Reflexivity

Louis H. Kauffman UIC, Chicago

"I am the observed link between myself and observing myself."

Heinz von Foerster



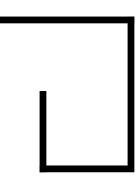
"Reflexive" is a term that refers to the presence of a relationship between an entity and itself. One can be aware of one's own thoughts. An organism produces itself through its own action and its own productions. A market or a system of finance is composed of actions and individuals, and the actions of those individuals influence the market just as the global information from the market influences the actions of the individuals. Here it is the self-relations of the market through its own structure and the

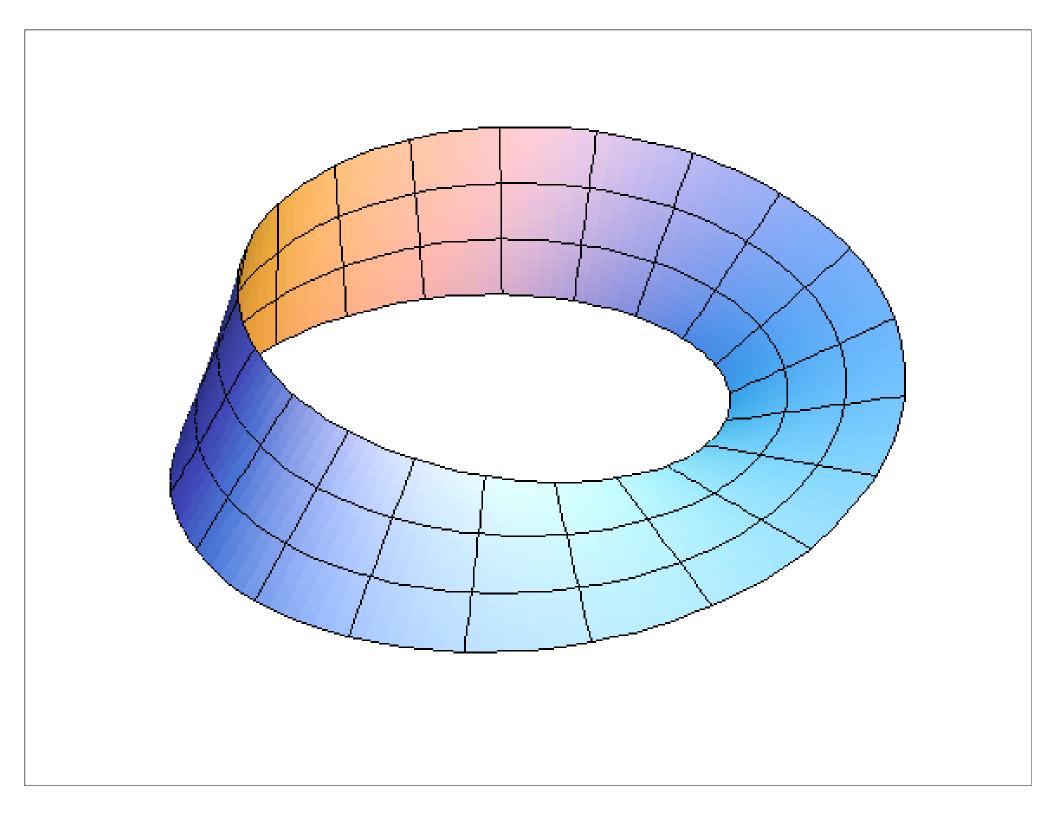
structure of its individuals that moves its evolution forward. Nowhere is there a way to cut an individual participant from the market effectively and make him into an objective observer. His action in the market is concomitant to his being reflexively linked with that market. It is just so for theorists of the market, for their theories, if communicated, become part of the action and decision-making of the market. Social systems partake of this same reflexivity, and so does apparently objective science and mathematics. In order to see the reflexivity of the practice of physical science or mathematics, one must leave the idea of an objective domain of investigation

