Reexamining an integrated inventory model with multi-retailer and rework using algebraic approach

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ABSTRACT

This paper reexamines an integrated inventory model with multi-retailer and rework using mathematical modeling and an algebraic method. We assume that a product is manufactured through an imperfect production process, and the reworking of random defective items is done right after the regular process in each cycle. After the entire lot is quality assured, multiple shipments will be delivered synchronously to \( m \) different retailers in each production cycle. The objective is to find the optimal production lot size and optimal number of shipments that minimizes total expected costs for such a specific supply chains system. The conventional approach uses differential calculus on system cost function to derive the optimal production-shipment policy (Chiu et al. [1]), whereas the proposed algebraic approach is a straightforward method that enables practitioners who may not have sufficient knowledge of calculus to understand and manage real-world systems more effectively.

Keywords: Production, Algebraic approach, Multi-retailer, Lot size, Rework, Multi-delivery

1. INTRODUCTION

Taft [2] first proposed an inventory model that employed mathematical techniques to determine the most economical production lot. It is also known as the economic production quantity (EPQ) model [3]. The EPQ model assumes a continuous inventory issuing policy to satisfy product demand. However, in real-life vendor-buyer integrated systems, multiple or periodic deliveries of end items are often used. Therefore, determination of the optimal number of delivery that minimizes the production-delivery cost becomes a critical management decision for such a system. Schwarz [4] first studied a one-warehouse N-retailer inventory system with the objective of determining the stock refilling policy that minimizes the expected total system cost per unit time. Studies related to different aspects of the supply chain optimization have since been extensively carried out [5-11]. Banerjee [6] studied a joint economic lot-size model for purchaser and vendor, with the focus on minimizing the joint total relevant cost. He concluded that a jointly optimal ordering policy, together with an appropriate price adjustment, could be economically beneficial for both parties, but definitely not disadvantageous to either party. Hoque [8] considered models of delivering a single product to multiple buyers when the set-up and inventory costs to the vendor are included. His models assume a close relationship between a manufacturer and buyers for a costless way of benefit sharing. Three models were developed, two of which transfer with equal batches and the third with unequal batches of the product. Optimal solution techniques are presented, a sensitivity analysis of the techniques is carried out, and several numerical problems are solved to support the analytical findings. A comparative study of the results shows that the supply by unequal batches performs better. This study also highlighted the limitation of methods used in obtaining the least minimal total cost in single-vendor single-buyer scenario.

Another special focus of the present study is the product quality obtained from the producer. The classic economic production quantity (EPQ) model assumes a perfect production, however, in a real-life production environment, due to various unpredictable factors it is inevitable to have random defective items produced. We consider that all nonconforming items are reworked and repaired in order to assure the entire lot has the expected quality. In past decades, many studies have attempted to address the issues of defective products and quality assurance in production systems [12-18]. Bielecki and Kumar [12] showed that there are ranges of parameter values describing an unreliable manufacturing system for which zero-inventory policies are exactly optimal even when
there is uncertainty in manufacturing capacity. As a result of their study, this provable optimality reinforces the case for zero-inventory policies, which was made on the separate grounds that it enforces a healthy discipline on the entire manufacturing process. Teunter and Flapper [14] studied a single-stage single-product production system. Items produced are categorized as the non-defective, the re-workable defective, or scrap. The system may switch between the production and the rework mode. After producing a fixed number (N) of units, all re-workable defective units are reworked. It was assumed that the rework time and the rework cost increase linearly with the time that a unit is held in stock. For a given N, they derived an explicit expression for the average profit, and using this expression the optimal value for N can be determined numerically.

In a recent paper, Grubbström and Erdem [19] proposed an algebraic derivation for solving the economic order quantity (EOQ) model with backlogging. Their method does not reference to the first- or second-order derivatives. Similar approaches were applied to solve various aspects of production and supply chain optimization [20-22]. This paper extends such an algebraic approach in order to reexamine a single-producer multi-retailer integrated inventory model with a rework process [1].

2. ASSUMPTION AND MODELING

In this study, we present an algebraic approach to reexamine a single-producer multi-retailer integrated inventory model with a rework process [1]. Such a specific supply chains model is described as follows. Consider that a product can be made at an annual production rate \( \lambda \), and the process may randomly generate an \( x \) portion of defective items at a rate \( d \). All items produced are screened and the inspection expense is included in the unit production cost \( C \). All defective items are assumed to be reworkable at a rate of \( P_1 \), and a rework process starts right after the end of regular production in each cycle (see Figure 1).

