Solving the Economic Load Dispatch Problem Using Crow Search Algorithm

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Abstract
Economic Load Dispatch (ELD) problem concern on scheduling the committed generating units outputs such that the load in demand can be provided with minimum operating cost while satisfying all units and system equality and inequality constraints. This paper proposes the use of Crow Search Algorithm (CSA) to solve the ELD Problem. CSA is yet another metaheuristic search algorithm that adopt the method of crows when they search, hide and retrieve food when needed. CSA is explored to solve the nonlinear ELD constrained optimization problem for a three units power system. The results obtained by CSA are compared with various results obtained in the literature. Simulation results shows that using CSA can lead to finding stable and adequate power generated that can fulfill the need of both the civil and industrial areas.

1. Introduction
ELD is one of the challenging optimization problems in power system planning. We need to find the optimal minimum for a nonlinear function with equality and non-equality continuations (1; 2). The general optimization problem is given in Equation 1.

\[
\begin{align*}
\text{minimize} & \quad f(x) \\
\text{subject to} & \quad \phi_j(x) = 0, \quad j = 1, \ldots, J \\
& \quad \psi_i(x) \leq 0, \quad i = 1, \ldots, M. \quad (1)
\end{align*}
\]

where \( f(x) \), \( \phi_j(x) \) and \( \psi_i(x) \) are functions of the design vector \( x = (x_1, x_2, \ldots, x_n)^T \). Economic load dispatch optimization helps in maintaining a high degree of cheap and reliable power for both civil and industrial use. The power plants must fulfill the demand with minimum transmission losses and lowest possible cost. Meanwhile, the system should also achieve the plant constraints (i.e. economic load dispatch). Solving the ELD problem means finding the optimal power generation schedule of all members of the power network such that the total fuel cost is minimized while sustaining operation requirements limits satisfied across the entire dispatch phases.

Conventional optimization techniques (3; 4) have long been applied to solve the ELD problem such as Quadratic Programming (5; 6), linear programming (7), sequential approach with a matrix framework (SAMF) (2), modified Lambda-iteration method (8), Newton Raphson and Lagrangian multiplier (LM) algorithms (9). In the real-design cases, the number of decision variables (i.e. power units) of the ELD area are very large. The objective criterion to be minimized could also have too many local minimum which might not lead to the minimum cost and the best generation schedule of power system units. Therefore, efficient search algorithms are needed.

Nature-inspired metaheuristic search algorithms gain a popularity due to their promising performance on solving many real-world optimization problems which are complex, nonlinear and multi-model. In the past two decades, the literature of metaheuristic search has expanded extensively. Some of the well-known metaheuristic approaches are Genetic Algorithms (10), Genetic Programming (11; 12; 13), Particle Swarm Optimization (14; 15), Simulated Annealing (16), Artificial Bee Colony (ABC) (17), Cuckoo Search (18; 19), etc.

A comprehensive review of various applications of Bio-Inspired Optimization (BIO) algorithms to solve complex ELD problems was presented in (20). Examples of techniques used to solve the ELD are Hybrid GA (21), Gravitational Search Algorithm (22), Seeker Optimization Algorithm (23), Differential Evolution (24), Evolutionary programming (25), etc. In (26), authors implemented a PSO algorithm to solve the ELD problem for number of power systems with various number of generators. The obtained PSO results were compared with GAs and QP algorithms. In (27), author presented an algorithm called biogeography-based optimization (BBO) to solve both convex and non-convex economic load dispatch (ELD) problems of thermal generators of a power system. The allowed constraints included transmission losses, ramp rate limits, multi-fuel options and prohibited operating zones.

