Integrated inventory model with multi-retailer and rework process

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ABSTRACT

This paper derives optimal production-shipment policy for a single-producer multi-retailer integrated inventory model with rework. We assume that a product is manufactured by a producer. All items are screened for quality assurance. Random defective items will be picked up and reworked in the same cycle when regular production ends. Upon entire lot is quality assured, multiple shipments are delivered synchronously to *m* different retailers in each production cycle. Each retailer has its own annual product demand, unit stock holding cost, and fixed and variable delivery costs. Mathematical modeling and analysis is used to derive the expected system cost. The Hessian matrix equations are employed to prove the convexity of cost function. A closed-form optimal production-shipment policy that minimizes total expected costs for such a specific integrated inventory model is obtained.

Keywords: Production, Lot sizing, Multiple shipments, Multiple retailers, Rework

1. INTRODUCTION

In real-life business environments, it is common to have a supplier who provides a product to its several retailer clients. In this type of supply chains, management will be pleased to figure out the best production-shipment policy in order to minimize the expected integrated system costs. Schwarz [1] first studied a one-warehouse N-retailer deterministic inventory system with the objective of deriving the stocking policy that minimizes the long-run average system cost per unit time. The optimal solutions along with a few necessary properties are derived for such a one-retailer and N identical retailer problems. Heuristic solutions for the general problem were also suggested. Production lot size was not considered in his model. Goyal [2] considered an integrated inventory model for a single supplier-single customer problem. A method was proposed for solving those inventory problems, wherein a product made by a single supplier is procured by a single customer. A numerical example was provided to verify his solution process. Banerjee [3] investigated a joint economic lot-size model for purchaser and vendor, with the focus on minimizing the joint total relevant costs. He concluded that a jointly optimal ordering policy, together with an appropriate price adjustment, could be economically beneficial for both parties, but definitely not disadvantageous to either party. Kim and Hwang [4] developed formulation of a quantity discount pricing schedule for a supplier. They assumed a single incremental discount system and proposed an algorithm for deriving an optimal discount schedule. They investigated cases in which both the discount rate and the break point are unknown and either one is prescribed, and used numerical example to illustrate their algorithm. Cetinkaya and Lee [5] presented an analytical model for coordinating inventory and transportation decisions in vendor-managed inventory (VMI) systems. They considered a vendor realizing a sequence of random demands from a group of retailers located in a given geographical region. They assumed that the vendor has the autonomy of holding small orders until an agreeable dispatch time with the expectation that an economical consolidated dispatch quantity accumulates. As a result, the actual inventory requirements at the vendor are partly dictated by the parameters of the shipment-release policy in use. The optimum replenishment quantity and dispatch frequency were simultaneously derived, and a renewal theoretic model for the case of Poisson demands was developed together with some analytical results for their model. Other studies in the related fields have also been extensively carried out to address various aspects of vendor-buyer supply chain issues [6-13].

Another special focus of the present study is the manufacturer's product quality. The classic economic production quantity (EPQ) model assumes a perfect production [14-16]. However, in a real life manufacturing environment, due to different unpredictable factors it is inevitable to have random defective items produced. In this study all nonconforming items are reworked and repaired in order to assure the entire finished lot has the expected quality. Many studies have been conducted during past decades to address different aspects of imperfect production systems with quality assurance issues [17-24].

The purpose of this study is to simultaneously determine the optimal production lot-size and optimal number of shipments that minimizes the total expected system costs for such a single-producer multi-retailer integrated inventory system with a rework process. Little attention has been paid to this specific issue, the present paper is intended to bridge the gap.

2. ASSUMPTION & MATHEMATICAL MODELING

This study examines a single-producer multi-retailer integrated inventory system with a rework process. We assume that a product can be made at an annual production rate P by the producer, and the production process may randomly generate an x portion of defective items at a production rate d. All items produced are screened and the inspection expense is included in the unit production cost C. All defective items are assumed to be re-workable at a rate of P_1, and a rework process starts right after the end of regular production, in each cycle. Under the normal operation, to prevent shortages from occurring, the constant production rate P must satisfies $(P-d-\lambda)>0$ or $(1-x-\lambda/P)>0$, where λ is the sum of annual demands of retailers and d can be expressed as $d=Px$.

