Uses of quantification and modelling category. The case of antimicrobial coating development.

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ABSTRACT

In order to provide elements for the construction of reference frameworks that allow the educational community to value and recover the uses of mathematical knowledge of people that occur in specific situations in their daily life, the aim of this research was to analyse the uses of quantification that emerge in a specific situation of measurement by a professional community. To address this objective, the uses of quantification of a chemical engineer were analysed by means of a debate between performance and form in a specific measurement situation whose argumentation was given by the quantification of grams of additive necessary to elaborate an antimicrobial coating. The results of this research allowed to account for the uses of quantification that emerged by the chemical engineer in this specific measurement situation. Among the conclusions, this research allowed the identification of epistemological aspects and contextualised rationalities that emerged in this situation and that are expected to provide elements for the construction of this type of reference frameworks.

Keywords: Modelling category, Engineering, Antimicrobial coating, Measurement situation, Uses of quantification.

1. INTRODUCTION

One of the purposes of mathematics teaching is to achieve a link with reality [1, 2]. Parra-Sandoval and Villa-Ochoa [3] point out that attending to this link implies recognising the existence of mathematical knowledge that is not institutionalised and that is used in different situations in everyday life. However, when making this type of link, schools often resort to the development of stereotyped tasks whose realities turn out to be artificial [4] and contribute little to the achievement of the link at issue [5]. Moreover, the cultural conception of mathematics as an abstract and infallible discipline is a factor that further complicates the achievement of this link, as it recognises school mathematics as something untouched, where the student is usually limited to learning in a mechanical way, leaving aside the functionality of mathematical knowledge [6], understood as knowledge that is useful for people in situations of everyday life, work and profession [7].

Mendoza and Cordero [8] indicate that the teaching of mathematics, in addition to being distanced from everyday reality, ignores the uses of mathematical knowledge by people that occur in non-school scenarios, where functional justifications prevail that do not necessarily respond to logical-mathematical reasoning. In this regard, when we speak of functional justification, we refer to the fact that the mechanisms in which the uses of knowledge in specific situations are developed are functional, that is, the justification is based on what is useful to humans [9].

A frame of reference for school mathematics is understood as a construct from the socioepistemological theory [10] that points out the elements that comprise the different possible perspectives and configurations that indicate guidelines and trace explanations about the ways of constructing and disseminating the teaching and learning of mathematical knowledge [11]. Considering that idea, this research aims to provide elements for the construction of frames of reference that allow the educational community to value and recover the uses of mathematical knowledge of the people that occur in specific situations in which they are involved. Along these lines, the concept of mathematical modelling in education is an attractive tool to support the construction of this type of frame of reference. Within the different approaches that exist to carry out the study and development of the concept of mathematical modelling (for example, Borromeo [12]; Lesh and Caylor [13]; Lesh and Doer [14]; Rodríguez and Quiroz [15], among others), we find the category of modelling [16], which is positioned as a process focused on people's uses of mathematical knowledge that occur in specific situations in which they are involved. For this reason, the category of modelling will be developed in this research to provide elements for the construction of the required frame of reference.

The structure of this article is as follows: Section 2 will show the theoretical framework that will support this research, where the modelling category will be presented. In addition, the research objective will be stated in this section. Section 3 will show the methodology that will make it possible to respond to the stated objective. Section 4 will report on the analysis and results, and finally, in section 5, the conclusions of this research will be presented.

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2. THEORETICAL FRAMEWORK

For the Socioepistemological Theory of Mathematics Education [10], a fundamental thesis to favour the learning of the meanings of mathematics is to know *the environment of reciprocal relations between mathematical knowledge and the reality of the learner* [2]. Socioepistemology focuses on studying people's uses of mathematical knowledge in specific everyday situations, in order, on the one hand, to recover the social character of mathematics, by focusing on the practices that regulate the social construction of mathematical knowledge [17], and, on the other, to contribute to a decentring of mathematical objects in the teaching of mathematics. It is important to highlight that, for socioepistemology, to speak of the social construction of mathematical knowledge refers to a notion that considers the interactions and processes of debate that communities experience, in order to carry out processes of institutionalisation of mathematical knowledge, together with its functionality in specific contexts and situations [6].

