Construction and Fostering of Mathematical Giftedness via Cybernetic Loops between Research and Practice

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ABSTRACT

According to the Universe of Engineering from the Royal Academy of Engineering, mathematics is the only discipline listed that appears in every application of engineering. Because of this major role, the question of a definition of engineering includes the question of a definition of mathematics. Going further to the educational field and looking at the participating human beings, the question alters to a question of a definition of Mathematical Giftedness. Modern concepts about that are very vague. We present an approach to an extensive definition that is based on cybernetic loops between research and practice, i.e. an iterative process started by entering the empirical field and performing developing cycles stemming from Design-Based-Research. The expected product is a systems-theoretical construct of Mathematical Giftedness, leading to innovative changes in the fostering of mathematical gifted students by the cybernetic most promising intervention at schools on the meta-level of beliefs and attitudes of teachers and learners in an incremental-evolutionary way. The paper is mainly focused on the theoretical background, supported by a brief excerpt from a current case study.

Keywords: Mathematical Giftedness, Systems Theory, Cybernetic Loops, Design-Based-Research

1. FROM "WHO ARE ENGINEERS?" TO "WHO IS MATHEMATICALLY GIFTED?"

Similar to I. Aleksander, who asks "what is engineering?" [2], we address the question "what is mathematical giftedness?". According to what the Royal Academy of Engineering named the Universe of Engineering, the discipline 'Mathematics' is the only one listed in the matrix in [28] that "has a dot" in every field referring to a special application. So, mathematics (once again) seems to be the most fundamental science when it comes to deal with disciplines like engineering. Hence, the fight for a "comprehensive, open and adaptive" [10, p. 14] definition somehow includes the definition of mathematics. So, when N. Callaos titles a paragraph with an underlined "Who are Engineers?" [10], to some account this implies the question "Who are Mathematicians?" – transferred to the educational sector this is: "Who is mathematically gifted?"
Being concerned with the didactics of mathematics in the first place, our main goal is to develop material to foster mathematically gifted students. But this has to be based on a solid concept of mathematical giftedness initially.

2. MATHEMATICAL GIFTEDNESS

The concept of mathematical giftedness seems hard to grab. There are even authors, who claim that there is no specific thing as mathematical giftedness ([15], for example). Others, mainly psychologists, claim that mathematical giftedness might be equivalent to some general kind of intelligence ([35], for example). Known for his research in the context of giftedness, D. Rost says "that the existence of an independent ‚mathematical' giftedness is highly controversial – mathematical giftedness is, as lots of empirical facts show, nothing else than general intelligence (problem solving within a numerical context)." [39, p. 94].
However, there is a large number of publications that imply that there is empirical evidence to the existence of such a disposition. Given that specific mathematical giftedness exists in some form, the common solution is to hand a list containing a collection of certain mathematical abilities and personal qualities (see among others [18], [22], [23], [24] and [36]). Loosely speaking there seem to be two necessary aspects of mathematical giftedness, i. e. abilities specific to mathematics on the one hand and general personality traits on the other. The first part contains mental skills like mathematical sensibility, memory, structuring, generalizing and reversion of mathematical processes, whereas the second part demands intellectual curiosity, willingness of exertion, joy in problem solving, perseverance and frustration tolerance. These items more or less show up in recent surveys concerning the topic of mathematical giftedness. However, by looking at the subject at different times and within different cultural contexts the definition changes somehow. There always remains some kind of vagueness that seems to be concept immanent. Hence, a conceptual approach by terms of systems theory was given in [5].

The cultural-philosophical point of view

The "Law of cultural differentiation" [20] refers to G. A. Ferguson who says that "cultural factors prescribe what shall be learned and at what age; consequently different cultural environments lead to the development of different patterns of ability" [14, p. 121]. So „the anthropological approach shows how the nature of a construct may vary across time as well as space." [34, p. 308]. This position leads to the objection of the common notion that mathematical giftedness is a definite human trait, which someone more or less has or has not. As B. Zimmermann puts it in [40]: the notion just mentioned is an one-dimensional perception that results from an often institutionally desired quantifying linear assessment of students' performance (by marks) classifying the better and the worse. In contrast, different notions of mathematics may not only be based on different kinds of mathematical giftedness but may also give rise to different concepts of mathematical giftedness. This has already been seen by Freudenthal: "The definition of mathematics varies. Each generation and each subtle mathematician within each generation formulates a definition that corresponds to his or her skills and insights." [22, p. 53]. For the following approach this perception is widened to a look not only at mathematics but also at mathematical giftedness as a social construct within time and culture. The conceptual framework therefore is delivered by systems theory and post-structuralistic sociology.