Unlike the classic EPQ model assumes a continuous inventory issuing policy for satisfying demand, this study considers a multi-shipment policy, and finished items can only be delivered to the retailers when the entire lot is quality assured in the end of rework process. Each retailer has its own annual demand rate λ_i. Fixed quantity n installments of the finished batch are delivered to multiple retailers synchronously at a fixed interval of time during the downtime t_3 (refer to Figure 1). Cost parameters used in this study are as follows: the production setup cost K, unit holding cost h, unit production cost C, unit cost C_R and unit holding cost h_1 for each reworked item, the fixed delivery cost K_{1i} per shipment delivered to retailer i, unit holding cost h_{2i} for item kept by retailer i, and unit shipping cost C_i for item shipped to retailer i. Additional notation is listed below:

H_1 = level of on-hand inventory in units when regular production process ends,
H = maximum level of on-hand inventory in units when the rework process ends,
t_1 = the production uptime for the proposed system,
t_2 = time required for reworking the nonconforming items produced in each cycle,
t_3 = time required for delivering all quality assured finished products to retailers,
Q = production lot size per cycle, a decision variable (to be determined),
n = number of fixed quantity installments of the finished batch to be delivered to retailers for each cycle, a decision variable (to be determined),
m = number of retailers,
t_n = a fixed interval of time between each installment of finished products delivered during downtime t_2,
T = production cycle length,
$I(t)$ = on-hand inventory of perfect quality items at time t,
$I_c(t)$= on-hand inventory at the retailers at time t,
$TC(Q,n)$ = total production-inventory-delivery costs per cycle for the proposed system,
$E[TCU(Q,n)]$ = total expected production-inventory-delivery costs per unit time for the proposed system.

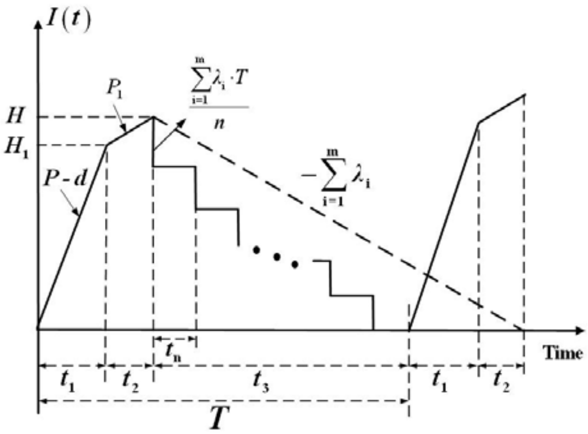

Fig. 1 On-hand inventory of perfect quality items in producer side

From Figure 1, the following equations can be directly obtained:

$$t_1 = \frac{Q}{P} = \frac{H_1}{P-d} \tag{1}$$

$$t_2 = \frac{xQ}{P_1} \tag{2}$$

$$t_3 = nt_n = T - \left(t_1 + t_2\right) = Q\left(\frac{1}{\lambda} - \frac{1}{P} - \frac{x}{P_1}\right) \tag{3}$$

$$T = t_1 + t_2 + t_3 = \frac{Q}{\lambda} \tag{4}$$

$$H_1 = \left(P-d\right)t_1 = \left(P-d\right)\frac{Q}{P} = (1-x)Q \tag{5}$$

$$H = H_1 + P_1 t_2 = Q \tag{6}$$

$$\lambda = \sum_{i=1}^{m} \lambda_i \tag{7}$$

The on-hand inventory of defective items during production uptime t_1 is

$$dt_1 = Pxt_1 = xQ. \tag{8}$$

Cost for each delivery to m retailers is

$$\sum_{i=1}^{m} K_{1i} + \frac{1}{n}\left(\sum_{i=1}^{m} C_i \lambda_i T\right) \tag{9}$$

