Localized Optical Modes in Spiral Photonic Media: Application to Lasing and Other Optical Phenomena

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ABSTRACT

A brief survey of the recent experimental and theoretical results on the low threshold distributed feedback (DFB) lasing in chiral liquid crystals (CLC) and new original theoretical results on the localized optical modes in spiral media (edge (EM) and defect (DM) modes) are presented. Because the CLCs demonstrate common for all spiral media optical properties the studying of the problem is performed for the certianty for CLCs.

Keywords: Localized optical modes in spiral media, anomalous absorption, low threshold lasing

1. INTRODUCTION

Recently there was a very intense activity in the field of localized optical modes, in particular, edge (EM) and defect (DM) modes in chiral liquid crystals (CLC) mainly due to the possibilities to reach a low lasing threshold for the mirrorless distributed feedback (DFB) lasing [1-4] in chiral liquid crystals. The EM and DM existing as a localized electromagnetic eigen state with its frequency close to the forbidden band gap or in the forbidden band gap, respectively, were investigated initially in the periodic dielectric structures [5]. The corresponding EM and DM in chiral liquid crystals, and more general in spiral media, are very similar to the defect modes in one-dimensional scalar periodic structures. They reveal abnormal reflection and transmission [1,2] and allow DFB lasing at a low lasing threshold [3]. The qualitative difference with the case of scalar periodic media consists in the polarization properties. The EM and DM in chiral liquid crystals are associated with a circular polarization of the electromagnetic field eigen state of the chirality sense coinciding with the one of the chiral liquid crystal helix. There are two main types of defects in chiral liquid crystals studied up to now. One of them is a plane layer of some substance differing from CLC dividing in two parts a perfect cholesteric structure and being perpendicular to the helical axis of the cholesteric structure [1]. Other one is a jump of the cholesteric helix phase at some plane perpendicular to the helical axis (without insertion any substance at the location of this plane) [2]. Recently, a lot of new types of defect layer were studied [8-14], for example, the CLC layer with the pitch differing from the pitch of two layers sandwiching the defect layer [8]. Almost all studies of the defect modes in chiral and scalar periodic media were performed by means of a numerical analysis with the excepting [15,16], where the known exact analytical expression for the eigen modes propagating along the helix axis [17,18] were used for a general studying of the defect mode associated with a jump of the helix phase. The used in [15,16] approach looks as a very fruitful one because it allows to reach easy understanding of the defect mode physics. In the present paper an analytical solutions of the EM and DM mode (associated with an insertion of an isotropic layer in the perfect cholesteric structure) are presented and some limiting cases simplifying the problem are considered.

2. BOUNDARY PROBLEM

The boundary problem for a CLC layer was solved in many papers (for example [19-21]) so we give here only expressions for the amplitude transmission T and reflection R coefficients of light incident at a CLC layer of thickness L along the spiral axis. We assume that the CLC is represented by a planar layer with a spiral axis perpendicular to the layer surfaces. We also assume that the average CLC dielectric constant ε_0 coincides with the dielectric constant of the ambient medium. This assumption practically prevents conversion of one circular polarization into another at layer surfaces [20,21] and allows taking into account only eigenwaves with diffracting circular polarization. The expressions for *R* and *T* take the form

$$R = i\delta \sin qL/\{(q\tau/\kappa^2)\cos qL + i[(\tau/2\kappa)^2 + (q/\kappa)^2 - 1]\sin qL\}$$

$$T = \exp[i\kappa L](q\tau/\kappa^2)/\{(q\tau/\kappa^2)\cos qL + i[(\tau/2\kappa)^2 + (q/\kappa)^2 - 1]\sin qL\},$$
(1)
(2)

where $q = \kappa \{1 + (\tau/2\kappa)^2 - [(\tau/\kappa)^2 + \delta^2]^{\nu_2}\}^{\nu_2}$, $\varepsilon_0 = (\varepsilon_{\parallel} + \varepsilon_{\perp})/2$, $\delta = (\varepsilon_{\parallel} - \varepsilon_{\perp})/(\varepsilon_{\parallel} + \varepsilon_{\perp})$ is the dielectric anisotropy, and ε_{\parallel} , and ε_{\perp} are the local principal values of the CLC dielectric tensor [16,17], $\kappa = \omega \varepsilon_0^{\nu_2}/c$, *c* is the speed of light, τ is the reciprocal lattice vector of the CLC spiral ($\tau = 4\pi/p$, where p is the cholesteric pitch).



