Optimum Controller Design of an Overhead Crane

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ABSTRACT

The optimum control problem of an overhead crane with uncertain parameters is considered. The crane parameters, e.g., the payload mass, the cable length, hoisting velocity, hoisting acceleration are assumed to be interval quantities in the sense that they are known only within specified lower and upper bounds. The Monte Carlo technique is applied to guarantee the desired performance of the considered overhead crane model. The proposed method is used to find optimum values of the controller gains based on minimizing a least square error function. Simulation results are provided to show the usefulness of the suggested approach to control overhead cranes.

Keyword: Overhead crane, Optimum Control, Monte Carlo Simulation

1. INTRODUCTION

The design problem of an optimum controller for dynamic systems with uncertain parameters has recently attracted considerable attention of researchers [1-4]. It is required to incorporate robustness in the controller design of such systems to overcome the uncertainties inherent in the model parameters. In overhead crane industry, the underlying model is based on the nominal operation conditions. However, in actual operation, the mass of the payload, the hoisting cable length, hoisting velocity and acceleration are interval quantities in the sense that they are only known within a priori defined lower and upper bounds.

In a recent work [2], the authors developed a Monte Carlo based technique to synthesize an interval controller for an overhead crane with only one interval parameter; namely, the cable length. In this work, the developed technique will be extended to a wider class of overhead cranes where four parameters are uncertain. These parameters are the payload mass, the cable length, the hoisting velocity and the hoisting acceleration. The values of these parameters are assumed to be known only within specified lower and upper bounds that depend on a given operating conditions of the underlying crane. The design objective is to synthesize a Proportional-Derivative (PD) feedback controller to minimize the overshoot of the swing angle response of the payload regardless of the inherent uncertainties of the crane parameters. Simulation results are provided to show the usefulness of the suggested approach to control overhead cranes.

The Monte Carlo approach [5-7] is utilized to find optimum values of the controller gains based on minimizing a least square error function. In this method, a large number of simulation experiments are run by using randomly chosen values of the controller gains generated from a uniform distribution of specified lower and upper bounds as defined by the operating conditions. For each randomly generated gain, the corresponding system variables of interest are simulated and the corresponding deviation from a desired trajectory is computed at each time instant. The optimum controller gain is then calculated as the value that minimizes the mean square error for all trajectories. Simulation results are provided to compare to illustrate the results.

2. MATHEMATICAL PRELIMINARIES

Interval analysis [8,9] is a method developed by mathematicians as an approach to putting bounds on mathematical computation and thus developing numerical methods that yield reliable results. Treatment is typically limited to real interval quantities defined as

$$[a,b] = \{x \in R, a \leq x \leq b\} \tag{1}$$

Instead of working with an uncertain real x we work with the two ends of the interval [a,b] which contains x: x lies between a and b, or could be one of them. Similarly, a function f when applied to x is also uncertain. In interval analysis, f produces an interval quantity which is all the possible values of f(x) for all $x \in [a,b]$.

To investigate the stability of an interval closed loop system, one needs to consider its interval characteristic polynomial, which is generally given by

$$\Delta(s) = [a_o, b_o] + [a_1, b_1]s + \cdots + [a_n, b_n]s^n + [1,1]s^{n+1} \tag{2}$$

Two theorems that are recently proved in [1] give a necessary condition and a sufficient condition for stability of the interval polynomial (2). These theorems are reproduced below for the convenience of the reader.

Theorem 1. The interval polynomial defined in (2) is stable if the following necessary conditions are satisfied:

$$b_i \geq a_i > 0, \ \ i = 0,1,2,\dots,n \ \ ,$$
$$a_i a_{i+1} \geq b_{i-1} b_{i+2} > 0, \ \ i = 1,2,\dots,n-2$$

Theorem 2. The interval polynomial defined in (2) is stable if the following sufficient conditions are satisfied:

$$b_i \geq a_i > 0, \ \ i = 0,1,2,\dots,n \ \ ,$$
$$0.4655 a_i a_{i+1} \geq b_{i-1} b_{i+2} > 0, i = 1,2,\dots,n-2$$

The stability conditions of Theorems 1 and 2 can be applied to the closed loop polynomial of Equation (2). This produces inequalities in terms of the controller parameters that can be solved to obtain their lower and upper bounds so that stability of the underlying system is ensured. In the next section, an overhead crane model with interval parameters is considered to illustrate the results.