This paper is organized as follows. In Section , we provide an overview of the mathematical formulation of the ELD problem. Details of the proposed CSA metaheuristic search algorithms and its mechanisms on solving optimization problems are presented in Section . Experimental results for a three units power system is presented in Section . Finally, we provide our conclusions.
2. Mathematical Formulation of ELD

In this section, we start by providing a formulation to the ELD problem. The economic dispatch problem objective is to maximize the economic welfare of a power network under various operation constraints. Assume we have a network with \( n \) buses (nodes). The unconstrained ELD problem can be formulated as:

\[
\begin{align*}
\text{Min } & \Delta_k(P_k) = C_1(P_1) + \cdots + C_n(P_n) \\
& = \sum_{k=1}^{n} \Delta_k(P_k)
\end{align*}
\]

\( \Delta_k(P_k) \) is the cost function of producing power at bus \( k \). A power system with this given configuration can be presented as in Figure 1 such that \( n \) thermal units are connected to a single bus-bar that is supplying a load power \( P_k \). The input to each unit is expressed in terms of cost rate (say $/h) \( P_k \), \( k = 1, \ldots, n \), \( n \) is the number of power generator units. The cost presented in Equation 2 can be approximated in a quadratic form as given in Equation 3 for minimization purposes (1; 28).

\[
\text{Min } \Delta_k(P_k) = \sum_{k=1}^{n} \alpha_k P_k^2 + \beta_k P_k + \gamma_k
\]

\( P_k \) is the generated power from generator unit \( k \), \( \alpha_k, \beta_k \) and \( \gamma_k \) are the fuel cost coefficients of unit \( i \). Two types of constraints shall be considered while solving this problem; equality constraints and inequality constraints.

2.1 Power Balance Equality Constraints

A real power system has to generate enough power such that it covers both the demand and the transmission lines power loss. It is known that the power produced at any power station go through large and complex networks such as transformers, transmission lines, cables and additional equipment to supply the end users of their demand. Therefore, it is always the case that the power units in a network always produce extra power not only to match the demand but also to recover the waste of transmission power. This difference in the generated and distributed power \( P_G \) is recognized as Transmission and Distribution loss power \( P_L \). Any lack in the generated power \( P_G \) will cause shortage in feeding the power in demand \( P_D \) which could be a reason for several problems for the system and loads (See Equation 4).

\[
P_G = \sum_{k=1}^{n} P_k = P_D + P_L
\]

where \( P_D \) is the load demand and \( P_L \) is the transmission lines loss, while \( n \) and \( P_k \) have the same definition as in Equation 3.

To consider the effect of transmission losses in our cost computation, we adopted the loss coefficient method which proposed by Kron and Kirchmayer (1; 28). In this method, a matrix \( \zeta \) is defined as “the transmission loss coefficients matrix” used to include the power loss. \( \zeta \) is a square matrix with a dimension of \( R^{n \times n} \) while \( n \) is the number of power generation units in the system. Equation 5 describes the definition of \( P_L \) based the transmission loss \( \zeta \)-matrix.

\[
P_L = \sum_{i=1}^{n} \sum_{j=1}^{n} P_i \zeta_{ij} P_j
\]

where \( P_L \) is the transmission power loss, \( P_i, P_j \) are the power generated from any two power generator units \( i, j \). Meanwhile, \( \zeta_{ij} \) is the elements of the matrix \( \zeta \) between \( i \) and \( j \) power generator units.

2.2 Generation Limit Inequality Constraints

The generated power from the power generation system should satisfy number of constraints based on the capacity of the generation unit. For instance, the generation units cannot exceed a certain power generation units since this may cause instability for the synchronous generators. Meanwhile, generating less power than a minimum limit may cause the rotor to over speed. These limitations for the \( k^{th} \) generator are described in Equation 6 and presented in (29).

\[
P_k^{\min} \leq P_k \leq P_k^{\max}, k = 1, \ldots, n
\]

where \( P_k^{\min} \) and \( P_k^{\max} \) are the limitation of generation for the \( k^{th} \) generation unit.

3. Crow Search Algorithm

Crow search algorithm is a newly provided metaheuristics search algorithm by A. Askarzadeh (30). The algorithm is inspired from the crow behavior while saving their excess food in hiding places and retrieve it when the food is needed. Crow used to show intelligent behavior. They have the capability to remember faces and send a warning to each other when dangerous is approaching. They also have many ways to hide their foods for several months. It was found that their brain-to-body ratio is close to the one in human.

The principles idea of CSA (30) can be summarizes as follows:

- Flock of Crows live together.
- Crows can memorize the places where they hide their food.
Crows committee thievery by following each other.

Crows protect their caches from being pilfered by a probability.

