Efficient experimental detection of milling stability boundary and the optimal axial immersion for helical mills

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ABSTRACT

In this paper, an optimal axial immersion is defined and it is calculated for the milling process. The analytical calculation is based on one degree of freedom model. A helical tool is considered during the calculation. Axial immersion values are found where large material removal rate can be achieved while negligible and tolerable vibration occurs. We explain this with the help of the Fourier series of the time-periodic cutting force. To support our analytical results measurements were carried out. Instead of making numerous measurements with distinct spindle speeds, we used wedge-shaped work-pieces to speed-up the lengthy experiments. In this way, the axial immersion was increased continuously during the cutting process. The stability boundary and the optimal axial immersions could be found with a single test at each spindle speed. We found that the vibration amplitudes were significantly reduced in case of the theoretically predicted optimal axial immersions.

Keywords: Chatter, Stability, Helical mills, Time delay, High-performance milling

Fig.1: Poor surface quality caused by self-excited vibration

1. INTRODUCTION

Vibrations in milling processes may cause poor surface properties and shorten the lifetime of the machine tool (see Fig.1.). There are two types of these vibrations which are essential. The first is the so-called “chatter” that is a kind of a self-excited vibration caused by the regenerative effect of the machined surface [1,2]. The chip thickness depends on the current position of the cutting edge and the past position of the previous/neighbouring cutting edge of the mill. Consequently, the governing equation of the model is a time delayed differential equation. In order to avoid this type of vibration, the so-called stability chart must be created as a function of the technological parameters. This calculation can be carried out in several ways like using the semi-discretization method [3,4,5], temporal finite elements [6], multifrequency analysis approach [7,8] or time domain simulation [9]. During our analysis, the semi-discretization method was used to detect the stability boundary of the milling process.

The second type of vibrations is caused by the periodic non-smooth cutting force. This forced vibration can have large amplitudes at resonant spindle speeds. There are some regions of the technological parameters where large material removal rates can be achieved, while regenerative chatter is also avoided. However, in most cases, forced vibrations can still occur at these parameters even close to resonance. These vibrations lead to poor surface quality. In [10,11,12] the quality of the milled surface was predicted by means of analytical calculation. The surface was generated in [13,14] according to numerical time domain simulation. In case of helical tools, we found that this kind of vibration can also be eliminated by using the optimal axial immersion that depends on the helix pitch of the tool and on the spindle speed [15,16].

In the present study measurement results are given to support our analytical approach and predictions. The method of the efficient experimental detection of milling stability boundary and the optimal axial immersion is also presented in details.

2. MODEL

In our analysis, a 1 DoF flexible tool and rigid work-piece model was used (Fig. 2.). In finishing operations, only the stable process is acceptable, which can be determined by a linear stability calculation. Thus, the cutting force $F_j$ of the $j$th tooth is approximated as a linear function of the chip thickness $h_j$ at the stationary cutting. The tangential and the radial cutting force components can be given as:

$$F_j^t(t) = \omega K_j h_j(t), \quad F_j^r(t) = \omega K_j h_j(t), \quad (1)$$
where \( w \) is the axial immersion, \( K_r \) and \( K_t \) are the specific cutting force coefficients, while \( k_r = K_t / K_r \). The chip thickness comprises two parts, the static and the dynamic chip thicknesses. The static part is generated by the constant feed motion \( v \), while the vibration of the tool described by the \( y \) coordinate generates the dynamic part, so the chip thickness can be estimated as

\[
h_j = v \sin(\phi_j) + (y(t) - y(t - \tau))\cos(\phi_j),
\]

(2)

Here \( \phi_j \) is the angular position of the \( j \)th tooth, the regenerative time delay is equal to the tooth-pass frequency (\( \tau = 2\pi / \Omega N \)), \( N \) is the number of the teeth and \( \Omega \) is the angular velocity of the tool. The term \( rv \) is equal to the feed per tooth.