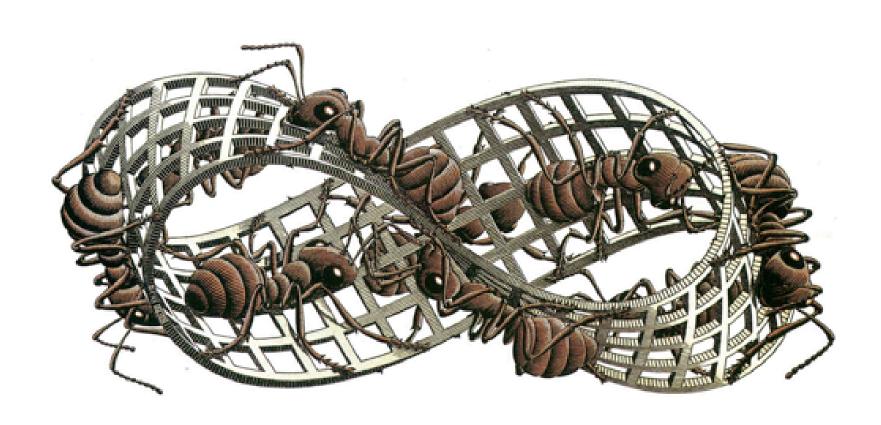
in brackets and see the enterprise as a wideranging conversation among a group of investigators. Then, at once, the process is seen to be a reflexive interaction among the members of this group. Mathematical results, like all technical inventions, have a certain stability over time that gives them an air of permanence, but the process that produces these novelties is every bit as fraught with circularity and mutual influence as any other conversation or social interaction.

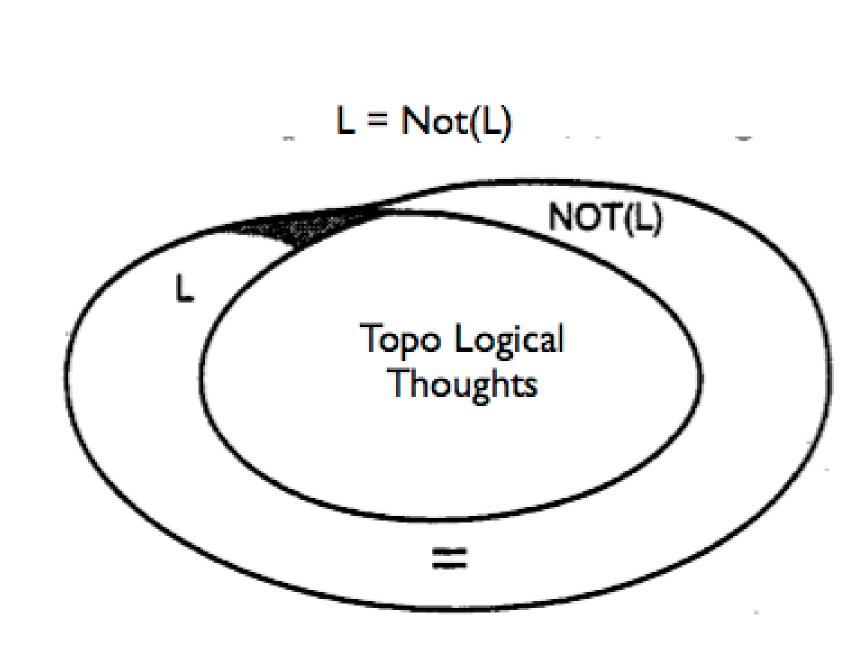
In an observing system, what is observed is not distinct from the system itself, nor can one make a complete separation between the observer and the observed. The observer and the observed stand together in a coalescence of perception. From the stance of the observing system, all objects are non-local, depending upon the presence of the system as a whole. It is within that paradigm that these models begin to live, act and enter into conversation with us.

A Form Re-enters its Own Indicational Space.