On solving an optimization problem using CSA, we assume that we have a $d$-dimensional vector representing a solution for the problem under study. The population of crow assumed to have $N$ individuals (i.e. population size).

The position of the crow $i$ at generation (i.e. iteration) $g$ is defined as $x_i^g; (i = 1, \ldots, N, g = 1, \ldots, g_{max})$. $g_{max}$ is the maximum number of generations. For example, if crow $i$ wants to visit its hiding place, we define it as $m_i^g$. Thus, the vector $x_i^g$ which represent the problem solution at generation $k$ can be represented as:

$$
\begin{bmatrix}
x_i^1 & x_i^2 & x_i^3 & \ldots & x_i^n \\
x_i^1 & x_i^2 & x_i^3 & \ldots & x_i^n \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
x_i^1 & x_i^2 & x_i^3 & \ldots & x_i^n 
\end{bmatrix}
$$

We run the algorithm to a fixed number of generations $g_{max}$. Each crow has a memory $m$ such that it can save the position where food was hidden. At generation $g$, the position of hiding place of crow $i$ is given by $m_i^g$. The memory location always saves the best obtained position of the crow so far. One of the main crow activity is to follow other crows such that they can allocate the places where they hide their food and hunt it. Thus, two scenarios could happen:

1. At generation $g$, crow $i$ decides to follow crow $j$ to find to the hiding place of crow $j$. Of course, crow $j$ does not know that crow $i$ is following. Thus, it is going to be easy for crow $i$ to find the hidden food and change its position to a new one as given in Equation 7.

$$
x_i^{g+1} = x_i^g + r_i \times f_i^g \times (m_i^g - x_i^g); r_i \geq AP_i^g
$$

where $r_i$ is a uniformly selected random value between $[0,1]$ and $f_i$ is defined as the flight length of crow $i$ while search for the food at generation $g$. $AP$ is defined as the awareness probability of crow $i$ at generation $g$.

2. Now, if crow $j$ realized that crow $i$ is following, crow $j$ has to mislead crow $i$ such that it will not locate its food position. Thus, crow $i$ will have to start again to find another crow to follow and so move to a new random position.

A flow chart which shows the detailed steps of the CSA algorithm is presented in Figure 2. In Figures 3 and 4, we show a flow chart of the crow based on two conditions of $f_l$ ($f_l < 1$, $f_l > 1$). Small values of $f_l$ leads to local search and large values results in global search. In fact, we need to make balance between local and global exploration of the search space. That is why adjusting $f_l$ will help in achieving convergences of the search algorithm.
3.1 Fitness Function

Our objective is to find estimate the optimal power units values $P_k$, $k = 1, \ldots, n$, $n$ is the number of power units, which minimize the objective criterion $L$ (see Equation 8).

$$L = \sum_{k=1}^{n} \Delta_k(P_k) + \lambda \times \sum_{k=1}^{n} P_k - P_D - P_L$$  \hspace{1cm} (8)

where $\Delta_k(P_k)$ is the cost of power generated from generator $P_k$, $P_D$ is the demand load, $P_L$ is the transmission lost power, $\lambda$ is an arbitrary chosen parameter with high value to penalize the losses in the cost computation. In our case, $\lambda$ was set to 100.

4. Experimental Results

In this section, we compare the results on solving the ELD problem using Genetic Algorithms (31), Particle Swarm Optimization (31) and Flower Pollination Algorithm (32) with the developed results based CSA.

A three units power unit system $P_k$ was selected from (31) to explore our idea on using CSA to find the optimal set of power generation of the system. The adopted system is expected to produce demand power of 400 MW. Table 1 shows the cost coefficient of the three generators, under study, while the matrix $\zeta$ is the loss coefficient matrix of the three units power system.