Fig.1 Reflection coefficient R calculated versus the frequency for a nonabsorbing CLC layer (δ =0.05, N=L/p=250). Here and in all figures below, δ (v–1) is plotted at the frequency axis (v=2(ω - ω_B)/($\delta\omega_B$) -1)).

The example of intensity reflection coefficient calculation is presented at Fig.1. If both amplitudes of the incident waves are equal to zero, no waves emerging from the layer exist if the dielectric tensor has a positive (or a very small negative) imaginary part. The calculations also show [23] that the values of the eigenwave amplitudes in the CLC layer excited by an incident light close to the stop-band edges are strongly oscillating functions of the frequency. At the points of maxima close to the stop-band edges their values are much larger than the incident wave amplitude. It turns out that the amplitude maxima frequencies coincide with the frequencies of zero reflection for a nonabsorbing CLC (see Fig.1).

3. EDGE MODE (NONABSORBING LC)

In a nonabsorbing CLC $\gamma=0$ in the general expression for the dielectric constant $\varepsilon=\varepsilon_0(1+i\gamma)$. The calculations of the reflection *R* and transmission *T* coefficients as functions of the frequency (Fig.1) give the well-known results [17-21], in particular, T+R=1 for all frequencies.

The mentioned above relation between the amplitudes of eigenwaves and incident waves at the specific frequencies shows that the energy of radiation in the CLC at the layer thickness for these frequencies is much higher than the corresponding energy of the incident wave at the same thickness. Hence, in complete accordance with [22], we conclude that at the corresponding frequencies the incident wave excites some localized mode in the CLC. To find this localized mode, we have to solve homogeneous system corresponding to the inhomogeneous system describing reflection and transmission of light [23]. The solvability condition for this homogeneous system determines the discrete frequencies of these localized modes:



Fig.2 The calculated EM energy (arbitrary units) distributions inside the CLC layer versus the coordinate (in the dimensionless units $z\tau$) for the three first edge modes (δ =0.05, N=16.5, n=1.2.3).

In the general case, solutions of Eq.(3) for the EM frequencies ω_{EM} can to be found only numerically. The EM frequencies ω_{EM} turn out to be complex quantities, which can be presented as $\omega_{EM} = \omega_{EM}^0(1+i\Delta)$, where Δ is a small parameter in real situations. Fortunately, an analytic solution can be found for a sufficiently small Δ ensuring the condition LImq<<1. In this case the ω_{EM} is determined by the conditions

$$qL=n\pi$$
 and $\Delta=-\frac{1}{2}\delta(n\pi)^2/(\delta L\tau/4)^3$, (4)

where integer n is the EM number, which increases as the frequency departs from the stop band edge.

The field intensity distributions in the layer for the EM numbers n = 1,2,3 are presented in Fig.2. The Fig.2 shows that the EM field is localized inside the CLC layer and its

energy density experiences oscillations inside the layer with the number of the oscillations equal to the EM number *n*. However, the EM energy is leaking from the layer through its surfaces and EM life-time τ_m is finite for a finite layer thickness L. For sufficiently thick CLC layers the EM lifetime τ_m can be found analytically [23]:

$$\tau_{\rm m} \approx 1/{\rm Im}(\omega_{\rm EM}) = (L/c)(\delta L/pn)^2. \tag{5}$$

4. ABSORBING LC

We assume for simplicity that the absorption in the LC is isotropic. We define the ratio of the imaginary part to the real part of the dielectric constant as γ , i.e., $\varepsilon = \varepsilon_0(1+i\gamma)$. In Fig.3 the 1-*R*-*T* dependence on the frequency is presented for a positive γ . In an absorbing LC *R*+*T*<1. It occurs that for each *n* the maximum absorption, i.e., maximal 1-*R*-*T*, occurs for

$$(n\pi)^2 = (\delta L\tau/4)^3 \gamma. \tag{6}$$

From the Eq.(6) follows that the maximum absorption occurs for a special relation between δ , γ , and *L*. Because of the assumed smallness of γ , this result corresponds to a strong enhancement of the absorption for weakly absorbing layers.