Total delivery costs of n shipments to m retailers in a production cycle are

$$n\sum_{i=1}^{m} K_{1i} + \sum_{i=1}^{m} C_i \lambda_i T \tag{10}$$

The variable holding costs for finished products kept by the manufacturer, during the delivery time t_3 where n fixed-quantity installments of the finished batch are delivered to customers at a fixed interval of time, are as follows [25].

$$h\left(\frac{n-1}{2n}\right)Ht_3 \tag{11}$$

Total stock holding costs for products kept by the retailers during the cycle are (see Figure 2).

$$\frac{1}{2}\sum_{i=1}^{m} h_{2i}\lambda_i\left[\frac{Tt_3}{n}+\left(t_1+t_2\right)T\right] \tag{12}$$

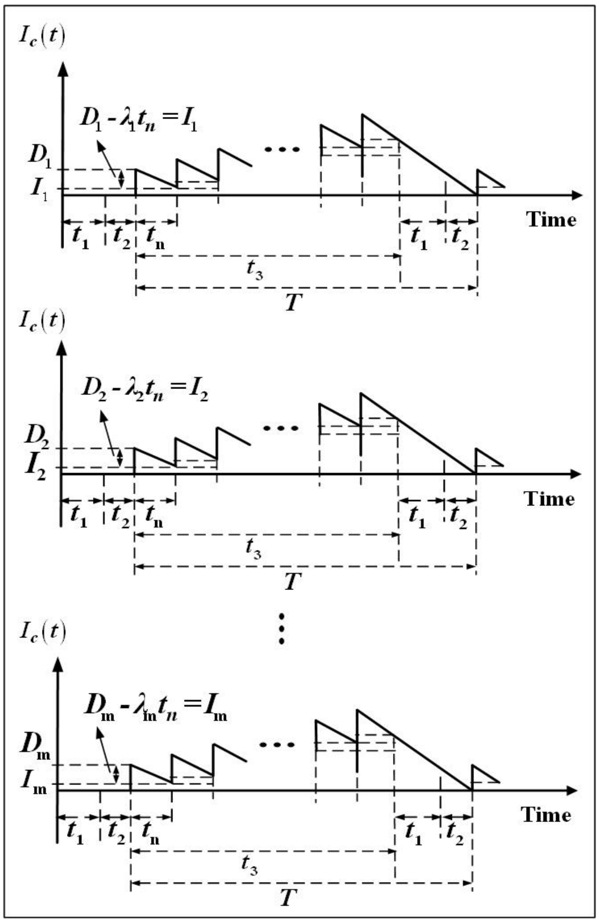

Fig. 2 On-hand inventory in *m* retailer sides

Total production-inventory-delivery cost per cycle $TC(Q,n)$ consists of the setup cost, variable production cost, the cost for the reworking, the fixed and variable delivery cost, holding cost during production uptime t_1 and reworking time t_2, and holding cost for finished goods kept by both the manufacturer and the customer during the delivery time t_3. $TC(Q,n)$ is

$$\begin{aligned}TC(Q,n) &= CQ + K + C_R\left[xQ\right] + n\sum_{i=1}^{m}K_{1i} + \sum_{i=1}^{m}C_i\lambda_i T\\ &+ h\left[\frac{H_1+dt_1}{2}\left(t_1\right)+\frac{H_1+H}{2}\left(t_2\right)+\left(\frac{n-1}{2n}\right)Ht_3\right]\\ &+ h_1\frac{P_1\cdot t_2}{2}\left(t_2\right)+\frac{1}{2}\sum_{i=1}^{m}h_{2i}\lambda_i\left[\frac{Tt_3}{n}+\left(t_1+t_2\right)T\right]\end{aligned} \tag{13}$$