Historical discourses

A suitable post-structuralistic view of mathematical giftedness is that as a result of epochal cultural discourses which were described first by M. Foucault in 1973. Analogously to the approach done in [10] for the case of engineering, we see this point of view in the same sense as R.L. Ackoff stressing the fact by which "historical analysis of the use of a concept can often reveal a trend in the evolution of the concept or a consistent theme of meaning which persist through numerous variations" [1, p. 148].

For illustrative aspects there are two extreme examples related to the description of being mathematically gifted:

a) Women: About 100 years ago in Germany there was the belief that mathematics is harmful to the female psyche, a fact that was based on the enormously successful book "About women's physiological amentia" [30]. This apparent absurdity kept up till National Socialism, where girls in secondary schools only received limited mathematical education [9, p. 49].

b) Asperger's Syndrome: This weakest form of autism was first described by H. Asperger in his habilitation thesis wherein he spoke of "autistic psychopaths" [3] which is a clearly medical or in modern terms even discriminating expression. However, nowadays Asperger's Syndrome (in conjunction with giftedness) is seen as a rather beneficial disposition of the "real heroes of our computer ruled world" ([33], [16]).

So, the aim is to formulate a "tentative definition based on the evolving core identified by a historical analysis" [10, p. 14].

Mathematical Giftedness in Systems Theory

Within the conceptual framework of systems theory the system mathematical giftedness is an open mental disposition construct [5]. It is surrounded by the environment mathematics that consists both of the mathematical truths (theorems, definitions, …) and the individual researchers making up these facts. Figure 1 illustrates the necessary structural couplings between the system and its environment that describe the influence of predominant philosophical notions about mathematics on a conception of mathematical giftedness. Hence, there is a structural change of the system because of the structural coupling with the environment. In terms of systems theory this means that "Mathematical Giftedness" is *viable*, i. e. however the system changes in structure, it will always adapt to the limiting conditions of the environment [25, p. 41]. Furthermore, viability in systems theory equates to the openness of constructs that is demanded by science theory and psychology ([7], [29]).

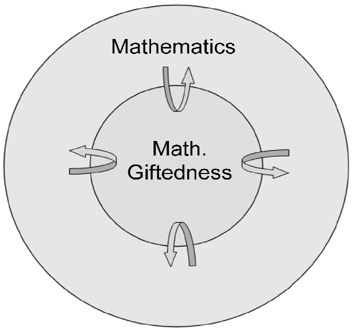

Figure 1: The system and its environment; structural couplings.

When it comes to environments, systems theory offers two types of. In this regard mathematics is "environment 2", which "is the world that creates sense for the system" [25, p. 81]. The further environmental factors that affect the system "Mathematical Giftedness" in a competitive way, too, but which do not necessarily endow the system with meaning or sense, are collected in "environment 1". An incomplete list for systems or powers, respectively, within environment 1 would be: educational system, sociology, philosophy, psychology, economy, fostering, family, media, … . So, "Mathematical Giftedness" manifests in different kinds and shapes depending on the notion of mathematics and the current state of the discourse concerning the authorities in environment 1. These items can only be detected by entering the field, like a certain school, and investigating the predominant notions and characteristics by qualitative and quantitative methods. Hence, from a sociologically point of view, the question "Is this person mathematically gifted?" should be altered to the question "Is this person mathematically gifted *within this specific environment(s)*?".

Identification and fostering of mathematically gifted students

For the purpose of fostering gifted students, the identification of those comes first. There has been done plenty of research emphasizing the quantitative aspect of different kinds of tests that historically range from a solely IQ-test over bringing in school marks up to modern multidimensional settings ([19], [31]). However, in day-to-day school life there is one dominant (non-) identifier and fosterer of giftedness, for example, in mathematics: the teacher – and because of the viability of the construct, the teacher's idea of mathematical giftedness may differ from that of a professional mathematician and a researcher in mathematics education, respectively.

Another question when it comes to fostering a certain talent is: "How should it be done?" Or more precise in the context of mathematics education: "How can innovative changes in the school curriculum or the teaching practise, respectively, aiming at the optimal fostering of mathematically gifted students, be gained?"

