

# Simulation based method for mathematical model design of LLC power converter

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## ABSTRACT

With increasing of microprocessors and DSP (digital signal processors) use as digital control systems in new, multiresonant power semiconductor converters, the new approaches for modeling and simulation of converters must be implemented. Designing the control system requires use of control to output transfer function for mathematical model of converter. For new, frequency controlled resonant converters, like LLC converter, standard averaging methods cannot be applied for revealing the transfer function. Instead of that, new simulation based methods or numerical methods using computing power of modern PC must be employed.

**Keywords:** mathematical model, simulation, resonant, converter, transfer function

## 1. INTRODUCTION

In the field of DC/DC converters for switching power supplies, the main focus is on implementation of digital control, which means use of microcontrollers or DSP with implemented control algorithms and functions for communication with user as well. For proper discrete controller design, the method named direct digital design can be used. This method uses transfer function of converter in discrete form, including effect of sampling and computation delay.

For above mentioned controller design purposes, transfer function of converter is necessary. This transfer function is based on model of power stage of controller. For PWM (Pulse Width Modulation) converters, such as buck, boost, etc. the averaging method is widely used, but for new resonant topologies such as LLC converter, with this averaging method, the control to output transfer function cannot be ascertained. In this paper, new simulation based method for obtaining the transfer function of converter is proposed. This method uses

MATLAB and OrCad PSPICE environments for revealing the transfer function. First, the circuit simulation of LLC converter has been made, which is discussed in chapter 2, next in chapter 3, the numerical discrete model based method for revealing the transfer function is discussed. This method uses values from simulation of dynamic states of converter in PSPICE. For identification of this dynamic system, a new, MATLAB based method is proposed. At the end of paper, the final form of mathematical model of converter is shown with comparison of its accuracy.

## 2. LLC POWER CONVERTER

Resonant converters are well-known due to their high efficiency and low EMI (Electromagnetic Interference) noise. Despite of that, their utilization in most of industry and consumer applications is still limited due to their disadvantages.

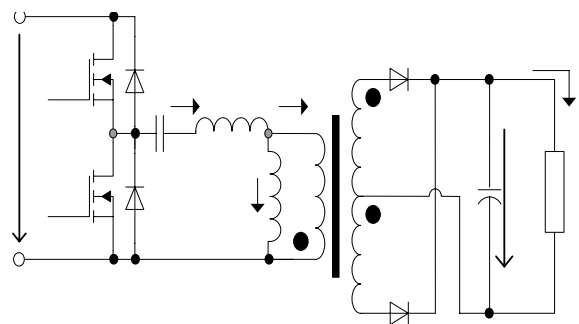


Fig.1. Principal scheme of LLC converter

Nowadays, many variations of resonant converter topologies are available, whereby basic topology are Series-Resonant Converter (SRC), Parallel-Resonant Converter (PRC) and Series-Parallel Resonant Converter (SPRC).

SRC has the character of current supply, which is characterized by high short circuit resistance. The disadvantage is the poor stability of the system in idle state, the need for spectrum management in a wide working range and the disappearance of soft commutation at low load. PRC has an excellent regulatory properties, but for power control throughout the operating range is also necessary to use complex frequency management. PRC is not unable to maintain conditions to ensure soft commutation in its regulatory scope. Addressing shortcomings mentioned topologies a series-parallel converter (SPRC).

The perspective topology is LLC resonant converter (fig.1), which exists for very long time, but because of lack of understanding of characteristic of this converter, it was used as a series resonant converter with passive load.

The benefit of frequency controlled LLC resonant converter is narrow switching frequency range, with light load and ZVS (Zero Voltage Switching) capability with even no load [1,2,3], also with wide input voltage range. LLC resonant converter is multi-resonant converter and is characterized by its unique DC - gain characteristic, which has two resonant frequencies ( $f_0$  and  $f_p$ ) see fig.2.