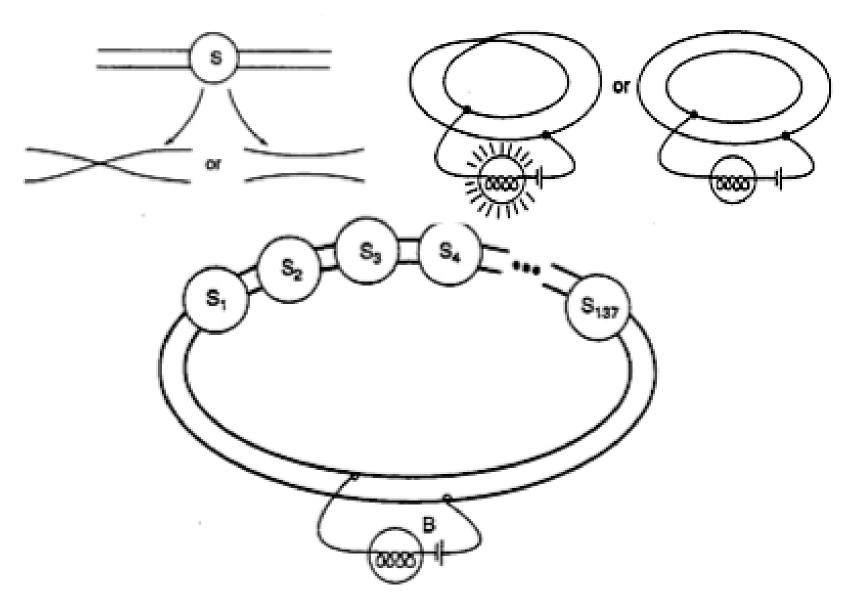


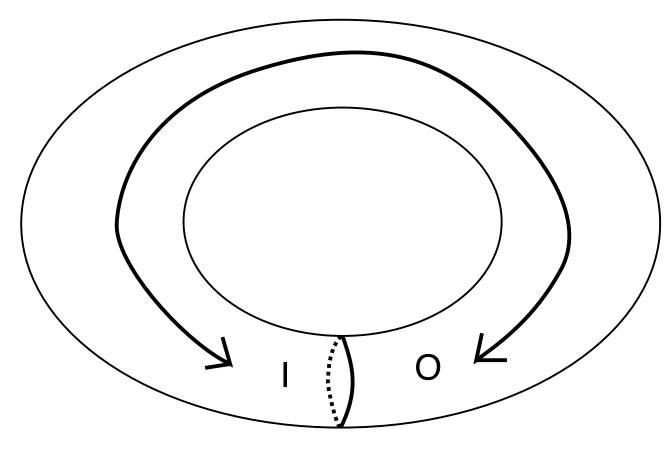






Mobius Applications Inc. (courtesy of Ricardo Uribe)

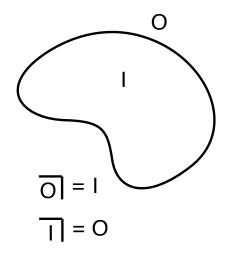


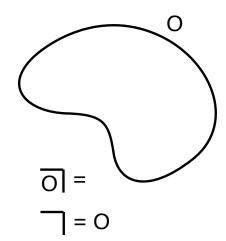


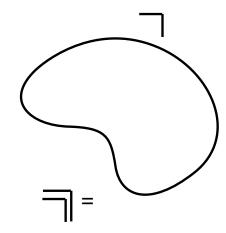
$$I = O = J$$

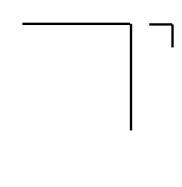
$$J = \overline{J}$$

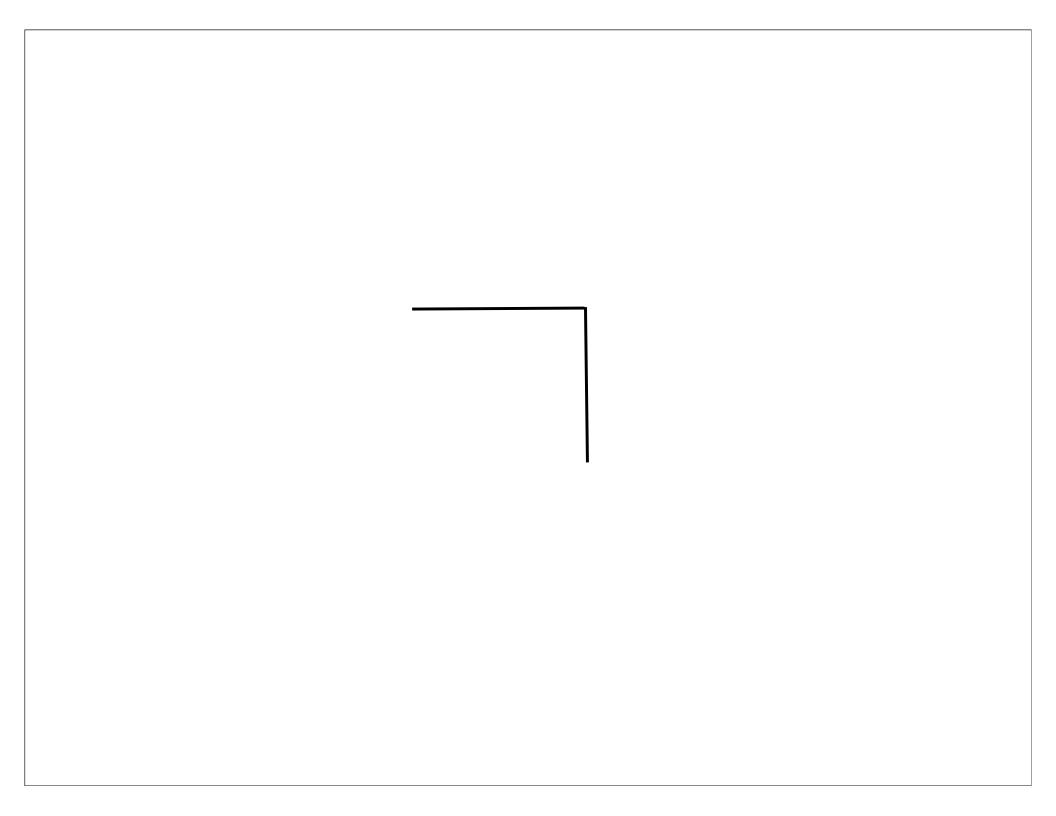
Descent Into the Form











The Form
We take to exist
Arises
From
Framing
Nothing.