Fig.3 The absorption 1-*R*-*T* calculated versus the frequency (l=300, $l=L\tau=4\pi N$, $\delta=0.05$) for $\gamma=0.001$.

As was shown in [20,24] just at the frequency values determined by (6), the effect of anomalously strong absorption reveals itself for an absorbing chiral LC (Fig.3).

5. AMPLIFYING LC

We now assume that $\gamma < 0$, which means that the CLC is amplifying. If $|\gamma|$ is sufficiently small, the waves emerging from the layer exist only in the presence of at least one external wave incident on the layer. In this case R+T>1 or 1-R-T<0 what just corresponds to the definition of an amplifying medium.

However, if the imaginary part of the dielectric tensor, i.e. γ reaches some critical negative value, the quantity R+T diverges (see Fig.4) and the amplitudes of waves emerging from the layer are nonzero even for zero amplitudes of the incident waves. The corresponding γ is the minimum threshold gain at which the lasing occurs. The equation determining the threshold gain (γ) coincides with Eq.(3). But, it must be solved now not for the frequency but for the imaginary part of the dielectric constant (γ).



Fig.4 *R* calculated versus the frequency (l=300, $l=L\tau$, $\delta=0.05$) (top) close to the threshold gain for the first lasing edge mode ($\gamma=-0.00565$), (bottom) close to the threshold gain for the second lasing edge mode ($\gamma=-0.0129$);

In the general case, this equation has to be solved numerically. However, for a very small negative imaginary part of the dielectric tensor the threshold values of the gain for the EM can be represented by analytic expressions in this limiting case.

For a very small $|\gamma|$ and *L*|Imq|<<1 the threshold values of γ can be found in analytic form:

$$\gamma = -\delta(n\pi)^2 / (\delta L\tau/4)^3 \tag{7}$$

It also follows from Fig.4: the different threshold values of γ correspond to the different edge lasing modes. This means that separate lasing modes can be excited by changing the gain (γ), i.e. the intensity of the pumping wave.

6. BOUNDARY PROBLEM FOR DM

The defect structure (DMS) which is under consideration here is shown at Fig.5. The solution of the boundary problem is carried out in the similar way as for a CLC layer above so we give below the final results (All simplifications accepted above for the CLC layer are accepted also for the DMS).



Fig.5 Schematic of the CLC DMS with an isotropic defect layer.

There is an option to obtain formulas determining the optical properties of the structure depicted at Fig.5 via the solutions found for a single CLC layer [23]. If one use the expressions for the amplitude transmission T(L) and reflection R(L) coefficient (1) for a single cholesteric layer (see also [20,21]) the transmission $|T(d,L)|^2$ and reflection $|R(d,L)|^2$ intensity coefficients for the whole DMS may be presented in the following form:

 $|T(d,L)|^{2} = |[T_{e}T_{d}exp(ikd)]/[1-exp(2ikd) R_{d}R_{u}]|^{2},$ (8)

 $R(d,L) |^{2} = | \{R_{e}+R_{u} T_{e}T_{u}exp(2ikd)/[1]$

$$-\exp(2ikd) R_{d}R_{u}]\}|^{2}, \qquad (9)$$

where $R_e(T_e)$, $R_u(T_u)$ and $R_d(T_d)$ are the amplitude reflection (transmission) coefficients of the CLC layers (1) (see Fig.5) for the light incidence at the outer (top) layer surface, for the light incidence at the inner top CLC layer surface from the inserted defect layer and for the light incidence at the inner bottom CLC layer surface from the inserted defect layer, respectively. It is assumed in the deriving of Eqs.(8,9) that the external beam is incident at the structure (Fig.5) from the above only.



Fig.6 R(d) versus the frequency for a nonabsorbing CLC (γ =0) at d/p=0.1 (top) and d/p=0.25 (bottom); δ =0.05, 1=200, 1=L τ =2 π N, where N is the director half-turn number at the CLC layer thickness L.