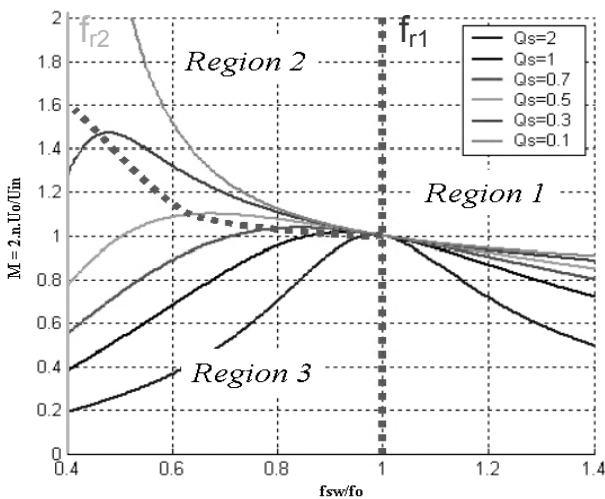


Fig.2. Gain characteristics of LLC converter

Region 1 and Region 2 are characterized by ZVS operating conditions. The operation of the converter in Region 3 is not recommended, due to the ZCS conditions. For operation of the converter with constant DC voltage, it is recommended to operate at the boundary between Region 1 and Region 2 as shown in fig.2. The region 2 is recommended for the operation when reduced DC-supply voltage occurs at the input of the converter. A detailed character of operation in each region is described in [1] [2] [3]. In this paper, we will focus only on the most complex operation, which is in the Region 2.

### Operation cycle of LLC converter

Depending on the state of the power switches T1 and T2, the operation of the LLC converter in Region 2 can be divided into six time intervals per cycle. Fig. 3 shows the relationship between currents and voltages in the LLC converter during each interval. Note, that for operation of the LLC converter in Region 1 the interval  $t_1 \rightarrow t_2$  will disappear.

**Interval  $t_0 - t_1$ :** During this period, switch T1 is ON while switch T2 is OFF. The primary current  $I_p$  (see fig.3) is greater than the current that flows through the parallel resonant inductor ( $L_m$ ). The difference current ( $I_p - I_m$ ) is the load current that flows through diode D1 when multiplied by turns ratio of the transformer. This interval ends at  $t_1$  when the load current becomes null ( $I_p = I_m$ ).

**Interval  $t_1 - t_2$ :** During this period, the switch T1 is ON while the switch T2 is OFF. At instant  $t_1$ :  $I_p = I_m$ , the secondary current is null. The voltage on the power transformer collapses (becomes lower than the output voltage). Neither of the output diodes on the secondary side (D1 and D2) is conducting. The load is supplied from the output capacitor  $C_{out}$ .