If helical tools are considered in the model, like Fig. 3., than the current angular position of the edge also depends on the axial coordinate of the tool \( z \).

\[
\phi_j(z, t) = t\Omega + \frac{2\pi z}{p},
\]

(3)

where \( p \) is the helix pitch.

The governing equation Eq. (5) can be separated into two parts by using the new variables \( \phi \) and \( \eta \).

\[
y(t) = \phi(t) + \eta(t)
\]

(8)

such that forced motion \( \phi(t) \) satisfies

\[
m\ddot{\phi}(t) + c\dot{\phi}(t) + k\phi(t) = F_{stat}(t),
\]

(9)

Substituting Eq. (8) and Eq. (9) into Eq. (4) we get the delayed parametrically forced part of the governing equation.

\[
m\ddot{\eta}(t) + c\dot{\eta}(t) + (k - A(t))\dot{\eta}(t) = -A(t)\eta(t - \tau)
\]

(10)

which describes the stability of the tool motion. If it is unstable, the tool vibration amplitude tends to infinity and there is no need to calculate the surface properties, this is already unacceptable.

When the trivial solution \( \eta(t) = 0 \) of Eq. (10) is stable, the particular motion of the tool center is given by the solution of the forced motion Eq. (9). This always has a periodic solution because of the small damping. By using this solution, we can compute the motion of the teeth, which generates the surface after the settled transient motion.

The parameter domains of stable cutting were calculated from Eq. (10) using the semi-discretization method, which is described in detail in [4,15].
3. OPTIMAL AXIAL IMMERSION

Efficient technical parameters for stable cutting can be chosen from the stability charts, but in most cases, forced vibrations still occur at these parameters. In these cases, the vibration is close to resonance because the frequencies of higher harmonics of the cutting force are close to the natural frequencies of the system \( \omega_n \). In case of helical tool we can define the optimal axial immersions where these harmonics disappear.

This phenomenon is explained below by the Fourier transformation of the cutting force signal, because the spectrum of the cutting force does not contain the natural frequency of the system. In other words, the resonant Fourier component of the cutting force is zero along these lines.

The local cutting force on a single elementary section of the helical tool is

\[
 f(\phi(z,t)) = \tau v \frac{K_1}{2} \sum_{j=1}^{N} g_j(\phi_j(z,t)) T_j(\phi_j(z,t)) dz
\]

(11)

where, the angular position \( \phi_j(z,t) \) of the edge is given by Eq. (3). Function \( f(\phi) \) is a periodic function in time and its period equals to the tooth-pass frequency \( 1/\tau \). By using Eqs. (3) and (5), the Fourier transformation of the cutting force is given by

\[
 \mathcal{F}_{cut}(\omega) = \int_{-\infty}^{\infty} f(t, \omega) e^{-i\omega t} dt
\]

(12)

The axial coordinate \( z \) is substituted by the new variable

\[
 z = (t-\theta)\frac{\Omega p}{2\pi},
\]

(13)

and the integration by substitution leads to the convolution integral.

\[
 \mathcal{F}_{cut}(\omega) = \int_{-\infty}^{\infty} \left[ \int_{-\frac{2\pi w}{\Omega p}}^{\infty} f(t, \omega) e^{-i\omega t} dt \right] e^{-i\omega \theta} d\theta
\]

(14)

with the step function

\[
 \hat{g}(\theta) = \begin{cases} 
 1 & \text{if } 0 < \theta < \frac{2\pi w}{p\Omega} \\
 0 & \text{otherwise.} 
\end{cases}
\]

(15)

The Convolution Theorem states that the Fourier transform transforms a convolution into a multiplication (see [17]), thus Eq. (14) can be written as

\[
 \mathcal{F}_{cut}(\omega) = \int_{-\infty}^{\infty} f(t, \omega) e^{-i\omega t} dt \cdot \hat{g}(\theta) e^{-i\omega \theta} d\theta
\]

(16)