SOLTSA Programme

This research is framed within the socioepistemological programme called *Forgotten Subject and Transversality of Knowledge* (SOLTSA, acronym in Spanish), whose main objective is to reveal the uses of mathematical knowledge and its resignifications that emerge in the mathematical knowledge communities of people at school, at work or profession and in their daily lives, where the functionality of mathematical knowledge predominates. Regarding the *uses of mathematical knowledge*, these are understood as the organic functions of situations (functionings), which are manifested by the "tasks" that make up the situation, and forms of use that refer to the kind of those "tasks". As for the tasks, these can be activities, actions, executions or alternations of domains proper to the organism of the situation [16]. When the alternation of tasks takes place, a new organic function originates, which debates with the forms of uses. This "act of use" is the *resignification of uses of mathematical knowledge* [9], which happen in specific situations that are part of that environment of reciprocal relations between mathematical knowledge and reality [18].

The argumentation of mathematical knowledge generated in a specific situation is a resignification of uses constructed in that situation, understanding by argumentation of mathematical knowledge the guiding thread of the specific situation from which mathematical knowledge emerges [6] Additionally, these resignifications are also constructed when transversalities of uses of mathematical knowledge occur between domains of knowledge, understanding by transversality of uses the resignification of uses of mathematical knowledge between scenarios or domains of knowledge, for example, between school and work; or between mathematics and engineering [18]. In other words, when the functionings and forms of the uses of mathematical knowledge that occur in a specific situation S_{in}, proper to a domain knowledge D_n, debate with other functionings and forms that occur in a specific situation S_{jm} $(i \neq j)$ proper to a domain S_{jm} $(i \neq j)$ thus generating new resignifications of uses.

The modelling category

The modelling category [16] has been worked on in the SOLTSA programme as the use of mathematical knowledge in a specific situation [6]. Several researches have developed this category - that is, they have promoted a resignification of uses of mathematical knowledge - by analysing the uses of mathematical knowledge that emerge from people in different types of specific situations, as is the case of the uses of optimisation in selection situations [18], the uses of compensation in weighting situations [19], the uses of accumulation in situations of change [20], the uses of anticipation in situations of periodisation [21] (Pérez-Oxté, 2021) and the uses of behaviour reproduction in situations of transformation [22], among others. These investigations have provided elements to build reference frameworks that serve as a basis for the design of school situations that allow the educational community to promote, in the teaching of mathematics, the uses of mathematical knowledge that emerge from the people in specific situations in which they are involved. In order to develop the category of modelling, and thus provide elements for the construction of reference frameworks that serve as a basis for the design of this type of situation, the objective of this research is to analyse the uses of quantification that emerge in a specific situation of measurement by a professional community.

3. METHODOLOGY

Context and participants

Delgado [23] developed an antimicrobial additive through the incorporation of copper-based particles (see Figure 1). This additive has the potential to be incorporated in different types of surface coatings as a complementary measure in the prevention of different diseases caused by pathogenic microorganisms, for example, those diseases caused by coliform bacteria contamination.

The additive consists of a copper-based antimicrobial agent supported on zeolite. Zeolite is an organic material considered to be a *carrier* because of its ability to absorb or support other materials, where *carrier* means an inert material to which other materials are added (adsorbed, impregnated or coated). The antimicrobial additive is in powder form and can range in size from 20 to 100 microns.

In order to test the potential of this additive, a professional community working at a Chilean university participated in a research project to validate *in situ* the antibacterial properties of a commercial epoxy coating containing an antimicrobial additive. In particular, it was evaluated whether it would significantly reduce the amount of coliform bacteria present on the walls of the prison cells of a police station located in Santiago, Chile.

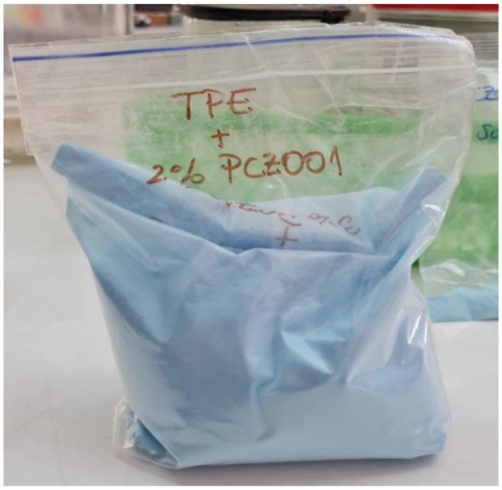

Figure 1. Antimicrobial additive used for the production of an antimicrobial coating.