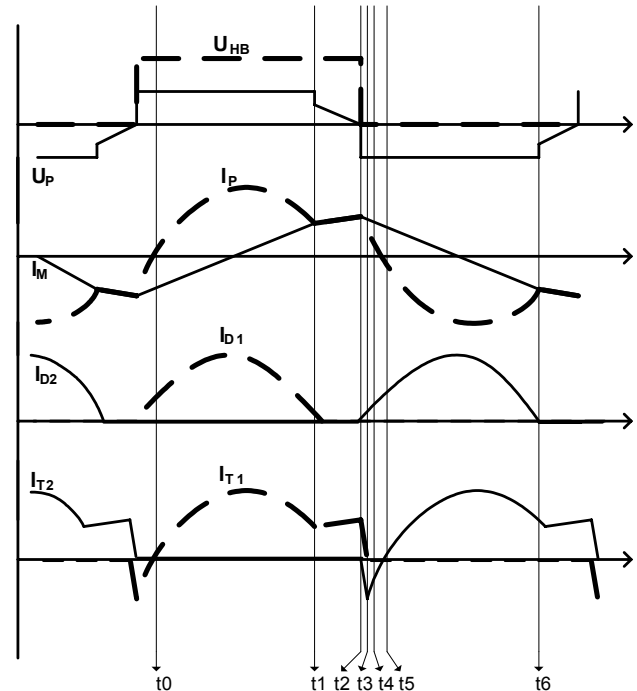


Fig.3. LLC converter timing diagrams for operation in region 2

**Interval  $t_2 - t_3$ :** This interval starts when T1 turns off (no current through the channel). The primary current  $I_p = I_m$  is supported by the resonant inductor  $L_r$ . This current discharges the parasitic capacitance of the switches T1 and T2 and drives the bridge voltage to 0 V. The Instant  $t_3$  corresponds to the bridge voltage reaching

0 V, the current starts to flow through the diode of the switch T2.

**Interval t3 – t4:** During this interval both switches T1 and T2 are OFF. At instant t3, the bridge voltage reaches 0 V and the current starts to flow through the diode of the switch T2. The voltage on the parallel inductor  $L_m$  is reversed and the secondary diode D2 starts to conduct. The primary current  $I_p$  is smaller than the  $I_m$  current which flows through the parallel resonant inductor  $L_m$ . During this interval the voltage across the switch T2 is 0 V and can be turned on with zero voltage switching (ZVS), thus eliminating switching losses. T2 is turned ON at instant t4.

**Interval t4 – t5:** At instant t4 T2 is turned ON with zero voltage switching. The primary current  $I_p$  still flows in the opposite direction through the switch T2. During this interval, the primary current  $I_p$  is smaller than the current  $I_m$  which flows through the parallel resonant inductor ( $L_m$ ). The current difference  $I_{out} = I_m - I_p$  is the load current which flows through the rectifier diode D2 when multiplied by the turns ratio of the transformer. At instant t5 the primary current  $I_p$  changes polarity.

**Interval t5 – t6:** The current flows in the opposite direction to interval [t0, t1]. The rectifier diode D2 is conducting and the load current is:  $I_{out} = I_m - I_p$ . At instant t6, the primary current  $I_p$  becomes the same as the parallel inductor current  $I_m$ . At this instant the load current ( $I_{out}$ ) is null. The output load starts to be supplied by the output capacitor. The set of the six intervals is then repeated.

### Simulation of LLC converter in OrCad PSPICE

For verification and mathematical model design, first, the simulation in Orcad PSPICE has been made. Data acquired from simulations in PSPICE are used in MATLAB for design of mathematical model. For best performance and accuracy of switching waveforms, all parasitic components were included into simulation.

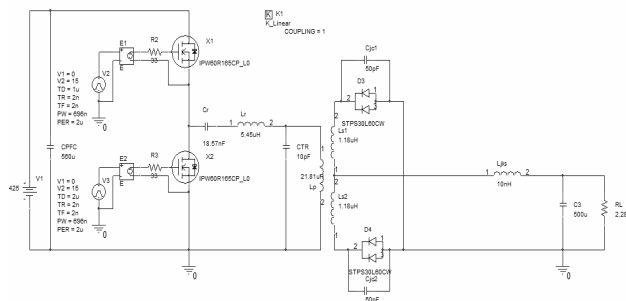


Fig.4. Simulation model of main circuit of proposed LLC resonant converter with parasitic components

For power transistors the IPW60R165CP model was used, and MUR10015CT was used for output diodes [3].

For transformer, the leakage inductance with all parasitic components was also simulated [7], [12], [13].

