The Analogical Way:
How Blind People Used to Do Mathematics Before the Computer Era

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ABSTRACT

This paper provides a brief history of the tools that people with low or no sight have used throughout the centuries to study or produce mathematical results before modern technologies, such as computers and screen-readers, were invented and changed the way people with visual disabilities approach the subject. Blind or partially sighted people who wanted to do mathematics have often created new tools and methodologies for themselves, often keeping them private for long periods, but eventually revolutioning the mathematical education and research with their inventions, many of which provided the inspiration for the technologies we have today.

Keywords: Assistive technologies, Blind people, Mathematics, Nemeth Code, Saunderson table.

1. INTRODUCTION

In the last decades, we have seen a steady improvement of assistive technologies for people with low or no sight. Mathematics was not forgotten by researchers and practitioners who have worked for more accessibility and inclusion, although some key problems have yet to be solved.
If nowadays people with low or no sight have advanced technologies to overcome many of the obstacles that their condition can provide whilst they approach mathematical studies or research, this minority group had to face tougher challenges in the past centuries, often having to craft new methods and tools from scratch to follow their mathematical ambitions. Surprisingly, the tools and the methods they invented are sometimes still relevant today as they transformed or provided the fundaments for some of today's technologies.
This paper follows the steps of the, often blind, mathematicians who designed new ways for people with sight disabilities to do mathematics in the analogical era, where computers did not exist. We can divide this long period in two distinct subperiods, the first one, namely the pre-braille era, that goes from the invention of the Saunderson tactile table of arithmetics to the creation of Braille coding, and the second one, roughly between the diffusion of Braille language and the arrival of computers. In the next section, we analyze the creation of the first device ever for conveying mathematics to the blind and the other tools that derived from it; in Section 3, we describe the invention of Braille language, its unsuitability in representing higher mathematics and its concepts, the birth of the Nemeth Code and its development into some of the most common assistive technologies of today.

2. THE PRE-BRAILLE ERA

English mathematician Nicholas Saunderson (1682-1739) is unanimously considered the first blind scientist in history. He was born in Thurlstone, South Yorkshire, from a middle class family and lost his sight because of an infection caused by smallpox at the age of one [1] [2]. However, his father, a tax collector for the British state, realized that his son deserved a decent level of education like many other children, so he started teaching him the letters of the alphabet when Nicholas Saunderson was only two years old, guiding the child's hands on the inscriptions and the engravings of the tombstones of the graveyard attached to St. John's Church in Penistone, South Yorkshire [1] [2].
The support of his father erased every form of pietism and self-loathing in young Saunderson, convincing the child that he could follow his passion: mathematics. He excelled at school, becoming fluent in French, Latin and ancient Greek and a good flute player, but, most of all, rapidly learning the mathematical concepts thanks to people who read him many of the texts available at that time [1]. We do not know the exact date of the invention, but we know that, during his youth, he crafted the first tool to allow people with low or no sight to execute complex arithmetical calculations only with the use of the sense of touch.
The so-called Saunderson tactile table or Saunderson Table of arithmetics was a simple designed, effective and innovative object for that time. It consisted of a square wooden tablet of approximatively 30 cm length, it was divided into 100 smaller squares thanks to the apposition of thin wooden stripes on its surface. Each one of the smaller squares presented 9 holes in a 3 for 3 grid, just like a tic tac toe table. Every row of the table used to represent a number, with the different columns associated to the different decimal bits of its representation. With the use of pins with two different kinds of heads to be installed into the holes, Nicholas Saunderson could easily represent large numbers, coding each decimal bit of each number putting his pins in a particular order in a small square, then representing numbers on the rows of the table [1].
He also created another version of the same tool to allow people with low or no sight to approach geometry. In this case, the table had no smaller squares defined on its surface, but only a great number of holes where he used to fix his pins. Then, to represent bidimensional geometrical figures, he used to stretch out a thin silk thread, setting it into the pins' heads, defining segments and straight lines [1].
Nicholas Saunderson did not publicize his invention, but there are sources that prove that he reached an incredible level of dexterity on these tools and that, especially whilst he was at Cambridge, he could interrupt his calculations whenever he

ISBN: 978-1-950492-71-8
ISSN: 2771-5914

https://doi.org/10.54808/IMCIC2023.01.35

wanted and restart hours later with no mistakes [3]. He became, despite his blindness and the fact he was largely self-taught, Lucasian professor at the University of Cambridge in 1712 [1] [2], the most prestigious chair in the mathematical world, the one Sir Isaac Newton had hold at the end of the 17th century and that, more recently, saw great names such as Paul Dirac and Stephen Hawking.