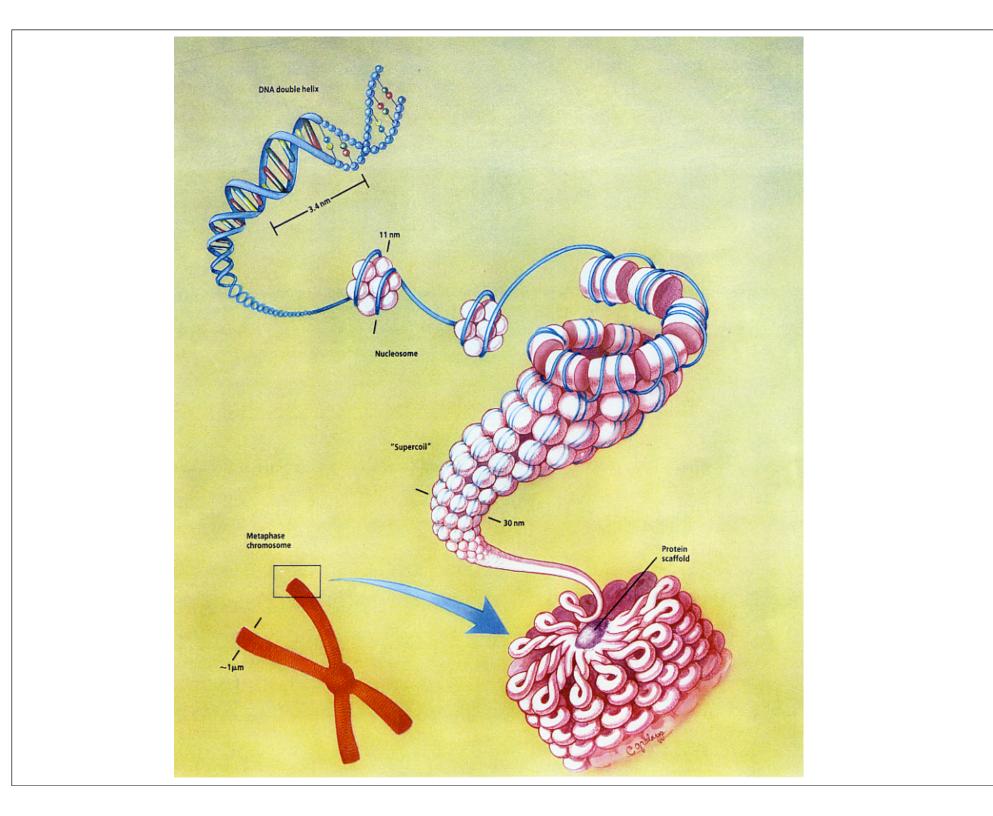
G. Spencer-Brown

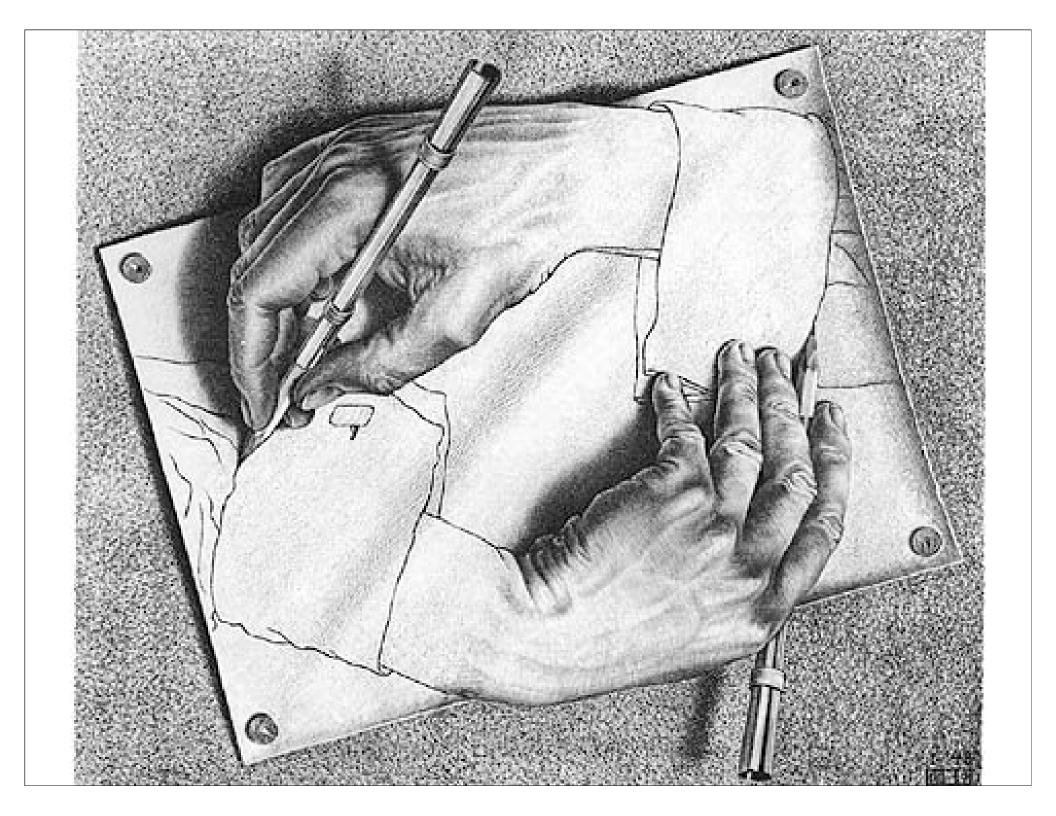
"[Seemingly]