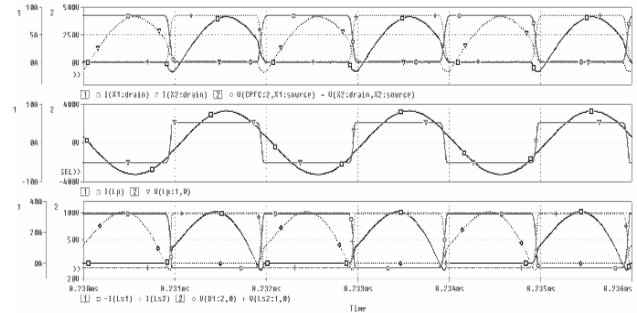


Fig.5. Time waveforms of voltages and currents during the simulation experiment:  $U_{in} = 425V$ ,  $P_{out} = 1008W$  (from top: transistor T1 and T2, transformer primary side, output diode D1 and D2)

In fig.5, the basic waveforms of LLC converter are shown. This simulation experiment was performed with full input voltage 425V and with output power 1kW. Fig.6. shows another waveforms, with lower input voltage at 325V and output power of 1kW. All operation conditions including soft start and overload were also verified in simulation experiments.

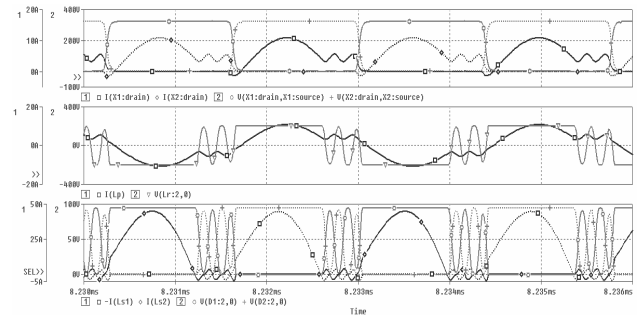


Fig.6. Time waveforms of voltages and currents during the simulation experiment:  $U_{in} = 325V$ ,  $P_{out} = 1008W$  (from top - transistor X1 and X2, transformer primary side, output diode D1 and D2)

From simulations above, it's clear to say, that design procedure was very accurate and results are showing acceptable performance.

## 3. TRANSFER FUNCTION OF LLC CONVERTER

Mathematical model of LLC converter requires control-to-output transfer function. Based on this transfer function, digital controller can be obtained. For PWM converters, standard "averaged" methods can be used for revealing the transfer function of converter. One of method with good results is "direct circuit averaging". This method can be easily implemented for standard PWM converters such as boost, buck, flyback etc. Transfer function obtained with this method has duty

cycle as an input value and output voltage as an output value.

### Identification of the system

Unlike the PWM converters, the control transfer function of frequency controlled resonant converters cannot be obtained by state space averaging method, due to different ways of energy processing. While state space averaging methods eliminates the information about switching frequency, they cannot predict dynamic properties of resonant converters, so the proper control-to-output transfer function cannot be evaluated.

There is a several methods for solving this problems, but some of them are too simplified and idealized, others are too complex and difficult to use [1], [2]. In this paper, new simulation based method for revealing the control transfer function is proposed.

This method is based on PSPICE simulation, which was discussed in previous chapter, and use of System Identification Toolbox in MATLAB environment. First, the simulation of main circuit in PSPICE must be created. Using of PSPICE simulation, the dependency of output (voltage, current) on input (switching frequency, duty cycle) can be simulated.

Another option for creation of the mathematical model is use of block called SLPS in MATLAB. This block creates interface between circuit model in PSPICE and mathematical models in MATLAB environment. Data acquired from simulation in PSPICE are used in MATLAB for creation of transfer function of LLC multiresonant converter. For obtaining of the transfer function in s-domain, the MATLAB System Identification Toolbox was used. Whole process is shown on fig.7. First, the simulation of dynamic step response of LLC converter on control value (switching frequency) was created in OrCad PSPICE environment. Result from this simulation was dynamic output voltage response (output value) on step of switching frequency (input value). This simulated results from OrCad PSPICE, were used in MATLAB as input values for identification of this dynamic system.