3. REDUCED ORDER INTERVAL MODEL

The model considered here is shown schematically in Fig. 1. It is a 3-degrees-of-freedom overhead crane model with a trolley mass m, and a payload material point with mass M hanging from a cable with length l [10-12]. The crane is driven by two actuating forces: the trolley force F_x and the hoisting force F_l. The generalized coordinates of the model are the trolley displacement x; the cable length l; and the payload swing angle ϑ.

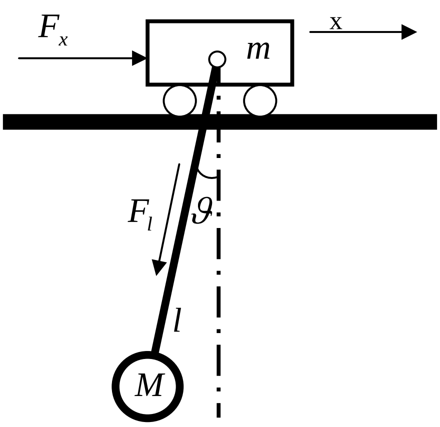

Fig.1. Schematic of the overhead crane

It can be shown that the completely controllable and observable part of the considered overhead crane model is the subsystem related to the dynamic behavior of the generalized coordinate $\delta\vartheta$ [13]. This subsystem is a 2nd order ordinary differential equation that can be written as

$$\begin{bmatrix} \delta\dot{\theta} \\ \delta\ddot{\vartheta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\beta & -\alpha \end{bmatrix} \begin{bmatrix} \delta\vartheta \\ \delta\dot{\vartheta} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -1 & \vartheta_o \end{bmatrix} \begin{bmatrix} \delta F_x^* \\ \delta F_h^* \end{bmatrix} \tag{3}$$

Where,

$$\beta = \frac{M}{m}\left[\frac{g}{l_o} - \frac{\ddot{l_o}}{l_o}\right] + \frac{g}{\ddot{l_o}} \qquad , \alpha = \frac{2\dot{l_o}}{l_o} \qquad (4)$$

$$\delta F_x^* = \delta F_x/(ml_o),\ F_h^* = \delta F_h/(ml_o) \qquad (5)$$

It is assumed that the parameters of the crane as well as the nominal operating conditions are interval quantities with known upper and lower bounds. In particular, the following interval quantities are defined, respectively, for the nominal operating cable length, velocity and acceleration

$$l_o \in [\underline{L},\ \overline{L}]\ ,\ \dot{l_o} \in [\underline{v},\ \overline{v}],\ \ddot{l_o} \in [\underline{a},\ \overline{a}] \qquad (6)$$

where $\underline{x}$ and $\overline{x}$ are, respectively, the lower and upper bounds of the corresponding interval quantity. The payload mass is similarly defined as

$$M \in [\underline{M},\ \overline{M}\ ] \qquad (7)$$

It should be noted that the bounds $\underline{v}$, $\underline{a}$ and $\underline{a_x}$ are negative real numbers and all other bounds are, on the other hand, positive real numbers.

