Assessing Learning Processes with a Gain-Loss Model

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ABSTRACT

Within the context of formative assessment, a probabilistic skill multimap model for assessing learning processes is proposed. The learning process of a student is modelled as a function of the interaction between the knowledge of the student and the effect of a learning object on specific skills. Model parameters are initial probabilities of the skills, effects of learning objects on gaining and losing the skills, careless error and lucky guess probabilities of the problems. An empirical application shows that the model is effective in detecting students’ knowledge and the effectiveness of learning objects on attaining specific skills. Practical implications for formative assessment are discussed.

Keywords: Formative Assessment, Learning Process, Learning Object, Knowledge Structure, Skill Map.

1. INTRODUCTION

Summative assessment points to grade the knowledge of students after the teaching is over through a score that summarizes their learning outcomes. Differently, formative assessment is ongoing throughout the teaching and aims to improve knowledge, skills and abilities of students by guiding teaching and learning at the individual level [1]. In addition, it helps the teacher to ascertain whether an educational intervention has been effective in promoting specific learning or not. Students’ specific strengths and weaknesses are pinpointed by assigning multidimensional skill profiles to them.

Within the context of formative assessment, we propose a probabilistic model for assessing the knowledge of students in the different steps of the learning process, and the effectiveness of the educational intervention in promoting specific learning. The theoretical framework is knowledge space theory [2,3] that, consistently with the aims of formative assessment, provides a non numerical, multidimensional representation of the characteristics of students.

In the following section, the theoretical model and its mathematical specification are presented. Then, an application on empirical data is provided. Finally, some practical implications for formative assessment of knowledge are discussed.

2. THE MODEL

In knowledge space theory, the knowledge state of a student is represented by the set of problems in a specific knowledge domain that this student is capable of solving [2,4,5,6]. Differently, we refer to the knowledge state of a student as the set of non directly observable skills possessed by the student and which underlie his/her observable responses (with the term "skills" we refer to a broad class of pieces of knowledge at both declarative and procedural knowledge, including notions, abilities, solution procedures, and so on).

In our work, the learning process of a student is modelled as a function of the interaction between the knowledge state of the student and the effect of an educational intervention, called learning object (with the term “learning object” we refer to a broad class of didactic tools at different levels of granularity). A probabilistic model is developed to assess the effect of learning objects on the attainment of skills required to solve problems in a given field of knowledge. Via the competency model, a skill multimap [4,5] is used to associate each problem with a collection of subsets of skills that are necessary and sufficient to solve it.

The model is characterized by five types of parameters. The parameter concerning the initial probability of the skills specifies what skills the students possess before the teaching begins. The gain and loss parameters respectively specify the effects of the learning object on gaining and losing specific skills. The careless error and lucky guess parameters respectively specify if a problem is failed by inattention or it is solved by guessing.

Model Specification

Let $S$ be a finite and non empty set of discrete skills, and $K$ be any subset of $S$. $K$ represents the unknown knowledge state of a student. Let $K_1$ and $K_2$ be two discrete random variables whose realizations are the knowledge states of a student at the pre-test and post-test, respectively. Let $Q$ be a finite and non empty set containing $n$ dichotomous problems, and $R_1$ and $R_2$ be two
discrete random variables whose realizations are the response patterns \( r \in \{0, 1\}^n \) of a student at the pre-test and post-test, respectively. Let \( m \) be the number of learning objects, and \( o \in \{1, 2, \ldots, m\} \) be the learning object the student is presented with.

The conditional probability that \( r_1 \) and \( r_2 \) are the response patterns of a randomly sampled student at the pre-test and post-test, given the learning object \( o \), is:

\[
P(r_1, r_2 | K, o) = \sum_{K_1, K_2} P(r_1 | K_1, o) P(r_2 | K_2, o) P(K_1, K_2 | K, o)
\]

where \( P(K_1 = K | K) \) is the initial probability of the state \( K \) at the pre-test, \( P(K_1 = L | K, o) \) is the transition probability from state \( K \) at the pre-test to state \( L \) at the post-test, \( P(r_1 | K_1, o) \) and \( P(r_2 | K_2, o) \) are the emission probabilities of response patterns \( r_1 \) and \( r_2 \) at the pre-test and post-test, respectively. Eq. (1) is the basic equation of the model. Model assumptions are that: a) the response patterns \( R_1 \) and \( R_2 \) are locally independent, given the states \( K_1 \) and \( K_2 \); b) the initial state \( K_1 \) does not depend on the learning object \( o \); and c) state \( K_1 \) depends on previous state \( K_1 \) and on the learning object \( o \).