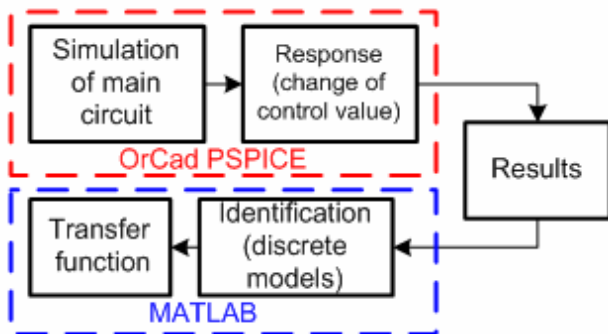


Fig.7. Simulation based method for identification of transfer function

Use of System Identification Toolbox offers several models for identification of dynamic system. With use of different models from System Identification Toolbox (SIT), identification of all converters is possible.

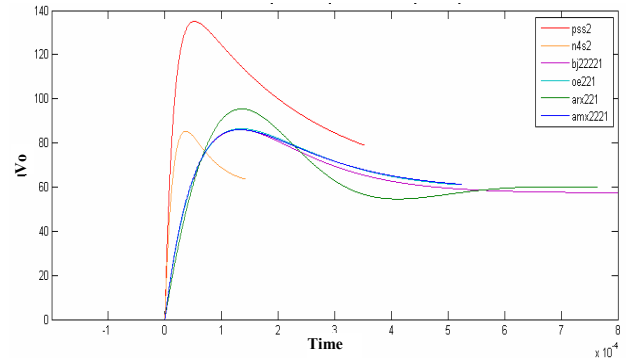


Fig.8. Accuracy of some model types from SIT with different degree (step response)

System Identification Toolbox includes number of models in continuous or discrete form, which can be used for identification of systems: ARX (Auto Regressive Exogeneous Input Model), ARMAX (Auto Regressive Moving Average Exogeneous Input Model) OE (Output Error Model), BJ (Box-Jenkins Model), SS (State-Space Model). Accuracy of the models depends on degree of polynomials used in transfer function. Fig.8 shows accuracy of identified transfer functions for different model types from simulated results of LLC converter. All models are in discrete form (z-domain), so the exact specification of sampling interval is necessary, which means, that the integration step which was used in OrCad PSPICE, must be also used for MATLAB. From fig.8 can be seen, that the best accuracy has ARX and ARMAX model.

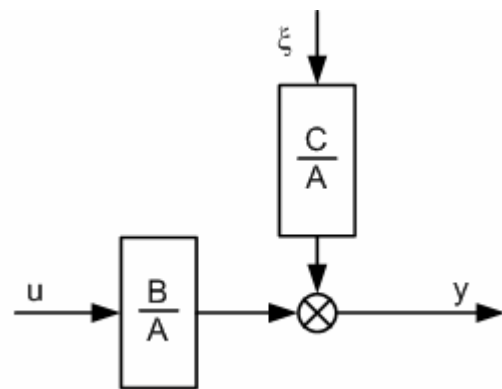


Fig.9. Structure of ARMAX model

The ARMAX model, whose structure is on fig.9, is similar to ARX model, but with better accuracy. It extends the ARX model with polynomial  $C(z)$  - equation of average value of perturbation function. The ARMAX model is suitable for those systems, where perturbation

function affects input variable  $u(n)$ . Searched transfer function has the same form as in ARX model, but the computing algorithm is different. The equation for ARMAX model is as follows:

$$A(z)y(n) = (B(z)u(n-k) + C(z)\xi(n)) \quad (1)$$

Structure of the model is shown on fig. 9. After adjusting, the form of Eq. (1) is:

$$y(n) = \frac{1}{A(z)}(B(z)u(n-nk) + C(z)\xi(n)) \quad (2)$$

Due to best accuracy of ARMAX model, only this model was used for revealing the transfer function. Another parameter necessary for the transfer function revealing, is degree of its polynomials. Fig. 10 shows accuracy of ARMAX model with different degree. Best accuracy offers model of 2<sup>nd</sup> degree, which means, that polynomials  $A(z)$ , and  $C(z)$  are just 2<sup>nd</sup> degree. We can say, that the control to output transfer function of LLC converter has same degree as ARMAX model.