The laws of physics, the so-called 'laws of nature', can be described by us.

The laws of brain functions - or ever more generally - the laws of biology, must be written in such a way that the writing of these laws can be deduced from them, i.e. that they have to write themselves."

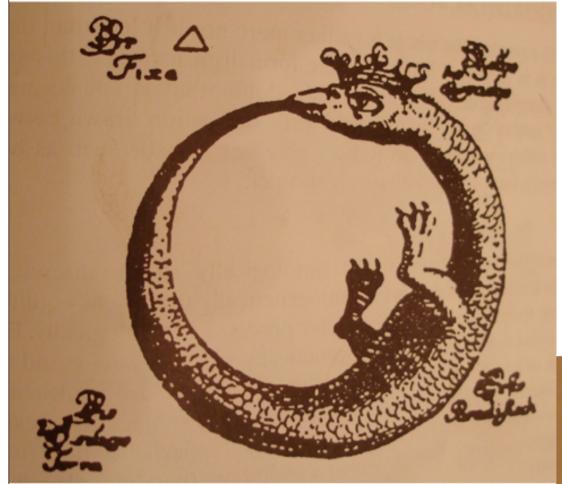
HVF, Cybernetics of Epistemology (1973).





ASC - New Logo





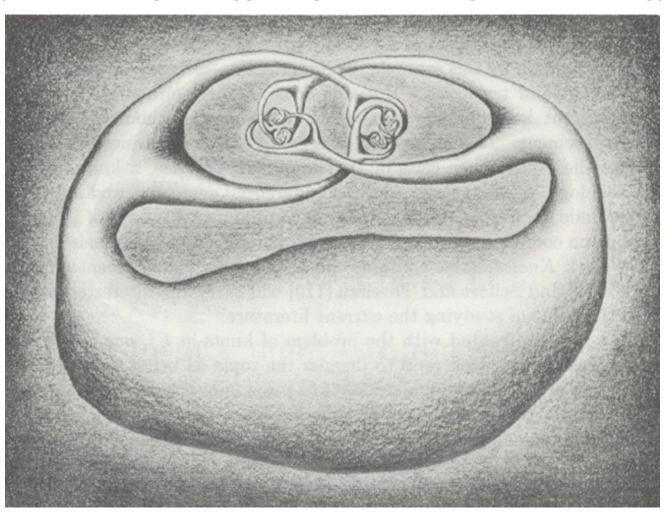
Ouroboros

Ourobori

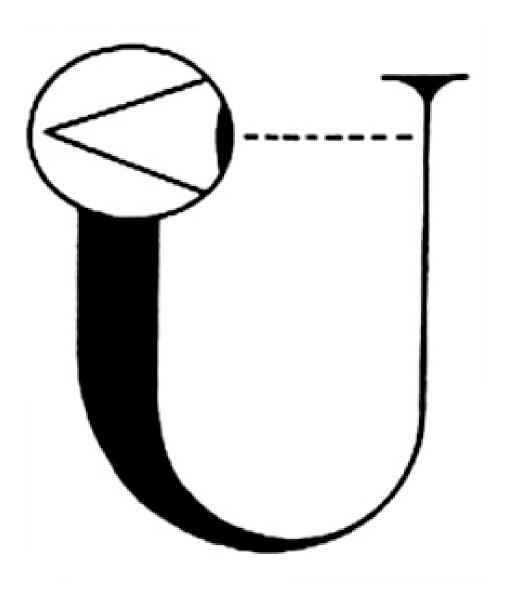




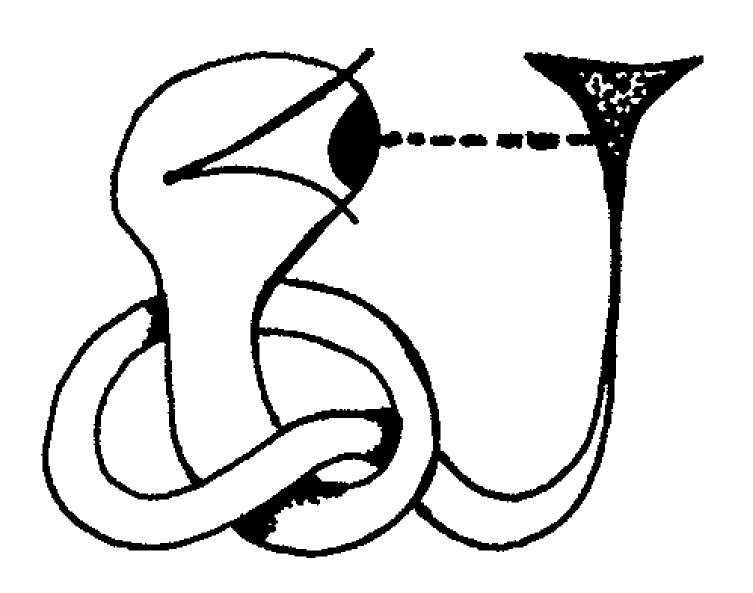
The Alexander Horned Sphere (from "Topology" by Hocking and Young)



John Wheeler's Universe as Self-Excited Circuit