For decades, Saunderson tactile table has been the only way blind people could approach mathematics with the sense of touch., furthermore, it inspired other inventions with the same goal. During the 1770s, Henry Moyes, a lecturer in chemistry at Manchester University, constructed a similar device, a sort of extension of the Saunderson tactile table that needed pins with three different kind of heads and a total of 576 squares arranged in a 24 for 24 grid [5]. Just like his illustrious predecessor, Moyes did not publicize his creation and kept it for his own use until 1788.

Whilst working at the Yorkshire School for the Blind between 1836 and 1883, Reverend William Taylor developed his "ciphering tablet", now called Taylor Slate [3] [6]. It consisted of a rectangular aluminum typing frame, 432 octagonal cells are stamped on the top in a 18 for 24 grid, with a recessed compartment at one end for holding the extra type when the frame is in use. The top part is riveted and braised into the frame, a black treated canvas or paper is between the top part and the base to serve as a sound buffer {6}.

Despite their differences, both the Taylor Slate and Moyes' device are clear descendants of Saunderson tactile table, but, surprisingly, many forms of abacus were inspired by the Taylor Slate; so many sighted children in the last decades have in fact learned the concept of quantities and the basic operations thanks to an effort made for people with low or no sight.

3. BRAILLE AND THE NEMETH CODE

Born totally blind because of the conjunction of retinitis pigmentosa and macular degeneration, descendant of a family of Hungarian immigrants, Abraham Nemeth (New York, 1918-Livonia, 2013) has been one of the most important figures in the history of the development of assistive technologies. His family never lost faith in the children's abilities and encouraged him to follow his talents; unfortunately, when starting university, he was discouraged from choosing the faculty of mathematics because of his blindness, so he went for a degree in psychology (7).

After a brief period at the American Foundation for the Blind, Nemeth went back to his original purpose and started to attend courses at the faculty of mathematics at Columbia University. During his childhood, he had rapidly mastered Braille, but this method, invented by Louis Braille more than a century earlier, could not provide a way of coding advanced mathematical formulas [7] [8] [9].

The author in [8] describes a key episode for the creation of the Nemeth Code. During the break of a calculus seminar at Wayne State University, Detroit, where he obtained a PhD in mathematics, Nemeth was approached by his professor who asked him a few questions about Braille coding. After receiving the answers, the professor commented that it seemed to him that blind people were taught to write at the mirror with invisible ink. In fact, Braille users have to punch the back of the paper, moving from right to left and reproducing the signs in a way that allows them to find understandable messages after turning the paper. Moreover, it appears clear that they cannot read what they write while they fix it on the paper. Things were even more difficult for the students with low or no sight that wanted to approach mathematical contents: in fact, Braille was initially designed to reproduce words; so, whilst most humanistic and literary subjects have always been accessible to Braille users, a representation of mathematics was very limited as adequate and coherent codes for trigonometry, calculus and other branches were incomplete or non-existent.

Since there were no screen readers during the '40s, Braille appeared to him as the most effective tool to convey mathematical contents at that time: after a period of practice to get good command of the technique, the blind or heavily visual impaired student gains such an immediateness in interpreting Braille coding as to rapidly catch up in speed with seeing people. Nemeth started to develop the principles of his code, which was eventually published in 1952 [7]. The need for such a system was perceptible by every blind or heavily visually impaired individual interested in mathematics; however, there were many possible ways to provide a Braille coding of mathematical symbols. Nemeth's research was focused on three key aspects [9]:

- Completeness: his code needed to include the representations of all possible symbols, letters and formulas.
- Coherence: the coding of all symbols, letters and formulas needed to be as close as possible to the standard ink equivalent.
- Accessibility: the code needed to avoid potential ambiguity that could have occurred when the same Braille coding represented more formulas or concepts.