The calculated reflection $|R(d,L)|^2$ spectra inside the stop band for the structure sketched at Fig.5 for nonabsorbing CLC layers are presented at Fig.6. The figures show minima in $|R(d,L)|^2$ at some frequencies inside the stop band at positions which depend on the defect layer thickness d. As it is known [1-3,15,16], the corresponding frequencies of minima of $|R(d,L)|^2$ and maxima of $|T(d,L)|^2$ correspond to the defect mode frequencies. For the layer thickness d =p/4, what is just one half of the dielectric tensor period in a cholesteric, these maxima and minima are situated just at the stop band center. In the d/p interval 0<d/p<0.5 the defect mode frequency value moves from the high frequency stop band edge to the low frequency stop band edge. At Fig.6 only R(d) are presented because for a nonabsorbing structure $|R(d,L)|^2 + |T(d,L)|^2 = 1$.

6. DEFECT MODE

Similarly to the case of EM the DM frequency ω_D is determined by the zero value of the determinant Det(d,L) of the system corresponding to the boundary value solution for the structure depicted at Fig.5 $\ [25]$:

 $Det(d,L)=4{exp(2ikd)sin^2qL-exp(-i\tau L)[(\tau q/\kappa^2)cosqL}$

+i(
$$(\tau/2\kappa)^2$$
+(q/ $\kappa)^2$ -1)sinqL]²/ δ^2]}. (10)

Note, that the Det(d,L) at a finite length L does not reach zero value for a real value of ω for a nonabsorbing CLC however reaches zero value for a complex value of ω . There is a leakage of the DM energy outwards through the external surfaces of the DMS. The ratio of the corresponding energy flow to the whole DM energy accumulated in the DMS determines the inverse DM life-time.

For nonabsorbing CLC layers the only source of decay is the energy leakage through their surfaces. The analysis of the corresponding expressions [25] shows that the DM lifetime τ_m is dependent on the position of the DM frequency ω_D inside the stop band and reaches a maximum just at the middle of the stop band , i.e. at $k=\tau/2$.

7. THICK CLC LAYERS

In the case of DMS with thick CLC layers (|q|L>>1) some analytic results related to DM can be also obtained by the same way as for EM. In particular, the defect mode life time τ reaches a maximum for the defect mode frequency at the stop band centre at fixed CLC layers thickness. For the DM frequency at the middle of the stoop band, i.e. at $k=\tau/2$, one finds for the DM life time τ_m :

$\tau_{\rm m} = [(1/8\pi\delta^2)(p/L)(L\epsilon_0^{\frac{1}{2}}/2c)(2\delta 2^{\frac{1}{2}}+1)\exp[2\pi\delta 2^{\frac{1}{2}}L/p].$

The expression reveals an exponential increase of $\tau_{\rm m}$ with increase of the CLC thickness L.

8. ABSORBING LC

To take into account the absorption we again accept $\epsilon = \epsilon_0(1+i\gamma)$. There are some interesting peculiarities of the optical properties of DMS (Fig.5). The total absorption $(1-|T(d,L)|^2 - |R(d,L)|^2)$ at the DM frequency (Fig.7) behaves itself unusually.



Fig.7 The total absorption for an absorbing CLC versus the frequency, γ =0.0003; d/p=0.1, δ =0.05, N=33.

At a small γ for the DM frequencies the absorption occurs to be much more than the absorption out of the stop band (see Fig.7). It is a manifestation of the so called "anomalously strong absorption effect" known for perfect CLC layers at the edge mode frequency [20,24]. So, one sees that at the DM frequency ω_D the effect of anomalously strong absorption similar to the one for EM [20,24] exists and more over the absorption enhancement for DM at small γ is higher than for EM. In the case of thick CLC layers (lq|L>>1) the dependence of γ , on L and other parameters ensuring maximal absorption may be found analytically. For the position of ω_D just in the middle of the stop band the expression for γ ensuring maximal absorption takes the following form

 $\gamma = [2(2^{\frac{1}{2}})/3\pi]((p/\delta L) \exp[-2\pi\delta 2^{\frac{1}{2}}(L/p)].$ (11)