Under the normal operation, to prevent shortages from occurring, the constant production rate \( P \) must satisfies \( (P-d)\lambda)\gt 0 \), where \( \lambda \) is the sum of annual demands of retailers and \( d\lambda+P_\lambda \). This study considers a multi-shipment policy and the finished items can only be delivered to the retailers when the entire lot is quality assured. Each retailer has its own annual demand rate \( I(t) \). Fixed quantity \( n \) installments of the finished batch are delivered to multiple retailers synchronously at a fixed interval of time during the downtime \( t_3 \) (Fig. 1).

All cost related parameters used in this study are as follows: the unit production cost \( C \), production setup cost \( K \), unit holding cost \( h \), unit cost \( C_k \) and unit holding cost \( h_i \) for each reworked item, unit disposal cost \( C_d \), the fixed delivery cost \( K_i \) per shipment delivered to retailer \( i \), unit holding cost \( h_i \) for item kept by retailer \( i \), and unit shipping cost \( C_s \). Other notation includes

\[
H_1 = \text{level of on-hand inventory in units when regular production process ends},
\]

\[
H = \text{maximum level of on-hand inventory in units when the rework process ends},
\]

\[
t_1 = \text{the production uptime for the proposed system},
\]

\[
t_2 = \text{time required for reworking the defective items produced in each cycle},
\]

\[
t_3 = \text{time required for delivering all quality assured finished products to retailers},
\]

\[
Q = \text{production lot size per cycle, a decision variable (to be determined)},
\]

\[
n = \text{number of fixed quantity installments of the finished batch to be delivered to retailers for each cycle, a decision variable (to be determined)},
\]

\[
m = \text{number of retailers},
\]

\[
t_n = \text{a fixed interval of time between each installment of finished products delivered during downtime \( t_3 \)},
\]

\[
T = \text{production cycle length},
\]

\[
I(t) = \text{on-hand inventory of perfect quality items at time } t,
\]

\[
I(t) = \text{on-hand inventory at the retailers at time } t,
\]

\[
T = \text{production cycle length},
\]

\[
E[TCU(Q,n)] = \text{total production-inventory-delivery costs per cycle for the proposed system.}
\]

From Figure 1 and with reference to [1], one notes that the total production-inventory- delivery cost per cycle \( TCU(Q,n) \) consists of the following (Eq. (1)): the setup cost, variable production cost, the cost for the reworking, disposal cost, the fixed and variable delivery cost, holding cost during production uptime \( t_1 \) and reworking time \( t_2 \), and holding cost for finished goods kept by both the manufacturer and the customer during the delivery time \( t_3 \).

\[
TC(Q,n) = CQ + K + Cx \sum_{i=1}^{n} K_i + \sum_{i=1}^{n} C \lambda_i T + h \sum_{i=1}^{n} \frac{H_i + d t_i(t_1)}{2} + \frac{H_i + H}{2} - \frac{n-1}{2n} H_t_3 \tag{1}
\]

Taking the randomness of defective rate into account, the expected values of \( E]\left[TCU(Q,n)\right]=\text{total expected production- inventory-delivery costs per unit time for the proposed system.}

Substituting all parameters [1] and with further derivations, the expected cost \( E[TCU(Q,n)] \) can be obtained as follows:

\[
E[TCU(Q,n)] = C \sum_{i=1}^{n} \lambda_i + \frac{Q}{2} \left( K + \sum_{i=1}^{n} K_i \right) \sum_{i=1}^{n} \lambda_i + Cx E[x] \sum_{i=1}^{n} \lambda_i + h \sum_{i=1}^{n} \lambda_i \sum_{i=1}^{n} \lambda_i + h \sum_{i=1}^{n} \frac{Q}{2} \sum_{i=1}^{n} \lambda_i + \frac{n-1}{2n} \sum_{i=1}^{n} \lambda_i + \frac{n-1}{2n} \sum_{i=1}^{n} \lambda_i + \frac{Q}{2} \sum_{i=1}^{n} \lambda_i + \frac{Q}{2} E[x] \sum_{i=1}^{n} \lambda_i + \frac{Q}{2} E[x] \sum_{i=1}^{n} \lambda_i \tag{2}
\]
3. THE PROPOSED ALGEBRAIC APPROACH