Incremental-evolutionary changes on the meta-level

When we look at environment 1 surrounding the systems Mathematics and Mathematical Giftedness, we are dealing with systems once again. According to the definitions given in [27] or [38] mathematics education in Europe and even mathematics education at a concrete school must be seen as "complex" systems. Such are networks of multiple connected components. One cannot change a component without influencing the character of the whole system.

Insights from the theory of cybernetics deliver a promising approach to fruitful innovation in the teaching process ([37], [6]). With reference to [27] two dimensions of steering complex systems can be distinguished. While the first dimension concerns the manner, the second one deals with the level of steering activities (see Figure 2).

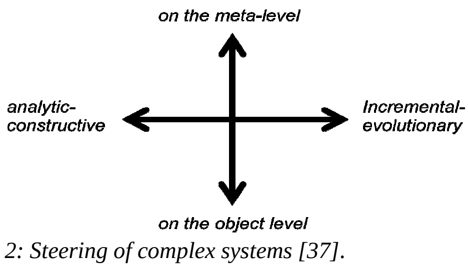

Figure 2: Steering of complex systems [37].

Hierarchical-authoritarian systems, for example, are founded on the method of *analytic-constructive* steering. This principle needs a controlling authority that defines ways for reaching certain aims. However, complex systems are defined as a network that can potentially be in so many states that nobody can cognitively grasp all possible states of the system and all possible transitions between the states. So this first approach fails by the fact that it would afford information about the system that cannot be gained in reality.

On the other hand one can try to focus on the natural growing and developing processes and claim that the changes in complex systems only result from those. This *incremental-evolutionary* steering tries to influence these systemic processes by accepting the fact that complex systems cannot be steered entirely in all details. Instead, this approach is satisfied with only little steps, i.e. incremental changes, in promising directions. And as to the metaphor of the butterfly's wings that may change the weather far away, every small step may cause unpredictable consequences. So with respect not to endanger the soundness of the whole system, only small changes are essential.

Perpendicular to this dimension, Figure 2 illustrates the dimension that distinguishes between the *object* and the *meta-level*. As the name implies, the object level consists of all concrete objects of the system. In the system of higher mathematical education these would be the teachers, the students, the materials as books, computers, software, the buildings and so on. In contrast, the meta-level comprehends organisational structures, social relationships, notions of the functions of the system etc. In the educational system e.g. notions of the subject mathematics and beliefs concerning learning and applying the subject matter are on this level.

For the question, how substantial innovations in the complex system "mathematics education" can be initiated successfully, the theory of cybernetic says: attempts of analytic-constructive steering will fail in the long term, since they ignore the complexity immanent in the system; changes on the object level do not necessarily cause structural changes of the system; in contrast, it is much more promising to initiate incremental-evolutionary changes on the meta-level of beliefs and attitudes of the teachers and learners, see Figure 3. On the one hand these are in accord with the complexity of the system and do not endanger its existence, and on the other hand they can cause substantial changes within the system by having effects on the meta-level, especially when they work cumulatively [37].

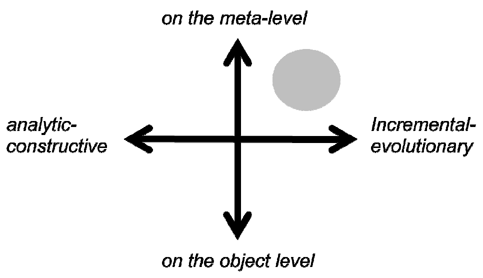

Figure 3: Innovations in complex systems [37].

Cybernetic loops between research and practice

Working in the area of mathematics education, we refer once again to the Universe of Engineering in [28] and restrict ourselves to the application 'education'. In order to construct a reliable definition of Mathematical Giftedness as well as to develop and implement changes in the mathematical education of mathematically gifted students at specific schools, we rely on an iterative process going from theory and research to practice and back. The (positive or negative) feedback and re-feedback forms a cybernetic loop establishing informing processes between research and practice, illustrated in Figure 4.

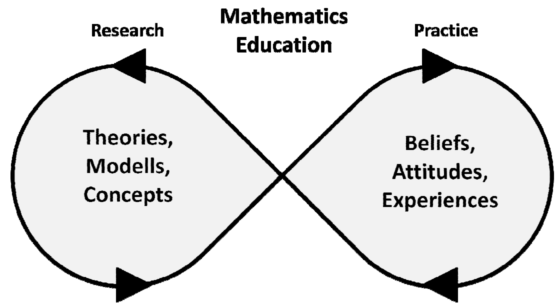

Figure 4: The cybernetic loop between research and (school) practice in mathematics education.