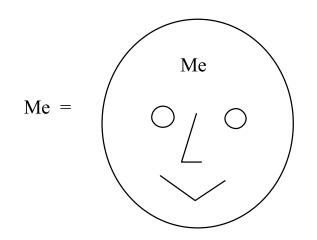
Knot Wheeler Universe

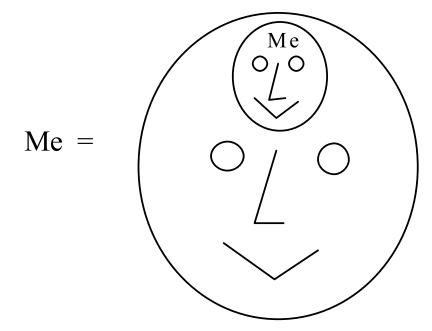


 Ω

$$\Omega = \{\Omega\}$$

$$\Omega \in \Omega$$





The Duplicating Gremlin Creates The Re-entering Mark.

$$A = \overline{AA}$$

$$A = \overline{AA}$$
Hence
$$A = \overline{AA}$$

How does Self-Reference Arise in Language?

Two Fundamental Operations

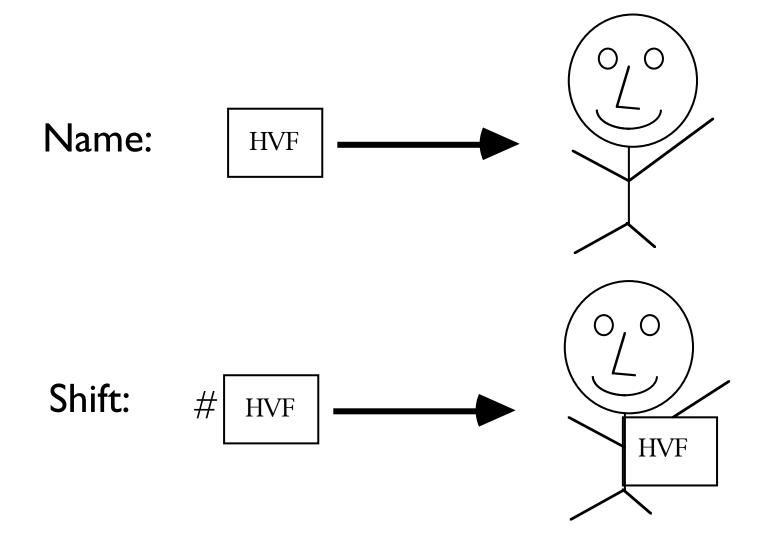
I. Naming.

A becomes $N(A) \longrightarrow A$.

2. Indicative Shift.

A ----> B becomes #A ----> AB

The Indicative Shift



When the meta-naming operation acquires a shifted name, that name refers to itself.