Since \( f(\phi) \) is a periodic function, its Fourier transform is a discrete function. It can be seen from Eq. (16) and also in Fig. 4, that the resonant Fourier component of the cutting force \( \mathcal{F}_{cut}(\omega) \) is zero if either

\[
 \omega_n \neq \frac{2\pi}{\tau} k = k\Omega N, \quad \Rightarrow \quad \Omega \neq \frac{\omega_n}{kN} \quad k \in \mathbb{N}
\]

(17)

or

\[
 \frac{\omega_n w}{\Omega p} = k \quad \Rightarrow \quad w = \frac{\Omega p}{\omega_n} k \quad k \in \mathbb{N}
\]

(18)

Fig. 4: Representation of the \( f(t, \omega) \) and \( \hat{g}(\theta) \) as a function in time and its Fourier transform.

If this component is zero then there is no resonance. The lines \( \Omega \) described by Eq. (18) are the same lines as the dotted slanting ones in Fig 9, while the other condition (32) is identical to (11) determined earlier and shown at \( p/Z \) in Fig 9.
4. EXPERIMENTAL VALIDATION

An experimental setup was established to confirm our analytical investigations. To realize this model, we manufactured a test rig with high flexibility in one direction (see Fig. 5) based on [8]. The mass of the upper part was 8.76 kg.

We carried out its modal analysis to check the natural frequency and the damping ratio of the test rig. We use Pulse Front-end and Pulse Labshop v11.0 software to detect the signal of the accelerometers (B&K4397) placed on the upper part. From the Impact test procedure, the measured natural frequency is \( \omega_n = 424.1 \text{ rad/s} \), and the damping ratio is \( \xi = 1.14 \% \).

In the test measurements, we used epoxy resin material. We used wedge-shaped work-piece to speed-up the lengthy experiments. In this way, the axial immersion was increasing continuously during the cutting process. So the stability boundary and the optimal axial immersions could be found with one single test along an \( \Omega = \text{const.} \) line.

During the measurements a 5-fluted tool was used with helix pitch \( p = 112.5 \text{ mm} \) and diameter \( D = 20 \text{ mm} \). Small radial immersion (0.5 mm) was used during the up-milling process, which is a usual value in case of surface finishing processes. The feed rate was set to \( \tau = 0.15 \text{ mm/tooth} \), where \( v \) is the feed velocity. The axial immersion is limited by the tool, so \( w_{\text{max}} = 35 \text{ mm} \).

In Fig.6 the theoretical optimal axial immersions are denoted by the dotted horizontal lines, where the total cutting force is constant in time, and continuous slanting lines, where the resonant Fourier component on the cutting force is zero. During the measurement the optimal immersions are detected as the local minimum point of the vibration amplitude. These points are denoted by white circles in Fig.6. We also detected the stability losing during the measurements. Those axial immersion where the stability losing occurs, are denoted by dark red circles. We can see that the measured optimal immersion points fit the theoretical ones very well, which support our analytical results.

5. CONCLUSION

In this report, we investigated the effect of the parameters of a helical tool model to predict the optimal axial immersions in high-speed milling. The most important observation is that small vibrations take place even for resonant angular velocities \( \left( \Omega = \omega_n / N, \omega_n / 2N, \omega_n / 3N, \omega_n / 4N, \ldots \right) \), if appropriate axial immersions are applied. The appropriate axial immersions are not just the trivial ones where the cutting force is time-independent, that is, where the axial immersions are equal to an integer multiple of the helix pitch \( p \) over the number of cutting teeth \( N \). Non-trivial appropriate axial immersions are also found which depend linearly on the cutting speed. The identification of these new appropriate axial immersions is useful since just the resonant cutting speeds are preferred to achieve high material removal rates with stable machining. If non-appropriate axial immersions are used at these cutting speeds, the machined surface quality parameters might be extremely poor.

We carried out measurements to support our theoretical results. Wedge-shaped work-piece was used to speed-up the lengthy experiments. We found that the vibration amplitude was significantly smaller in case of the optimal axial immersions, which confirmed our theoretical predictions.
6. ACKNOWLEDGEMENT

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7. REFERENCES