One notes that Eq. (2) contains two decision variables, namely \(Q\) and \(n\). Moreover, there are several different forms of decision variables in the right-hand side of Eq. (2), such as \(Q, Q^-, nQ^-,\) and \(Qn^-\). Hence, one can rearrange Eq. (2) as

\[
E[TCU (Q, n)] = C \sum_{i=1}^{m} \lambda_i + C_x E[x] \sum_{i=1}^{m} \lambda_i + \sum_{i=1}^{m} C_i \lambda_i + h \sum_{i=1}^{m} \lambda_i (Q^-) \]  
\[+ \frac{1}{2} \left( \frac{h}{P_1} \right) \left( E[x] - \left( E[x] \right)^2 + \frac{1}{\sum \lambda_i} \right) \sum \lambda_i \]  
\[+ \frac{1}{2} \left( \frac{h}{P_1} \right) \left( E[x] \right)^2 \sum \lambda_i + \sum_{i=1}^{m} h \lambda_i \left( nQ^- \right) \]  
\[+ \left( \sum_{i=1}^{m} K_{hi} \right) \sum \lambda_i (nQ^-) \]

Let

\[
\beta_i = C \sum_{i=1}^{m} \lambda_i + C_x E[x] \sum_{i=1}^{m} \lambda_i + \sum_{i=1}^{m} C_i \lambda_i \]

\[
\beta_2 = h \sum_{i=1}^{m} \lambda_i \]  
\[+ \frac{1}{2} \left( \frac{h}{P_1} \right) \left( E[x] \right)^2 \sum \lambda_i + \sum_{i=1}^{m} h \lambda_i \left( nQ^- \right) \]  
\[
\beta_3 = \left( \sum_{i=1}^{m} K_{hi} \right) \sum \lambda_i \]  
\[
\beta_4 = \sum_{i=1}^{m} \left( \frac{1}{P_1} \right) - E[x] \sum \lambda_i + \sum_{i=1}^{m} h \lambda_i \left( nQ^- \right) \]  
\[
\beta_5 = \left( \sum_{i=1}^{m} K_{hi} \right) \sum \lambda_i \]  
\[
\beta_6 = \sum_{i=1}^{m} \left( \frac{1}{P_1} \right) - E[x] \sum \lambda_i + \sum_{i=1}^{m} h \lambda_i \left( nQ^- \right) \]

Then Eq. (3) becomes

\[
E[TCU (Q, n)] = \beta_1 + \beta_2 (Q^-) + \beta_3 (Q) + \beta_4 (nQ^-) + \beta_5 (n^-Q^-) \]

Rearrange Eq. (9) one has

\[
E[TCU (Q, n)] = \beta_1 + (Q^-) \left( \sqrt{\beta_1} \beta_2 - \sqrt{\beta_2} \right) \]
\[+ \left( n^-Q^- \right) \left( \sqrt{\beta_1} \beta_3 - \sqrt{\beta_3} \right) \]
\[+ 2 \sqrt{\beta_1} \beta_4 + 2 \sqrt{\beta_2} \beta_5 \]

One notes that Eq. (10) will be minimized if its second and third terms equal zero. That is

\[
Q = \frac{\beta_1}{\beta_2} \]

(11)

and

\[
n = \sqrt{\frac{\beta_1}{\beta_3}} \]

(12)

Substituting Eqs. (5) and (6) in Eq. (11), and substituting Eqs. (7), (8), and (11) into Eq. (12), the optimal number of shipments \(n^*\) is

\[
n^* = \left( \frac{K \left( \sum_{i=1}^{m} \lambda_i - \frac{1}{P_1} \right) \left( \frac{1}{P_1} - E[x] \right)^2 + \frac{1}{P_1} \left( E[x] \right)^2 \right)}{\left( \sum_{i=1}^{m} \lambda_i - \frac{1}{P_1} \right) \left( \frac{1}{P_1} - E[x] \right)^2 + \frac{1}{P_1} \left( E[x] \right)^2} \]

(13)

One notes that Eq. (13) is identical to that obtained by using the conventional differential calculus method [1]. Now, in order to find the integer value of \(n^*\) that minimizes the expected system cost, the two adjacent integers to \(n^*\) must be examined respectively for cost minimization [23]. Let \(n^*\) denote the smallest integer less than or equal to \(n\) (derived from Eq. (13)) and \(n\) denote the largest integer less than or equal to \(n\). Because \(n^*\) is either \(n^*\) or \(n\), we can first treat \(E[TCU(Q,n)]\) as a cost function with a single decision variable \(Q\), and do the following rearrangements.