A statement F that talks about the indicative shift, becomes a statement that talks about its own name.

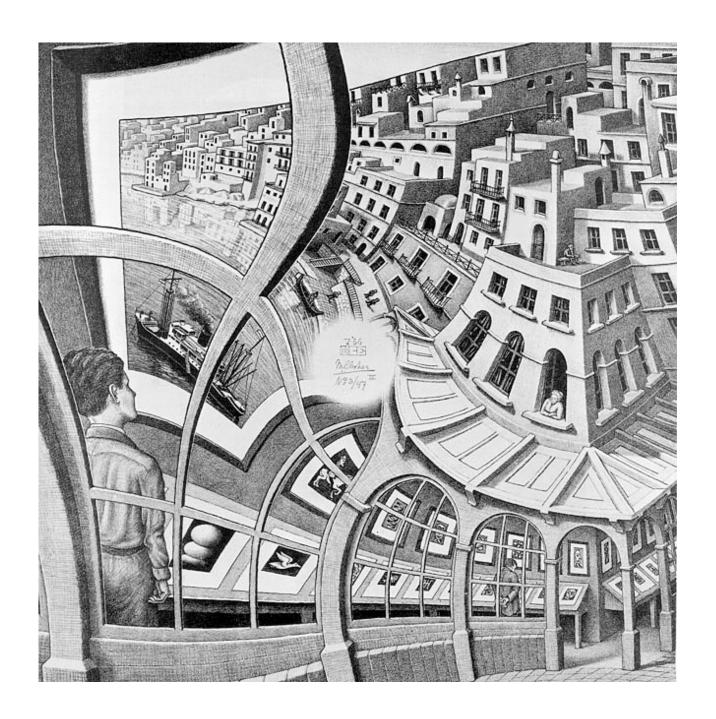
This statement declares its own validity.

Anyone reading this statement will be unable to verify its truth.

This statement is false.

If this statement is true then Unicorns exist.

There is no proof of this statement within the formal system in which it is written.





A reflexive space S is a space where the points in S are in I-I correspondence with the mappings of S to itself.

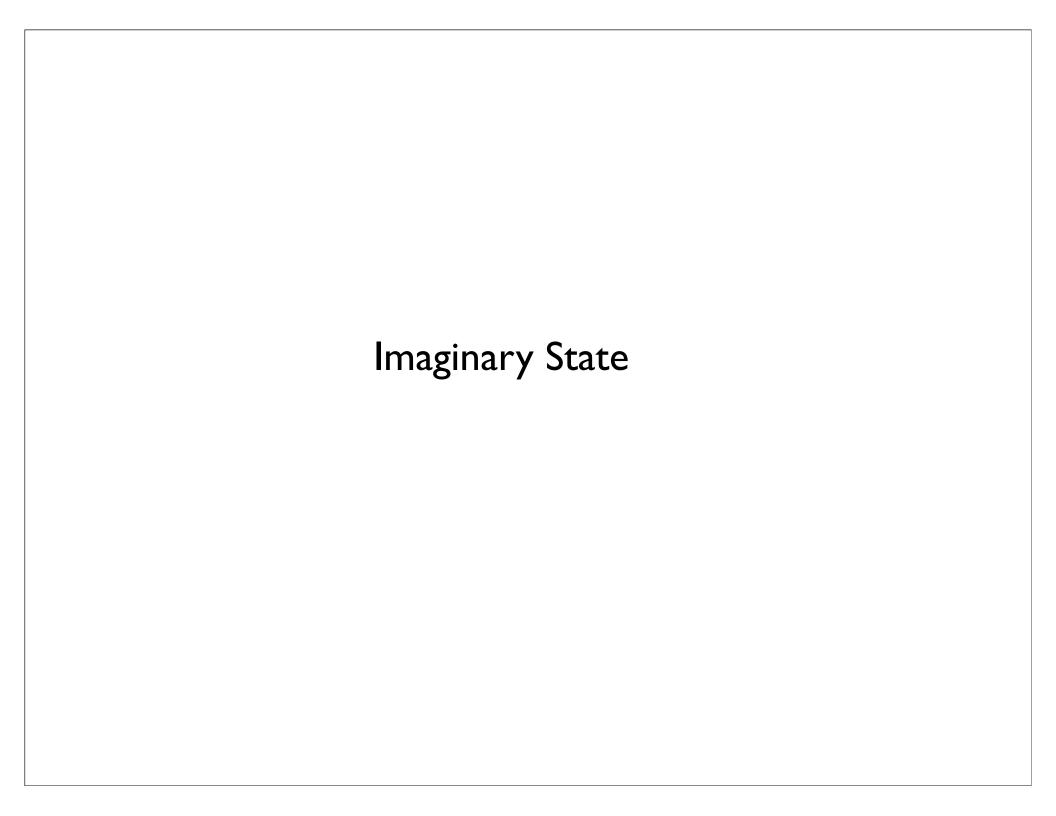


The Reflexive Existence of Fixed Points and Self-Reference.