An antimicrobial coating is the one that inhibits or eliminates the growth of bacteria adhering to its surface. To evaluate the antimicrobial activity of a coating it is necessary to quantify the number of bacteria adhering over time. If the number of bacteria decreases, it means that there is antimicrobial activity.

The professionals who were part of this community were four: a Civil Chemical Engineer with a PhD in Engineering Sciences, with specialisation in Materials Science (P1), a Chemical Engineer (P2), a Chemist with a PhD in Chemistry (P3) and a Biochemist with a PhD in Biotechnology (P4). The immersion with this community comprised four moments (M1, M2, M3, M4): M1: Preparation of an antimicrobial coating based on an antimicrobial additive, M2: Painting of the walls of the prison cells with an antimicrobial coating, M3: Sampling of coliform bacteria on the walls of the prison cells painted with an antimicrobial coating and M4: Counting of coliform bacteria. However, this research will report the uses of quantification that occurred during M1, which was conducted by P2. Specifically, this immersion was carried out in a specific measurement situation (situation in which we searched for measuring the amount, in grams, of additive needed to make an antimicrobial coating) which had the purpose of quantifying the grams of additive to be mixed with the grey oil-based coating[1] that came in a 20 litre bucket (see Figure 2), in order to transform it into an antimicrobial coating that could be applied to the walls of the police station's prison cells.

[1] This paint is traditionally used by Carabineros (Chilean national law enforcement police) to paint the walls of the police station's cells.

Figure 2. Paint used by Carabineros to paint the walls of the cells.

Data collection

During this immersion, data collection was carried out by means of: (a) audiovisual and photographic records to support the observation carried out by the researchers, (b) semi-structured interviews with the professionals in this community who participated in the process of making an antimicrobial coating, in order to better understand this process of elaboration. In order to avoid introducing possible obstacles to the objectivity of the data collected through the researchers' participation in this community, this research chose the researchers to play the role of observers [24]. However, according to Guber [25], the researchers took on the role of participants only when necessary and always remembering that their primary role was that of observers. The above allowed us to identify the routine situation where this community made use of quantification.

Data analysis

The uses of quantification that emerged in this specific measurement situation were analysed according to the analysis of uses of mathematical knowledge carried out in different socio-epistemological works (for example, see [18]; Opazo and Cordero [26]; Tuyub and Zapata [27]; Morales and Cordero [28], who carried out this type of analysis by means of a debate between functioning and form, where functioning responds to *what knowledge is used for* and form responds to *how knowledge is used* [29]).

4. ANALYSIS AND RESULTS

The preparation process of the antimicrobial coating was carried out in the chemistry laboratory of the university where the community professionals worked. To prepare the antimicrobial coating, P1, the project leader, decided that one kilogram of mixture should be made up of 95% paint and 5% additive. This percentage was determined on the basis of experience and the background of an investigation in which hospital intravenous poles (IV poles) were coated with paint containing 3% by mass of the antimicrobial additive, which did not result in a significant decrease in the number of bacteria [30]. Therefore, P1 decided that the 3% should be increased to 5%.

Based on this indication, P2 undertook to develop an antimicrobial coating. For this purpose, he made use of a graduated glass container that allowed him to take out the paint in a smaller volume so that he could handle the paint more easily and comfortably. His initial idea was to take one litre of paint out of the bucket with this container, put it in another plastic container and there make one litre of antimicrobial coating, and then repeat this process 20 times in order to make 20 litres of antimicrobial coating. Before starting this process, P2 used an electronic balance to find out the mass of the plastic container, so that he could tare the mass. Thus, when calculating the mass of the plastic container with paint, the electronic balance would only indicate the mass of the paint and would not consider the mass of the plastic container. In this regard, P2 stated the following:

P2: This weighs 66.86 grams [referring to the plastic container] and what we are going to do is tare it, leave the balance at zero [grams] to eliminate the factor [referring to the mass of the plastic container] (see Figure 3).

Figure 3. Calculation of the mass of the plastic container on the electronic balance.

However, due to convenience, P2 decided to work with 800 ml instead of one litre, as otherwise he would stain his glove with paint and it would be uncomfortable for him to take the graduated glass container. In this regard, an extract of P2's statement is shown:

P2: We are going to leave it at 800 [ml] in fact....
Researcher: It's not going to be the litre any more?
P2: No...because otherwise it will touch my glove [referring to the paint] and make it dirty. We'd better use the 800 [ml] and then I can transfer it better (see Figure 4).