### Transfer function

As mentioned above, the sampling interval used for models in SIT must be equivalent with integration step used for simulation in OrCad PSPICE. If sampling interval used in SIT differs from integration step in PSPICE, the dynamic results (from step response) will have different time representation in MATLAB, so the searched transfer function will have incorrect form.

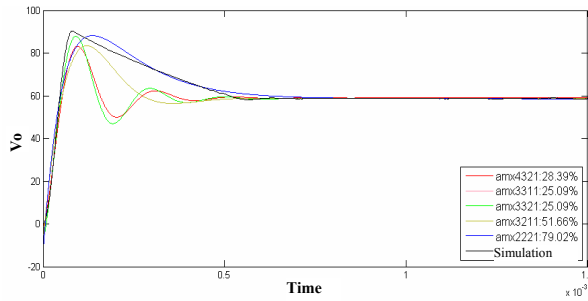


Fig.10. Accuracy of ARMAX models with different degree (up to down 4,3,2)

Requested discrete transfer function is in form:

$$G(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1z^{-1} + b_2z^{-2} + \dots + b_{nb}z^{-nb}}{a_0 + a_1z^{-1} + a_2z^{-2} + \dots + a_{na}z^{-na}} \quad (3)$$

For better view, fig.11 shows comparison of ARMAX 2221 model, used for transfer function, with simulated dynamic step response from OrCad PSPICE.

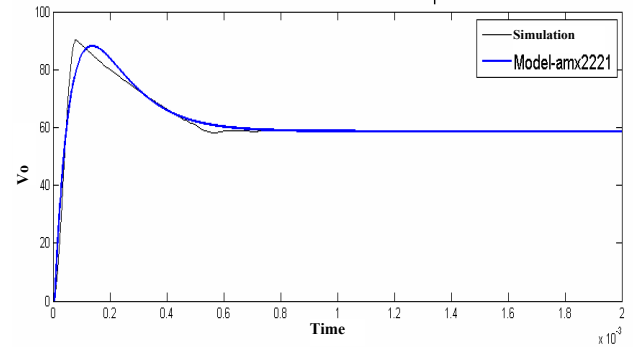


Fig.11. Comparison of simulated results (PSPICE) with ARMAX model of 2nd degree (MATLAB).

For LLC converter, whose waveforms are in fig.5 and fig.6, resulting transfer function is in Eq. (4).

$$G(s) = \frac{2.054 \cdot 10^6 s + 6.895 \cdot 10^9}{s^2 + 2.29 \cdot 10^4 s + 1.176 \cdot 10^8} \quad (4)$$

After acquiring of transfer function, we can obtain basic mathematical model of LLC converter. This mathematical model can be used for design of discrete model, with all effects from discrete control system.

## 4. CONCLUSIONS

The method for creation of mathematical model discussed in this paper is based on circuit model of LLC converter in OrCAD PSPICE and use of MATLAB System Identification Toolbox. This approach eliminates the problems of transfer function obtaining, whose results from use of standard “averaging” methods. Instead of that, this approach uses simulation in OrCad PSPICE, based on circuit model of converter, for acquiring the results and subsequent use of MATLAB for creation of accurate control to output transfer function (in s- or z-domain). Advantage of this method is in use of all parasitic components (resistors, capacitances, inductors) in OrCad PSPICE simulation, so the transfer function is very accurate. Also, the approach for identification of the system, proposed in this paper, uses new, discrete model based method. This method allows rapid acquiring of mathematical model of converter with use of computing power of modern PC. Another advantage of this method is possibility of use for any type of converter without limitation in structure of converter.

### Acknowledgements

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