Applying Theorems 1 and 2 of Section 2 to the characteristic polynomial of Equation (3) yields that the necessary and sufficient conditions for the reduced order overhead crane model to be stable is that the coefficients α and β must be positive values. However, it is obvious that there is no guarantee that they are positive during all operating conditions of the crane. Coefficient α is clearly negative when $\dot{l_o}$ is negative, i.e., when the load is being hoisted. The same argument is also valid for the coefficient β that could be negative if the hoisting acceleration is higher than the gravitational one. It is, therefore, necessary to employ an appropriate feedback control to guarantee the stability of the above model for all operating conditions. Such a controller should be robust even if the nominal operating cable length, hoisting/lowering velocity and acceleration are uncertain and known within given known intervals as discussed above. Moreover, the controller gains are to be selected in such a way to satisfy the desired performance characteristics. One main interest of this paper is to achieve this task by applying a Monte Carlo based approach to select the gains to minimize a function of the least square error. This method is a generalization of the technique recently developed by the authors in [2] where only the cable length is considered an interval quantity.

4. MONTE CARLO OPTIMUM CONTROL

In this section, an optimum controller is designed by applying the Monte Carlo technique [2,5-7] where a large number of simulation experiments are run by using randomly chosen values for the controller gain. These values are generated from a uniform distribution of specified lower and upper bounds as defined by the operating conditions. For each randomly generated gain, the corresponding swing angle trajectory is simulated and the corresponding deviation from a desired trajectory is computed at each time instant. The optimum controller gain is then calculated as the value that minimizes the mean square error of the swing angle for all trajectories. Therefore, the optimum controller gain is calculated as

$$k_{d-opt} = \min_k\{MSE\} \qquad (8)$$

Where the mean square error is defined as

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\int_{t_o}^{t_f}[(\theta_i(t,k_{di}) - \theta_d(t)]^2 dt\ ] \qquad (9)$$

In the above equation, $\theta_i(t,k_{di})$ is the swing angle trajectory corresponding to the i[th] randomly generated controller gain k_{di}; $\theta_d(t)$ is the desired swing angle trajectory; n is the number of Monte Carlo experiments; and t_o and t_f are the initial and final simulation time, respectively.

It should be noted that, for nominal operation, the swing angle, ϑ_o, is usually kept small and the trolley mass, m, is relatively large. As a result, the contribution of δF_h to the control of $\delta\vartheta$, as seen from Equations (3) and (5) is almost negligible. The anti swing controller proposed in this paper will be based, therefore, on the actuation of the trolley force δF_x. Toward that end, the following feedback control scheme is proposed to eliminate the payload swing, $\delta\vartheta$, as fast as possible and without overshoot.

$$\delta F_x^* = \left[-\frac{Mg}{ml_o} + k_P\right]\delta\vartheta + k_D\delta\dot{\vartheta} \qquad (10)$$

$$\delta F_h^* = 0 \quad (11)$$

Where k_P and k_D are, respectively, the proportional and derivative controller gains. These gains will be selected in such a way that the desired characteristics for the decay of the swing angle are achieved. Substituting Equations (10) and (11) in (3), one can obtain

$$\delta\ddot{\vartheta} + \left[\frac{2\dot{l_o}}{l_o} + k_D\right]\delta\dot{\vartheta} + \left[\frac{g}{l_o} - \frac{M\ddot{l_o}}{ml_o} + k_P\right]\delta\vartheta = 0 \quad (12)$$

Let the desired payload swing characteristics be define by

$$\delta\ddot{\vartheta} + 2\xi\omega_n\delta\dot{\vartheta} + \omega_n^2\delta\vartheta = 0 \quad (13)$$

Where ξ and ω_n are, respectively, the desired interval damping factor and interval natural frequency for the payload swing dynamics. Comparing Equations (12) and (13), the following interval equations must be satisfied to achieve the desired payload sway performance.