In order to take the randomness of defective rate into account, the expected values is used in cost analysis. Substituting all parameters from equations (1) to (12) in $TC(Q,n)$ and with further derivations the expected cost E[$TCU(Q,n)$] can be obtained as follows:

$$\begin{aligned}E\left[TCU(Q,n)\right] &= C\sum_{i=1}^{m}\lambda_i + \frac{1}{Q}\left(K+n\sum_{i=1}^{m}K_{1i}\right)\sum_{i=1}^{m}\lambda_i + C_R E[x]\sum_{i=1}^{m}\lambda_i\\ &+\sum_{i=1}^{m}C_i\lambda_i+\frac{h}{2}\left(Q\sum_{i=1}^{m}\lambda_i\right)\left[\frac{1}{P}+\frac{1}{P_1}\left(2E[x]-\left(E[x]\right)^2\right)\right]\\ &+\left(\frac{n-1}{2n}\right)\left(hQ\sum_{i=1}^{m}\lambda_i\right)\left[\frac{1}{\sum_{i=1}^{m}\lambda_i}-\frac{1}{P}-\frac{E[x]}{P_1}\right]+\frac{h_1}{2}\left(\frac{1}{P_1}\right)\left(E[x]\right)^2 Q\sum_{i=1}^{m}\lambda_i\\ &+\left(\frac{n-1}{2n}\right)\left(\sum_{i=1}^{m}h_{2i}\lambda_i Q\right)\left[\frac{1}{P}+\frac{E[x]}{P_1}\right]+\left(\frac{1}{2n}\right)\frac{1}{\sum_{i=1}^{m}\lambda_i}\left(\sum_{i=1}^{m}h_{2i}\lambda_i Q\right)\end{aligned} \tag{14}$$

3. DERIVING THE OPTIMAL OPERATING POLICY

The Hessian matrix equations [26] is employed here to prove the convexity of E[$TCU(Q,n)$], that is to verify whether Eq. (15) holds.

$$\begin{bmatrix}Q & n\end{bmatrix}\cdot\begin{pmatrix}\dfrac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q^2} & \dfrac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q\partial n}\\ \dfrac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q\partial n} & \dfrac{\partial^2 E\left[TCU(Q,n)\right]}{\partial n^2}\end{pmatrix}\cdot\begin{bmatrix}Q\\ n\end{bmatrix}>0 \tag{15}$$

From equation (14), one obtains

$$\begin{aligned}\frac{\partial E\left[TCU(Q,n)\right]}{\partial Q} &= -\frac{\left(K+n\sum_{i=1}^{m}K_{1i}\right)\sum_{i=1}^{m}\lambda_i}{Q^2}\\ &+\frac{h}{2}\sum_{i=1}^{m}\lambda_i\left[\frac{1}{P}+\frac{1}{P_1}\left[\left(2E[x]-\left(E[x]\right)^2\right)\right]\right]\\ &+\left(\frac{n-1}{n}\right)\left(\frac{h}{2}\right)\left[1-\sum_{i=1}^{m}\lambda_i\left(\frac{1}{P}+\frac{E[x]}{P_1}\right)\right]\\ &+\frac{h_1}{2}\left(\frac{1}{P_1}\right)\left(E[x]\right)^2\cdot\sum_{i=1}^{m}\lambda_i\\ &+\left(\frac{1}{2n}\right)\left(\sum_{i=1}^{m}h_{2i}\lambda_i\right)\left[(n-1)\left[\frac{1}{P}+\frac{E[x]}{P_1}\right]+\left(\sum_{i=1}^{m}\lambda_i\right)^{-1}\right]\end{aligned} \tag{16}$$

$$\frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q^2}=\frac{2\left(K+n\sum_{i=1}^{m}K_{1i}\right)\sum_{i=1}^{m}\lambda_i}{Q^3} \tag{17}$$

$$\begin{aligned}\frac{\partial E\left[TCU(Q,n)\right]}{\partial n} &= \frac{\sum_{i=1}^{m}K_{1i}\cdot\sum_{i=1}^{m}\lambda_i}{Q}\\ &-\left(\frac{Q}{2n^2}\right)\left(\sum_{i=1}^{m}h_{2i}\lambda_i-h\sum_{i=1}^{m}\lambda_i\right)\cdot\left[\frac{1}{\sum_{i=1}^{m}\lambda_i}-\frac{1}{P}-\frac{E[x]}{P_1}\right]\end{aligned} \tag{18}$$