Table 1: Cost Coefficient of Three Units System

<table>
<thead>
<tr>
<th>$P_1$ (MW)</th>
<th>$P_2$ (MW)</th>
<th>$P_3$ (MW)</th>
<th>$\alpha_1$ (S/MW$^2$)</th>
<th>$\alpha_2$ (S/MW$^2$)</th>
<th>$\alpha_3$ (S/MW$^2$)</th>
<th>$\beta_1$ (S/MW$^2$)</th>
<th>$\beta_2$ (S/MW$^2$)</th>
<th>$\beta_3$ (S/MW$^2$)</th>
<th>$\gamma_1$ (S/MW$^2$)</th>
<th>$\gamma_2$ (S/MW$^2$)</th>
<th>$\gamma_3$ (S/MW$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>150</td>
<td>150</td>
<td>0.035546</td>
<td>0.02111</td>
<td>0.001797</td>
<td>88.3053</td>
<td>38.32762</td>
<td>38.27041</td>
<td>124.3311</td>
<td>165.5696</td>
<td>135.06592</td>
</tr>
</tbody>
</table>

$$\zeta = 10^{-4} \begin{pmatrix} 0.71 & 0.3 & 0.25 \\ 0.3 & 0.69 & 0.32 \\ 0.255 & 0.32 & 0.8 \end{pmatrix}$$

We run CSA for 100 generations for 50 experiments and collecting the values of the $P_k$ for each power station. The tuning parameters for CSA are given in Table 2. The estimated $P_k$ for each generator, the power loss, the total generated power and the computed cost based GAs, PSO, Flower Pollination Algorithm (FPA) and CSA are also shown in Table 3. In Figure 5, we show the convergence of the meta-heuristic search process based CSA in both the best and average cases. In Figure 6, we show the convergence of the estimated loads $P_1$, $P_2$ and $P_3$ using CSA in the best case. The statistical results obtained for the 50 experiments is provided in Table 4. It can be seen that CSA provided the minimum fuel cost in this case compared to other reported methods in the literature. This shows that the CSA is more effective in finding the best load for the three generator system.

Table 2: Tuning parameters of the CSA

<table>
<thead>
<tr>
<th>Problem</th>
<th>Population Size</th>
<th>Number of Generations</th>
<th>$f^*$</th>
<th>$AP$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three Unit System</td>
<td>100</td>
<td>100</td>
<td>2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 3: Estimated output power for Three Units Power System

<table>
<thead>
<tr>
<th>Power outputs</th>
<th>GA (31)</th>
<th>PSO (31)</th>
<th>FPA (32)</th>
<th>CSA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$ (MW)</td>
<td>102.617</td>
<td>102.612</td>
<td>102.4468</td>
<td>82.054756</td>
</tr>
<tr>
<td>$P_2$ (MW)</td>
<td>153.809</td>
<td>153.8341</td>
<td>175.124962</td>
<td></td>
</tr>
<tr>
<td>$P_3$ (MW)</td>
<td>151.1321</td>
<td>151.1321</td>
<td>150.394976</td>
<td></td>
</tr>
<tr>
<td>$P_4$ (MW)</td>
<td>7.41324</td>
<td>7.41733</td>
<td>7.4126</td>
<td>7.574696</td>
</tr>
<tr>
<td>Load demand</td>
<td>20840.1</td>
<td>20838.38</td>
<td>20838.1</td>
<td>20812.574934</td>
</tr>
<tr>
<td>Fuel Cost ($/h)$</td>
<td>20840.1</td>
<td>20838.38</td>
<td>20838.1</td>
<td>20812.574934</td>
</tr>
</tbody>
</table>

Table 4: Statistical results obtained for the fuel cost

<table>
<thead>
<tr>
<th></th>
<th>Worst</th>
<th>Mean</th>
<th>Best</th>
<th>Std</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20812.603886</td>
<td>20812.587410</td>
<td>20812.574934</td>
<td>0.009634</td>
</tr>
</tbody>
</table>

5. Conclusions

In this paper, we explored the use of CSA to solve the ELD problem for power systems planning. The practicality of the proposed metaheuristics CSA was tested for three power generators test case. The gained results were compared to existing results based on GAs, PSO and FPA. It was shown that CSA are superior in obtaining a combination of power loads that fulfill the problem constraints and minimize the total fuel cost. CSA found to be efficient in finding the optimal power generation loads. CSA was capable of handling the non-linearity of ELD problem. The evolved power using CSA minimized both the cost of generated power, the total power loss in the transmission and maximize the reliability of the power provided to the customers.

References

Figure 5: Three Units System: Convergence of evolutionary process of CSA

Figure 6: Three Units System: Estimated Power Load based CSA


