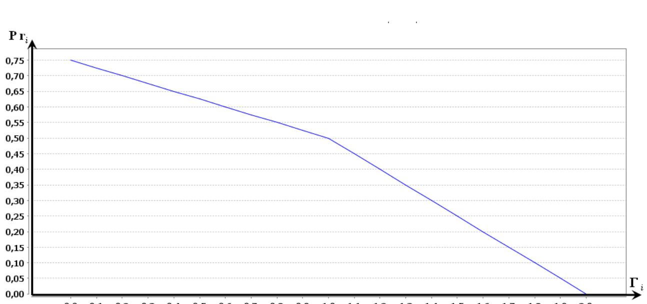

Figure 4. Violation probability

References

[1] P. Palensky and R. Posta, "Demand side management in private home using lonworks," in *Proceedings.1997 IEEE International Workshop on Factory Communication Systems*, 1997.

[2] G. Zhou and M. Krarti, "Parametric analysis of active and passive building thermal storage utilization," *Journal of Solar Energy Engineering*, vol. 127, pp. 37–46, 2005.

[3] R. Z. Freire, G. H. Oliveira, and N. Mendes, "Predictive controllers for thermal comfort optimization and energy savings," *Energy and Buildings*, vol. 40, pp. 1353–1365, 2008.

[4] M. Muselli, G. Notton, P. Poggi, and A. Louche, "Pv-hybrid power system sizing incorporating battery storage: an analysis via simulation calculations," *Renewable Energy*, vol. 20, pp. 1–7, 2000.

[5] D. L. Ha, S. Ploix, E. Zamai, and M. Jacomino, "A home automation system to improve the household energy control," in *INCOM2006 12th IFAC Symposium on Information Control Problems in Manufacturing*, 2006.

[6] G. D. Oliveira, M. Jacomino, D. L. Ha, and S. Ploix, "Optimal power control for smart homes," in *The 18th World Congress of the International Federation of Automatic Control (IFAC), Milano, Italy*, 2011.

[7] D. Bertsimas and M. Sim, "The price of robustness," *Operations Research*, vol. 52(1), pp. 35–53, 2004.

[8] G. Wood and M. Newborough, "Energy-use information transfer for intelligent homes: Enabling energy conservation with central and local displays," *Energy and Buildings*, vol. 39, pp. 495–503, 2007.

[9] C. Artigues, P. Lopez, and A. Haït, "Scheduling under energy constraints," in *International Conference on Industrial Engineering and Systems Management*, 2009.

[10] AFNOR, "Ergonomie des amniances thermiques , détermination analytique et interprétation du confort thermique par le calcul des indices PMV et PDD et par des critères de confort thermique local," *Norme européenne, norme française*, 2006.