$$\frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial n^2}=$$

$$\left(\frac{Q}{n^3}\right)\left(\sum_{i=1}^{m} h_{2i}\lambda_i - h\sum_{i=1}^{m}\lambda_i\right)\left[\frac{1}{\sum_{i=1}^{m}\lambda_i}-\frac{1}{P}-\frac{E[x]}{P_1}\right] \tag{19}$$

$$\frac{\partial E\left[TCU(Q,n)\right]}{\partial Q\partial n}=-\frac{\sum_{i=1}^{m}K_{1i}\cdot\sum_{i=1}^{m}\lambda_i}{Q^2} -\left(\frac{1}{2n^2}\right)\left(\sum_{i=1}^{m} h_{2i}\lambda_i - h\sum_{i=1}^{m}\lambda_i\right)\left[\frac{1}{\sum_{i=1}^{m}\lambda_i}-\frac{1}{P}-\frac{E[x]}{P_1}\right] \tag{20}$$

Substituting Eqs. (17), (19) and (20) in Eq. (15) one has

$$[Q \quad n]\cdot\begin{pmatrix}\frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q^2} & \frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q\partial n}\\ \frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q\partial n} & \frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial n^2}\end{pmatrix}\cdot\begin{bmatrix}Q\\ n\end{bmatrix} =\frac{2K\sum_{i=1}^{m}\lambda_i}{Q}>0 \tag{21}$$

Equation (21) is resulting positive, because K, λ, and Q are all positive. Hence, $E[TCU(Q,n)]$ is a strictly convex function for all Q and n different from zero. Therefore, the convexity of $E[TCU(Q,n)]$ is proved, and there exists a minimum of $E[TCU(Q,n)]$. To simultaneously determine the production-delivery policy for the proposed single-producer multi-retailer integrated inventory model with a rework process, one can solve the linear system of Eqs. (16) and (18) by setting these partial derivatives equal to zero. Further derivations one obtains

$$Q^*=\sqrt{\frac{2\left(K+n\sum_{i=1}^{m}K_{1i}\right)\sum_{i=1}^{m}\lambda_i}{\left\{\begin{array}{l} h\sum_{i=1}^{m}\lambda_i\left[\frac{1}{P}+\frac{1}{P_1}\left[\left(2E[x]-\left(E[x]\right)^2\right)\right]\right]+h_1\left(E[x]\right)^2\sum_{i=1}^{m}\lambda_i\frac{1}{P_1}\\ +\left(\frac{n-1}{n}\right)\left[h+\left(\sum_{i=1}^{m}h_{2i}\lambda_i-h\sum_{i=1}^{m}\lambda_i\right)\left(\frac{1}{P}+\frac{E[x]}{P_1}\right)\right]+\sum_{i=1}^{m}h_{2i}\lambda_i\left(n\sum_{i=1}^{m}\lambda_i\right)^{-1}\end{array}\right\}}} \tag{22}$$

and

$$n^*=\sqrt{\frac{K\left(\sum_{i=1}^{m}h_{2i}\lambda_i-h\sum_{i=1}^{m}\lambda_i\right)\left[\left(\sum_{i=1}^{m}\lambda_i\right)^{-1}-\left(\frac{1}{P}+\frac{E[x]}{P_1}\right)\right]}{\sum_{i=1}^{m}K_{1i}\left\{\begin{array}{l} h\sum_{i=1}^{m}\lambda_i\left[\frac{1}{P}+\frac{1}{P_1}\left[2E[x]-\left(E[x]\right)^2\right]\right]+h\\ +\left(\sum_{i=1}^{m}h_{2i}\lambda_i-h\sum_{i=1}^{m}\lambda_i\right)\left(\frac{1}{P}+\frac{E[x]}{P_1}\right)+h_1\left(E[x]\right)^2\sum_{i=1}^{m}\lambda_i\left(\frac{1}{P_1}\right)\end{array}\right\}}} \tag{23}$$

It is noted that in real-world situation the number of deliveries takes on integer values only, however Eq. (23) results in a real number. In order to determine the integer value of n^* that minimizes the expected system cost, two adjacent integers to n must be examined respectively [27]. Let n^+ denote the smallest integer greater than or equal to n (derived from Eq.(23)) and n^- denote the largest integer less than or equal to n. Substitute n^+ and n^- respectively in Eq. (22), then applying the resulting (Q, n^+) and (Q, n^-) in Eq. (14) respectively. By selecting the one that gives the minimum long-run average cost as the optimal replenishment-distribution policy (Q^*, n^*). An example is provided in next section to show the practical usage of the obtained results.