Braille dots can be arranged in 63 different positions to compose letters and words; thus it appears evident that it is impossible to represent all different mathematical symbols plus all the letters of both the Latin and the Greek alphabets with this very limited number of dots combinations. Therefore, Nemeth created strings of consecutive Braille symbols to represent even the most advanced concepts such as double integrals and matrix determinants. This was the only accessible method to provide such coding in a compact and non-ambiguous way. From letters of the Latin alphabet and numbers, of which coding already existed and that was effortlessly incorporated with no modification, the learner that approaches the Nemeth Code goes through capital letters of the Latin alphabet, coded as their small equivalents preceded by a specific sign, and up to more complex notation, including indexes, subscripts, relation signs, logarithms and functions [9]. As far as subscripts and exponents are concerned, the author in [9] underlines that locating them on a different line than the main one, as it is usually done with standard ink writing, could result in inevitable ambiguities for the blind or partially sighted reader; so the Nemeth Code provides the much needed univocal representations for them.

The Nemeth Code was explicitly designed to meet the need for a Braille representation of all the concepts that previous methods used to ignore. The effort was successful as it is now possible to represent all symbols, letters and formulas in a number of branches including linear algebra, combinatorics, statistics, analysis, algebra, geometry and logics.

However, if the completeness and the accessibility requirements were met adopting the most economical strategy, finding a number of all different strings to be associate to symbols, letters and concepts in a non-ambiguous way, the hardest task was to keep the code coherent. Since its first version, the real strength point of the Nemeth Code was the very close similarity between the coding of mathematical symbols and their ink equivalent. This plays a key role in increasing the shared ground between the student with low or no sight and the seeing teacher, reducing the time they must spend discussing the mechanics of coding and allowing them to focus only on the mathematical content [9].

The Nemeth Code has gone through four revisions and a number of minor updates over the years and still represents one of the most useful tools to convey mathematical knowledge and culture to blind and partially sighted people all over the world. It improved the cultural standards of Braille users, allowing them to gain access to an entirely new range of scientific topics and closing the gap with the humanistic subjects. Moreover, it upgraded previous methodologies for conveying mathematical contents to people with low or no sight and increased their autonomy.

4. CONCLUSIONS

It is very interesting to note how the results of few enlightened people work overtime. The methodologies and the instruments conceived and created by Saunderson, Braille and Nemeth have inspired the tools and the instruments of modern days. They are inherited and used by modern Braille displays and Braille printers and other instruments that improve capabilities of blind and partially sighted people in reading and, more in general, in studying, allowing them to contribute, also in a significant way, to the progress of science and society.

Another interesting aspect is that, while in the past it was the user who had to adapt to technologies and tools, today the approach has shifted and we try to create interfaces that can adapt to the characteristics of the user. Therefore, by separating semantics from presentation, it is possible to have tools and materials for study and work that can facilitate the sharing of knowledge and collaboration regardless of the characteristics of the participants. Even in these processes, the discoveries of Braille, Nemeth, Saunderson and the once who followed their steps find their own practical space. Through a fairly simple coding it is possible to linearize mathematical formulas, so that they can be reproduced on a Braille display. Using a similar principle, scientific contents can be represented with alternative formalisms (for example in LATEX) to allow their reading and manipulation, and all of this descends from the work and the effort of these figures.

Formulas and numbers can thus be represented in different ways and this, through a computer equipped with Braille displays and screen reader, can also take place in real time, so that the same text can also be used and manipulated simultaneously by sighted and blind people.

Nevertheless, at a closer look, the most important of their achievements is that they have shown, through their life and their daily works, that it's possible also without sight to study, even in the scientific field, and to give a relevant contribution, if the context is inclusive, using the correct instruments, today more performing and more efficient than in the past.

In conclusion, we can say that, despite the fact that some problems still exist and the various solutions are not perfect, we are in a period full of possibilities, which, also thanks to the contribution of these great men of the past, could favour full inclusion processes, also in the study of mathematics and, more in general, scientific subjects.

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