Let \( x \) be the probability that skill \( s \) belongs to the initial knowledge state. Assuming total independence among the skills, the probability \( P(K_1 = K) \) is resolved according to Eq. (2):

\[
P(K_1 = K) = \prod_{s} \gamma_s^{x} (1 - \pi_s) \beta_s^{1-x},
\]

where \( \gamma_s \in \{0, 1\} \) is equal to 1 if skill \( s \) belongs to state \( K \).

Let gain \( \beta_{0s} \) be the probability that students presented with learning object \( o \) gain the skill \( s \) going from the pre-test to the post-test, and loss \( \lambda_{os} \) be the probability that the same students lose it. The conditional probability of state \( L \) at the post-test, given state \( K \) at the pre-test and learning object \( o \), is:

\[
P(L | K, o) = \sum_{s} \left[ \prod_{s} \gamma_s^{x} (1 - \beta_{0s}) \beta_s^{1-x} \right] \prod_{s} \left[ \prod_{s} \lambda_{os}^{x} (1 - \beta_{0s}) \beta_s^{1-x} \right]^{1-x},
\]

where \( \lambda_{os} \in \{0, 1\} \) (resp. \( \beta_{0s} \)) is equal to 1 if skill \( s \) belongs to state \( K \) (resp. \( L \)).

Let careless error \( \alpha_q \) be the probability that students fail problem \( q \) given that it is solvable by their knowledge state, and lucky guess \( \beta_q \) be the probability that the same students solve problem \( q \) given that it is not solvable by their knowledge state. Assuming responses to the problems are locally independent, given students’ knowledge state, the conditional probability of response pattern \( r \), given the state \( K \), is:

\[
P(r | K, o) = \prod_{q} \left[ \prod_{s} \gamma_s^{x} (1 - \alpha_q) \beta_s^{1-x} \right]^{1-x} \left[ \prod_{s} \beta_q^{x} (1 - \beta_q) \beta_s^{1-x} \right]^{1-x},
\]

where \( t \in \{1, 2, 3\} \) and \( \alpha_q, \beta_q \in \{0, 1\} \) is equal to 1 if problem \( q \) is solvable by state \( K \), and \( \beta_q \) is equal to 1 if problem \( q \) is solved. Eq. (4) is the BLIM [5,7] and the DINA Model [8,9]. Initial probabilities are estimated for each skill, gain and loss for each learning object and each skill, careless error and lucky guess for each problem. Maximum likelihood estimates of the parameters can be computed by an application of the Expectation-Maximization algorithm [10].

3. EMPIRICAL APPLICATION

Sixty-seven university students were presented with a collection of 13 open response problems in elementary probability theory through a computer-based testing procedure. Table 1 represents the problems in a concise format. Four skills (stochastic independence, law of total probability, conditional probability, probability of the complement of an event), and their combinations, were required to solve all the problems.

<table>
<thead>
<tr>
<th>Table 1. Problems and conjunctive skill map</th>
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<tbody>
<tr>
<td>Problem</td>
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Note. cd = conditional probability; cp = complement of an event; tt = stochastic independence; id = total probability.
To test the model, a $2 \times 2$ experimental design with two learning objects (effective vs. ineffective) and two assessment steps (pre-test and post-test) was planned. The effective learning object (i.e., instructions and information concerning the four skills) was supposed to be useful to learn the skills required to solve the problems, whereas the ineffective learning object (i.e., concepts of elementary probability theory that were not relevant for solving the problems) was not supposed to be useful. After responding to the problems the first time (pre-test), students belonging to a first group (Group E, $N = 36$) were presented with the effective learning object, and those belonging to a second one (Group I, $N = 31$) with the ineffective learning object. Then, a post-test with the same problems took place.

In the present application, each problem was associated with the skills that are necessary and sufficient for its mastery according to the conjunctive skill map represented in Table 1. The knowledge structure $K$ on the collection of problems delineated by the given skill map is:

\[
K = \{(2, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2, 5\}, \{1, 3, 6\}, \{1, 4, 7\}, \{2, 3, 8\}, \{2, 4, 9\}, \{3, 4, 10\}, \{1, 2, 3, 5, 6, 8, 11\}, \{1, 2, 4, 5, 7, 9, 12\}, \{1, 3, 4, 6, 7, 10, 13\}, \{2, 3, 4, 8, 9, 10\}, \emptyset\}.
\]

Goodness-of-fit (i.e., how well the model fits the data) and goodness-of-recovery (i.e., how well model parameters are recovered by the estimation algorithm) of the model based on the knowledge structure $K$ were tested using a parametric bootstrap [11]. Pearson’s Chi-square statistic was used as a goodness-of-fit index. Model identifiability was tested by estimating the parameters 100 times from different initial points of the parametric space.

4. RESULTS

Goodness-of-fit of the estimated model ($p = .12$), as well as goodness-of-recovery ($0.1 \leq SE \leq 0.19$) were good. Parameter estimates did not change by varying their initial values. Table 2 contains the estimates of the parameters $\pi$, $\gamma$ and $\lambda$.