Actually, giving feedback to the studied empiric field, may be seen as 'consulting' in the educational sector. This "consulting is research [and] offers the researcher one of the most effective channels from achieving impact." [17, p. 485].

By specializing the loop to the development of new materials for the fostering of mathematical gifted students the informing process leads to the so called Design-Based-Research (DBR). In order to achieve a "sustainability of innovation" [32] the original motivation and basic idea is to study learning phenomena not inside artificial surroundings but in real situations [8]. By systematic development, testing, evaluation and re-design, a postulated complexity of learning processes shall be approximated in a better way. This means in practice that in front of the background of theoretical thoughts a teaching-learning-scenario is developed and implemented into a concrete learning situation. After testing and evaluation of this scenario in the field, the learning unit is optimized [21]. The DBR-approach again can be illustrated in form of a loop [13, p. 153] as done in Figure 5.

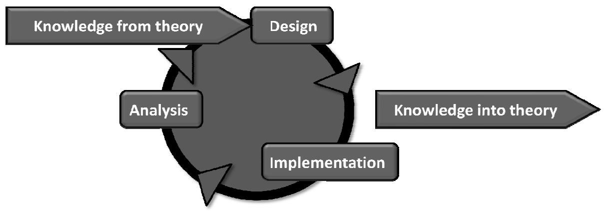

Figure 5: n-th step of the iterative innovation process within the DBR-approach.

As results from the iterative process there are both a theoretical output concerning the know-how for the design process and a practical output in form of concrete improvements for the evolvement of innovative potentials in day-to-day educational practice ([11], [12], [4]). In the context of the theoretical output and by generalization of design solutions, so-called "design frameworks" are gained which represent coherent guidelines for the design of learning environments.

3. EXAMPLE: EXCERPT FROM A CASE STUDY

In order to establish a custom-made material-based program that suits – and, in case, changes – the predominant concept of Mathematical Giftedness at a specific German school for high attaining pupils, the manifestation of the construct Mathematical Giftedness at this specific school was investigated by qualitative teacher interviews. Furthermore, for reasons of validation and to cover the beliefs and problems of risk groups, group discussions were done in a non-directive way with each group consisting of about five low and high, respectively, attaining students from the 11th and 12th grade. Additionally, a questionnaire was given to the students in order to evaluate the beliefs, notions, self-perceptions and interests in mathematics in a quantitative way, supporting and grounding the qualitative data (and vice versa).

Description of field and sample

The school is a German boarding school for higher secondary education. So the students at this school are about between 15 and 18 years old. The classes are small and consist of 16 students with equally distributed gender. All students that want to apply for the school must have at least the mark "good" in the main subjects German, mathematics, one foreign language and one natural science. Furthermore, the average mark in the last two school reports has to be at least "good", too. Students that fulfill the requirements with respect to marks have to pass a further selection process that consists of the intelligence structure test I-S-T 2000 R [26] and a two-day assessment centre concerning their social skills.
All (eight) mathematics teachers were interviewed in an individual, problem-centered and episodic way with focus on their beliefs concerning their notion of mathematics, Mathematical Giftedness and the students at their school. From the transcript excerpts an empirically grounded conclusion on the predominant concept of Mathematical Giftedness in practice can be drawn and set in contrast to the predominant concept in research.

Illustrative data from a teacher interview

Teacher A is 47 years old. He has worked at the school for six years, earned a PhD in a non-math subject and had a several years experience of working with gifted students at different secondary schools before entering the current school. The interview lasted 55 minutes.

Teacher A: notion of mathematics: Teacher A addresses several aspects. His answers can be separated according to different topics. So, the main themes are listed, followed up by some quotes stemming from the interview (translated into English):

- leisure: "[…] thinking it over relaxed and quietly, this and then concentrating on it, that gives a better return"
- philosophical thoughts: "[…] it must make sense. I mean that philosophically, too. Because you said what pupils should answer to what is mathematics. Yes. (.) That is a matter of taste either way, but that it makes sense somehow."
- communicative process: "Mathematics is a communicative process."
- logically clearly defined and also communicative intellectual game: "I hope they don't see mathematics as deterrent but as (.) a logically clear defined (.) and also (.) communicative intellectual game."
- intellectual appealing world of thought: "[…] as an appealing, intellectually appealing, but that's also a matter of taste, intellectual appealing mhh (..) world of thoughts"
- also applications: "That is a matter of taste either way, but that it makes sense somehow. ((hm)) (..) Either just cognitively or by application"
- intellectual joy, happiness: "The goals? ((yes)) Well, that the pupils have intellectual fun […] Joy, actually that's the highest goal."