9. AMPLIFYING LC

The calculation results for the reflection $|R(d,L)|^2$ coefficient at $\gamma < 0$ are presented at Fig.8. For a small absolute value of γ the shape of the transmission T(d,L) and reflection $|R(d,L)|^2$ coefficients is qualitatively the same as for zero amplification (γ =0). For a growing absolute value of γ at some point a divergence of $|T(d,L)|^2$ and $|R(d,L)|^2$ happens (Fig.8)) with no signs of noticeable maxima at other frequencies. The corresponding value of γ may be considered



Fig.8 R(d) (top) for an amplifying CLC versus the frequency, γ =-0.00117; (bottom) γ =-0.0045 d/p=0.1, δ =0.05, N=33.

as a close to the threshold value of the gain (γ) for the DFB lasing at the DM frequency. Continuing the increase of the absolute value of γ one finds that diverging maxima for $|R(d,L)|^2$ at the edge mode frequencies appear (without no traces of maximum at the defect mode frequency) for the gain being approximately four time more than the threshold gain for the DM (Fig.8). The observed result show that the DM lasing thresholds gain is lower than the corresponding threshold for the EM. Another conclusion following from this study is the revealed existence of some interconnection between the LC parameters at the lasing threshold which for thick CLC layers was found above analytically for DM and for the EM. Really, a continuous increase of the gain results in consequential appearance of a lasing at new EM with

disappearance of lasing at the previous ones corresponding to more low thresholds (what was experimentally observed [3]). The mention above interconnection between the LC parameters at the lasing threshold in the case of thick CLC layers (|q|L>>1) may be found analytically. If the DM frequency ω_D is located at the stop band centre the corresponding interconnection for the threshold gain (γ) is given by the formula:

 $\gamma = -[2(2^{\frac{1}{2}})/3\pi](p/\delta L) \exp[-2\pi\delta 2^{\frac{1}{2}}(L/p)].$ (12)

The following from Eq.(12) exponentially small value of $|\gamma|$ for thick CLC layers confirms mentioned above statement about more low lasing threshold for DM compared to EM.

10. CONCLUSION

The performed in the previous sections analytical description of the EM and DM neglecting the polarization mixing at the boundaries of CLC in the structures under consideration allows one to reveal clear physical picture of these modes which is applicable to the defect modes in general. For example, more low lasing threshold and more strong absorption (under the conditions of anomalously strong absorption effect) at the DM frequency compared to the EM frequencies are the features of any periodic media. Note, that the experimental studies of the lasing threshold [3] agree with the corresponding theoretical result obtained above. Moreover, the experiment [3] confirms also the existence of some interconnection between the gain and other LC parameters at the threshold pumping energy for lasing at the DM and EM frequencies. For a special choice of the parameters in the experiment the obtained formulas may be directly applied to the experiment. However, in the general case one has take into account a mutual transformation at the boundaries of the two circular polarizations of opposite sense. In the general case the EM and DM field leakage from the structure is determined as well by the finite CLC layer thickness so by the leakage due to the polarization conversion. Only for sufficiently thin CLC layers or in the case of the DM frequency being very close to the stop band frequency edges the main contribution to the frequency width of the EM and DM is due to the thickness effect and the developed above model may be directly applied for the describing of the experimental data. The defect type considered above is a homogenous layer. The developed approach is applicable also to the defect of "phase jump" type [2,3,15,16] and so the corresponding results are practically the same as above. Namely, the equation related to the case of a "phase jump defect" one gets from the equations presented above by the substitution in the factor exp(2ikd) instead 2ikd the quantity $2\Delta \phi$, where $\Delta \phi$ is the spiral phase jump at the defect plane. It should be mentioned also that the localized DM and EM reveal themselves in an enhancement of some inelastic and nonlinear optical processes in photonic LCs. As examples the corresponding experimentally observed effects for the enhancement of nonlinear optical second harmonic generation [27] and lowering of the lasing threshold [28] in photonic LCs have to be mentioned along with the theoretically predicted enhancement of Cerenkov radiation (section 4 in [20] and chapter 5 in [21]).

In the conclusion should be stated that the results obtained here for the EM and DM (see also [25] and [26] for the EM) clarify the physics of these modes and manifests a complete agreement with the corresponding results of the previous investigations obtained by a numerical approach [22]. The work is supported by the RFBR grants 09-02-90417-Ukr_f_a, 10-02-00417-a and 10-02-92103-JP_a.

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