Table 2. Maximum likelihood estimates of the parameters $\pi$, $\gamma$ and $\lambda$

<table>
<thead>
<tr>
<th>Skill</th>
<th>Initial prob.</th>
<th>Gain $\pi$</th>
<th>Loss $\lambda$</th>
<th>Gain $\gamma$</th>
<th>Loss $\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compl. of event</td>
<td>.79</td>
<td>&lt; .01</td>
<td>&lt; .01</td>
<td>&lt; .01</td>
<td>&lt; .01</td>
</tr>
<tr>
<td>Total probability</td>
<td>.49</td>
<td>.80</td>
<td>.09</td>
<td>&lt; .01</td>
<td>&lt; .01</td>
</tr>
<tr>
<td>Cond. probability</td>
<td>.36</td>
<td>.23</td>
<td>.02</td>
<td>&lt; .01</td>
<td>&lt; .01</td>
</tr>
<tr>
<td>Stochastic ind.</td>
<td>.35</td>
<td>.48</td>
<td>.10</td>
<td>&lt; .01</td>
<td>&lt; .01</td>
</tr>
</tbody>
</table>

The learning object presented to Group E has been effective in promoting the attainment of the skills. “Total probability” is the skill attained with highest probability ($\pi_{\text{E}} = .80$), followed by “stochastic independence” ($\pi_{\text{D}} = .48$), and “conditional probability” ($\pi_{\text{E}} = .23$). The skill concerning the complement of an event is not further acquired ($\delta_{\text{E}} < .01$). Since it is the skill having the highest initial probability, it is difficult to assess the effectiveness of the learning object in the few students who did not possess it in the pre-test. Unlike the learning object presented to Group E, the one presented to Group I has not been effective in promoting the attainment of the skills ($\gamma < .01$ for all). All differences between parameters across the two groups were significant at a level $p < .001$. Unexpectedly, in Group E the probability of losing three of the four skills is greater than in Group I, even if these probabilities are quite small (see Table 2).

Table 3 contains the estimates of the parameters $\alpha$ and $\beta$.

Table 3. Maximum likelihood estimates of the parameters $\alpha$ and $\beta$

<table>
<thead>
<tr>
<th>Problem</th>
<th>Careless error $\alpha$</th>
<th>Lucky guess $\beta$</th>
<th>Careless error $\alpha$</th>
<th>Lucky guess $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.02</td>
<td>.31</td>
<td>.02</td>
<td>.06</td>
</tr>
<tr>
<td>2</td>
<td>.22</td>
<td>.04</td>
<td>.51</td>
<td>.03</td>
</tr>
<tr>
<td>3</td>
<td>.02</td>
<td>.35</td>
<td>.07</td>
<td>.17</td>
</tr>
<tr>
<td>4</td>
<td>&lt; .01</td>
<td>.20</td>
<td>.63</td>
<td>.02</td>
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<tr>
<td>5</td>
<td>.09</td>
<td>&lt; .01</td>
<td>.69</td>
<td>&lt; .01</td>
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<td>6</td>
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<td>7</td>
<td>.25</td>
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</table>

5. CONCLUSIONS

The model assessed the initial knowledge state of the students and the change in this state due to the learning process. In particular, it has permitted to observe the effect of the learning objects on gaining and losing specific skills. Estimated probabilities of acquiring the skills were greater in the group presented with the effective learning object than in the other one.

Model parameters provide the teacher with information for organizing and evaluating learning programs. Initial probabilities of the skills help the teacher to assess what the students already know and what they are ready to learn. Gain and loss parameters enable the teacher to select the best learning object for the specific weaknesses of the students, given their knowledge state. Careless error and lucky guess parameters inform the teacher about the effectiveness of each problem in detecting the underlying skills. The problems with the smallest careless error and lucky guess probabilities are with confidence solved only by students who possess the underlying skills. On the contrary, the problems with the highest careless error and lucky guess probabilities do not reliably inform on the presence of the underlying skills. The latter could suggest failings in the specification of the skill map or in the wording of the problems.

Once the model has been estimated and validated on the group of students, posterior Bayesian estimates of the parameters, as well as those of the knowledge states, can be computed for each student.

By focusing on the skills which underlie the problems, the model can suggest to the teacher which skills should be taught so that a previously unsolvable problem becomes solvable. In this respect the model is not dissimilar to other ones [8,9,12,13]. However, unlike existing models, our model allows to compare...
different learning objects by directly measuring their effect on the acquisition of specific skills. In this way, it helps the teacher to select the best material and instructions for the specific weaknesses of the students. Assessing the knowledge of the students and the effectiveness of the educational interventions in promoting specific learning can directly guide teaching and learning. This is consistent with the aims of formative assessment.

6. REFERENCES