Teacher A: manifestation of Mathematical Giftedness: When teacher A was asked for his idea of mathematical giftedness, the interview again delivered some main points:

- intuition; sensing the overall train of thoughts: "Well, I see that in fact that they actually sense the point of it all without that I would have spent many words about the problem. […] a kind of intuition, aha, so now there is this and that and this. That they sense the overall train of thoughts where the method will lead us"
- interest in alternative definitions and the corresponding consequences: "[…] it is the permanent attempt (.) can't we play differently? ((yes, yes)) And if yes, where would this lead us to? If no, why doesn't it work out?"
- aesthetical sensation and joy: "Those have more fun than usual. […] Over the fact that this is a beautiful construction of the theory ((ah, yes, certainly)) Or: what a wonderful exactness! ((mhm)) So this joy over a good construction […] they really feel it."

Teacher A: looking at the structural coupling: Teacher A has a very high-level and sophisticated notion of mathematics. Because of the structural couplings between the system Mathematical Giftedness and its environment of mathematics this leads to strong correlations with teacher A's idea of Mathematical Giftedness, see Table 1.

Notion of Mathematics	*Math. Giftedness*
• leisure • philosophical thoughts • communicative process • logically clearly defined and also communicative intellectual game • intellectual appealing world of thought • also applications • intellectual joy, happiness	• intuition: sensing the overall train of thoughts • interest in alternative definitions and the corresponding consequences • aesthetical sensation and joy about that

Table 1: Notion of mathematics and Mathematical Giftedness of teacher A; parts corresponding to each other are indicated by arrows.

A bunch of insights from the ongoing process

Proceeding further on the cybernetic loop in Figure 4 between the two worlds of research and practice there were several unexpected findings that made their way into the re-design of theoretical ideas and constructs. We list some examples:

- From the other teachers' interviews came the consensus that in spite of the more or less equalizing selection process and the school's official focus on mathematics, there is still an enormous heterogeneity in mathematics classes.

- Although all teachers are extremely qualified professionals, most with international or academic background, their notions of mathematics are quite different. The extreme counterpart to teacher A's notion, for example, may be teacher B, who claims that mathematics is nothing more than a useful tool to solve equations and formulas.
- Almost all teachers confessed that the main problem when it comes to the students' performance (in mathematics, too) may not be their potential giftedness, but the psychological hindrance of a narcissistic wound that comes from being confronted with just best-of-students in class and the eventual loss of this status for oneself.

A parallel survey was the questionnaire addressing the students:
- Some big differences could be seen in the answers to several questions concerning the students' attitudes towards mathematics and the subsequent notion of the subject between the 11^{th} and the 12^{th} grade.
- The evaluation of the answers concerning the students' interest and self-perceptions with regard to mathematics revealed an always higher valuation of the subject by themselves in contrast to their meaning about the interest and self-perception by the rest of the students in class.

With respect to the first item, the discussion groups with selected students showed for example that the teacher's and the classroom colleagues' behavior have big influence on the motivation and the attitude of the students towards mathematics. Those secondary environmental factors have to be considered according to a model explaining the relationship between a giftedness potential and the quantifiable assessment in mathematics as illustrated in [36], for instance.
In order to answer open questions, to validate the extracted data from the qualitative and quantitative surveys, and to "keep the cybernetic loop moving", the author visited the field again and presented the preliminary results to the teachers. The feedback from the research side led to a (re-)feedback from the consulted practical side which will influence the design of applicable material. Further iterative loops between research and practice will eventually reveal deficits within the learning units which will lead to improvements once again.

4. SUMMARY

The goal of the whole research process shown in this paper is that stepping down from a bird-eye's view from research onto practical mathematical education in schools and moving instead iteratively on a synergetic cybernetic loop will on the one hand hopefully bring up a modern, "comprehensive, open and adaptive" [10, p. 14] constructed theory of Mathematical Giftedness, which is on the other hand accompanied by an adequate catalogue of fostering materials for mathematically gifted students at specific schools and beyond. According to the theory of cybernetics, careful interventions in the field via incremental-evolutionary acts on the level of notions and beliefs will be the most promising way to establish success in the education of mathematically gifted students.

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