\[
E[TCU (Q, n)] = \beta_1 + [\beta_2 + \beta_3 (n^-)] (Q^-) \]
\[+ [\beta_4 + \beta_5 (n^-)] \]

(14)

or

\[
E[TCU (Q, n)] = \beta_1 + 2 \sqrt{\beta_1 + \beta_3 (n^-)} \cdot \sqrt{\beta_2 + \beta_4 (n^-)} \]
\[+ (Q^-) \left( \sqrt{\beta_1 + \beta_3 (n^-)} - \sqrt{\beta_2 + \beta_4 (n^-)} \right)^2 \]

(15)

Upon derivation of Eq. (15), one notes that \(E[TCU(Q,n)]\) will be minimized if the second term of Eq. (15) equals zero. That is

\[
Q = \sqrt{\frac{\beta_1 + \beta_3 (n^-)}{\beta_2 + \beta_4 (n^-)}} \]

(16)

Substituting Eqs. (5), (6), (7), and (8) in Eq. (16), the optimal production lot size is

\[
Q^* = \left( \frac{2 + \frac{K + \sum_{i=1}^{m} K_{hi} \sum \lambda_i}{n \sum \lambda_i}}{\left( \sum_{i=1}^{m} \lambda_i - \frac{1}{P_1} \right) \left( \frac{1}{P_1} - E[x] \right)^2 + \frac{1}{P_1} \left( E[x] \right)^2} \right) \]

(17)
It is noted that Eq. (17) is identical to that obtained by using the conventional differential calculus method [1]. Finally, to find the optimal replenishment-delivery \((Q^*, n^*)\) policy, one can substitute all related system parameters, along with \(n^*\) and \(n\), in Eq. (17). Then, applying the resulting \((Q^*, n^*)\) and \((Q, n)\) in Eq. (3) respectively, and selecting the one that gives the minimum expected system cost as the optimal \((Q^*, n^*)\) policy.

4. NUMERICAL EXAMPLE

The aforementioned algebraic approach and its resulting equations (13), (17), and (3) are verified in this section using the same numerical example [1]. Consider in a single-producer multi-retailer integrated inventory model that a product can be made at a rate \(P=60,000\) units per year and its annual demands \(i\) from 5 different retailers are 650, 350, 450, 800, and 750 respectively (total demand \(i=3000\) per year). The random defective rate \(x\) follows a uniform distribution over the range of \([0, 0.3]\). All nonconforming items are reworked and repaired at a reworking rate \(P_r=3600\) per year.

Other values of parameters are \(C_i=$100, \(K_i=$35,000, \(h_i=$25, \(h_2=$60, \(C_2=$60, \(K_i\) for retailer \(i\) are $400, $100, $300, $450, and $250 respectively, \(h_i\) for retailer \(i\) are $70, $80, $75, $60, and $65 respectively, \(C_i\) for retailer \(i\) are $0.5, $0.4, $0.3, $0.2, and $0.1 respectively.

We first determine the optimal number delivery by computing Eq. (13), one has \(n=4.51\). Then, examine the two adjacent integers to \(n\) and applying Eq. (17), one obtains \((Q, n)=(2310,5)\) and \((Q, n)=(2228,4)\). Finally, substituting these \((Q, n)=(2310,5)\) and \((Q, n)\) in Eq. (3) respectively. Choosing the one that gives the minimum system cost as our optimal policy, one obtains \((Q, n)=(2310,5)\) and total expected cost \(E(TCU(Q^*,n^*))=438,211\).

The research results were confirmed to be identical to those obtained by the traditional method (Chiu et al. [1]).

5. CONCLUDING REMARKS

This study proposes an algebraic approach for determining the optimal production-delivery policy for a single-producer multi-retailer integrated inventory model with a rework process. Unlike the conventional method which uses differential calculus on the system cost function to find the optimal policy [1], the proposed algebraic approach is a straightforward method that may enable practitioners with little knowledge of calculus to understand and manage such a real-life supply chains system more effectively.

ACKNOWLEDGEMENTS

The authors appreciate the support of National Science Council (NSC) of Taiwan under grant number: NSC 100-2410-H-324-007-MY2.

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