Figure 4. Using the glass container to remove 800 ml of paint.

Once the 800 ml of paint was placed in the plastic container, P2 proceeded to determine its mass using the electronic balance, determining that 800 ml of paint had a mass of 703.78 grams (see Figure 5).

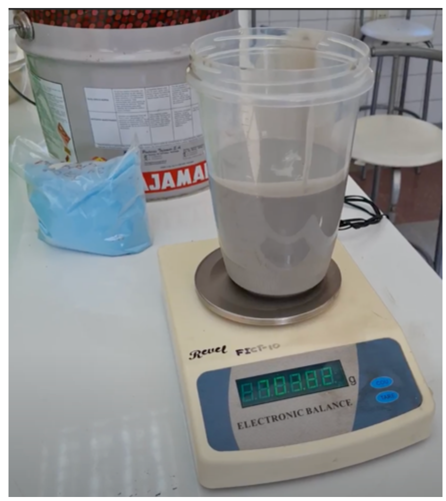

Figure 5. Calculation of the mass of 800 ml of paint using an electronic balance.

To quantify the amount of grams of additive that should be in 703.78 grams of an antimicrobial coating, P2 made use of a simple rule of three, where he considered as 100% to 703.78 grams and as 5% to a x (amount of grams of additive), determining the latter equal to 35.189 grams (see Figure 6).

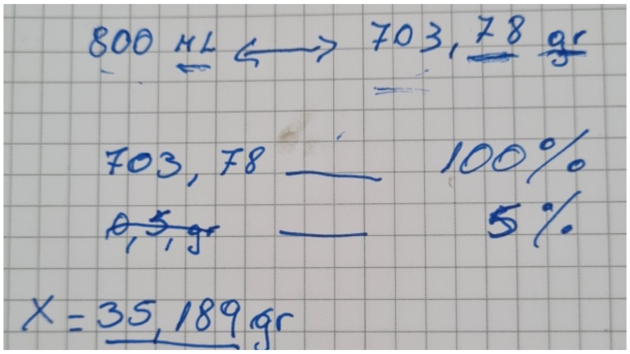

Figure 6. Quantification of the grams of additive to be used to make 800 ml of an antimicrobial coating.

The process of homogenising the mixture of the additive with the paint was first carried out manually and then mechanically, with the aid of mechanical stirrers operating at an average speed of 80 revolutions per minute for 10 hours (see Figure 7).

Figure 7. Use of mechanical stirrers to homogenise the mixture of the additive with the paint.

Once the homogenisation process was completed, P2 repeated this process 25 times to produce 20 litres of antimicrobial coating, which was subsequently used to paint the walls of the police station cells.

This specific situation corresponded to a *measurement situation* as it sought to measure the amount of grams of additive to be used to prepare 20 litres of antibacterial coating. Table 1 shows a use of P2 quantification in this specific measurement situation.

Table 1. Uses of P2 quantification.

Functioning of the use of quantification (*what is quantification used for?)*	To determine the amount of grams of additive to be used to make 800 ml of an antimicrobial coating.
Form of the use of quantification (*how is quantification used?*)	By means of a simple rule of three.

5. DISCUSSIONS AND CONCLUSIONS

The modelling category emerged in this research as the use of quantification by P2 in this specific measurement situation. The common thread of this situation that allowed the mathematical knowledge to emerge was given by the quantification of the grams of additive that were necessary to elaborate an antimicrobial coating. This allows us to point out that the argumentation generated from this specific measurement situation was given by the quantification, which in turn is a resignification of uses constructed in this situation.

This research provides elements to understand the mathematical knowledge that emerges from specific communities based on their functionality. The emphasis was not on paying attention to what people in the communities do not know and should learn, but on what they do, how they do it and what they do it for. The emphasis is then on the uses of mathematical knowledge put into play in this specific situation and not on the mathematical objects. In particular, this research has made it possible to identify epistemological aspects and contextualised rationalities that emerged in this situation and which are expected to provide elements for the construction of frames of reference that will allow the educational community to value and recover the uses of quantification that emerge from people in specific measurement situations. This idea will be possible by designing school situations that, once implemented in mathematics classes, can contribute to transforming school mathematics with the aim of generating a reciprocal relationship between people's daily lives and school mathematics, taking into account the functionality of mathematical knowledge.

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