4. NUMERICAL EXAMPLE

Consider that a product can be made by a producer at a production rate (P) of 60,000 units per year and the annual demands λ_i of this product from 5 different retailers are 650, 350, 450, 800, and 750 respectively (total demand is 3000 per year). There is a random defective rate during production uptime which follows a uniform distribution over the interval [0, 0.3]. All nonconforming items are repairable during the rework process at a rate (P_1) of 3600 per year. Values for additional parameters are

- K = \$35000 per production run,
- C = \$100 per item,
- h = unit holding cost at the producer side, \$25 per item per year,
- h_1 = unit holding cost for reworked item, \$60 per item per year,
- C_R = \$60, cost for each items reworked,
- K_{1i} = the fixed delivery cost per shipment for retailer i, they are \$400, \$100, \$300, \$450, and \$250 respectively,
- h_{2i} = unit holding cost for item kept by retailer i, they are \$70, \$80, \$75, \$60, and \$65 per item respectively,
- C_i = unit transportation cost for item delivered to retailer i, they are \$0.5, \$0.4, \$0.3, \$0.2, and \$0.1 respectively.

We first determine the optimal integer number delivery for the proposed model by computing Eq. (23), one has n=4.51. Then by examining the aforementioned two adjacent integers to n and applying Eq. (22), one obtains (Q, n^+)=(2310,5) and (Q, n^-)=(2228,4). Finally, substituting these (Q, n^+) and (Q, n^-) in Eq. (14) respectively, and choosing the one that gives the minimum system cost, one obtains the optimal number of delivery n^*=5, the optimal replenishment Q^*=2310, and total expected cost E[TCU(Q^*,n^*)]=\$438,211.

Variation of Q and n effects on the optimal E[TCU(Q^*,n^*)] are illustrated in Figure 3. It is noted that as the random defective rate x increases, the optimal production lot size Q^* decreases, but the expected system cost E[TCU(Q^*,n^*)] increases significantly. One also notes that the optimal number of delivery n^* decreases as Q^* decreases.

5. CONCLUDING REMARKS

This paper studies a single-producer multi- retailer integrated inventory model with a rework process. In real world supply chains environments, it is usual to have a vendor supplies a product to multiple retailers, and during the production process generation of nonconforming items seems to be inevitable. Management of such an intra supply-chain system would certainly like to figure out the best replenishment-distribution policy so that the long-run average system cost is minimized. We proposed a solution procedure by the use of mathematical modeling and analysis to deal with the aforementioned supply chain system and a closed-form solution of the optimal production- shipment policy is obtained. Effects of various system parameters on the optimal solution are investigated in order to provide management with some insights of this specific single-producer multi-retailer integrated inventory model.

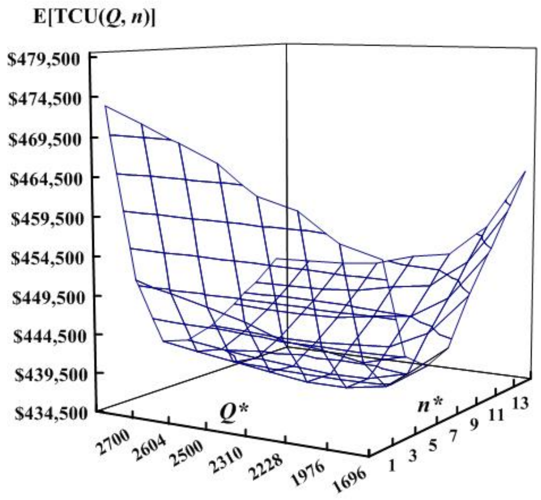

Fig. 3 Variation of Q and n effects on the optimal $E[TCU(Q^*,n^*)]$

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