$$2\xi\omega_n = \frac{2\dot{l_o}}{l_o} + k_D \quad (14)$$

$$\omega_n^2 = \frac{g}{l_o} - \frac{M\ddot{l_o}}{ml_o} + k_P \quad (15)$$

It can be shown that $k_P = 0$ and $\xi = 1$ give the optimum performance with no overshoot [1]. Using Equations (12) and (13) above, one can derive the required interval derivative gain as

$$k_D = \left[\underline{k_D},\ \overline{k_D}\right] \quad (16)$$

Where the lower and upper bounds are

$$\underline{k_D} = 2\left[\left(\frac{g}{\overline{L}} - \frac{\overline{M}\overline{a}}{m\,\underline{L}}\right)^{0.5} - \frac{\overline{v}}{\underline{L}}\right] \quad (17)$$

$$\overline{k_D} = 2\left[\left(\frac{g}{\underline{L}} + \frac{\overline{M}|\underline{a}|}{m\,\underline{L}}\right)^{0.5} + \frac{|\underline{v}|}{\underline{L}}\right] \quad (18)$$

Practical considerations for crane operations require that the maximum lowering acceleration of the cable to be much less than the gravitational acceleration. Thus, the lower bound of the derivative gain of the controller as given by Equation (18) is guaranteed to be a real number in practice.

5. SIMULATION RESULTS

We consider an overhead crane with the lower and upper bounds of its parameters are given in Table 1.

Table 1. Crane Interval bounds

Variable	Lower Bound	Upper Bound
Cable Acceleration	-0.2 m/s2	0.2 m/s2
Cable Velocity	-0.5 m/s	1.2 m/s
Cable Length	1 m	20 m
Payload Mass	0 kg	500 kg
Trolley Mass	1500 kg	1500 kg

For the parameter values given in Table 1, a simulation run is carried out, for crane parameters and gain values randomly generated from a uniformly distributed population. The responses due to step initial disturbance for 50 runs are shown in Fig. 2. It can be observed that all runs are stable as expected. However, the overshoot of many of them is relatively large.

It is suggested, in this work, to use an optimum controller gain that corresponds to the minimum value of the mean square error of the swing angle. This optimum gain is found off-line by using a large number of runs, simulated using randomly generated crane parameters that cover the possible operating conditions. The error from the desired swing angle (zero) is then calculated for each run and the optimum gain is determined as 5.33. This optimum gain is then used on-line to control the crane under any operating conditions within the given lower and upper bounds of the parameter values. The simulation for the case of using the optimum gain is illustrated in Fig. 3. It is clear that all responses are stable with no overshoot as desired.

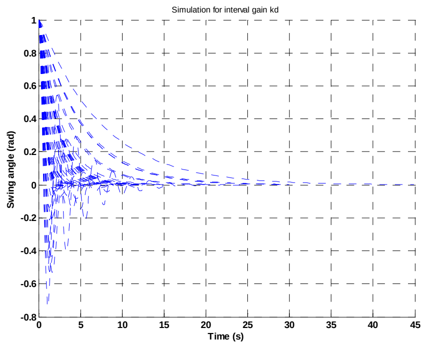

Fig. 2 Response for random crane parameters and controller gain

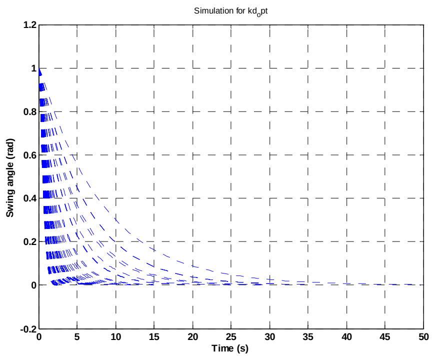

Fig. 3 Response for random crane parameters and optimum gain (k_d = 5.33)

6. CONCLUSIONS

The control problem of an overhead crane with uncertain parameter is considered in this paper. The parameters of the considered model are assumed to be interval quantities rather single valued ones. The Monte Carlo approach is applied to control the considered overhead crane model. The design objective is to synthesize a Proportional-Derivative (PD) feedback controller for a reduced order interval crane model such that the desired performance can be ensured despite of the parameter uncertainty. A simulation study is performed to illustrate the results. It is shown that the control performance of the payload swing angle is reasonable despite the presence of uncertain parameters.

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