(In a domain where entities are processes and new processes become new entities.)

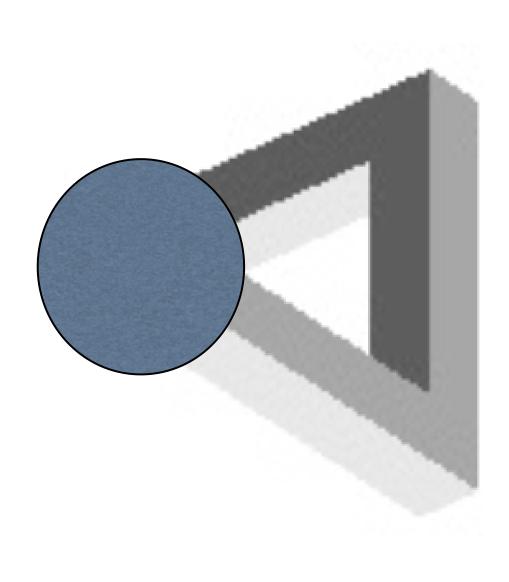
In a reflexive domain, self-reference and the logic of self-reference arise inevitably and must be taken into account. All logics and all systems of modeling that avoid this issue are incomplete reflections of the whole.

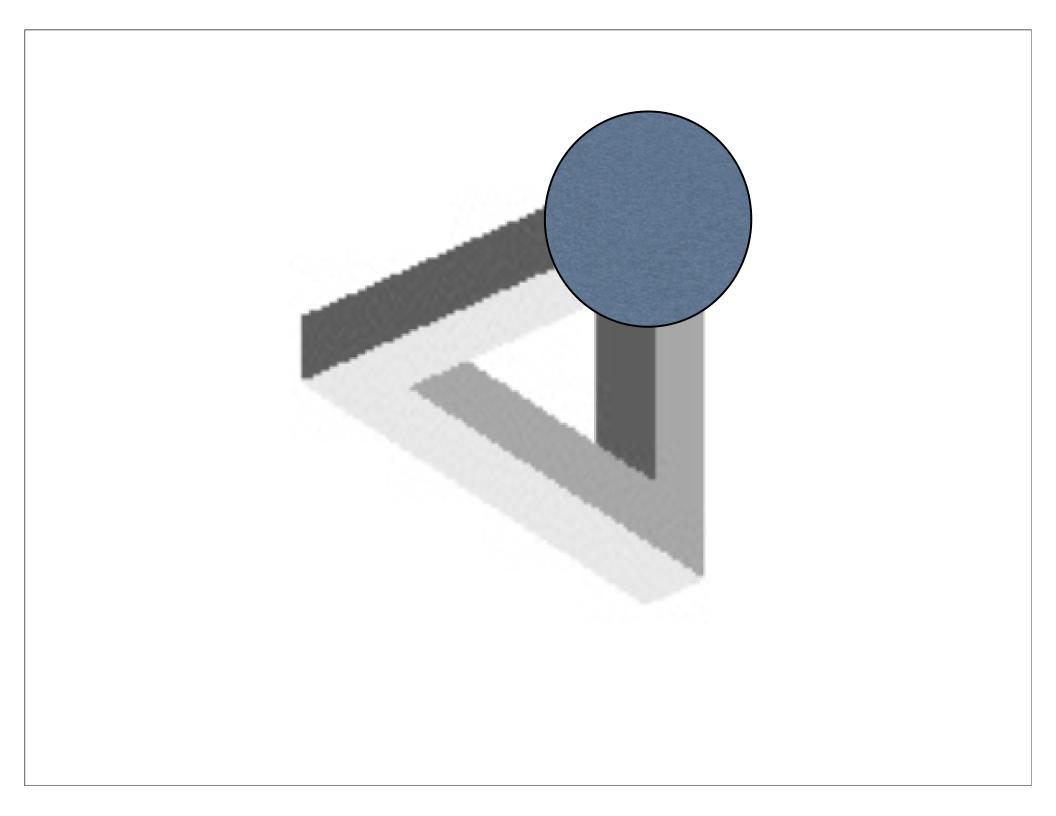
This understanding is only the beginning, a first step in the direction of creating and designing a reflexive world where the world and its models are part of a larger whole that is that world.

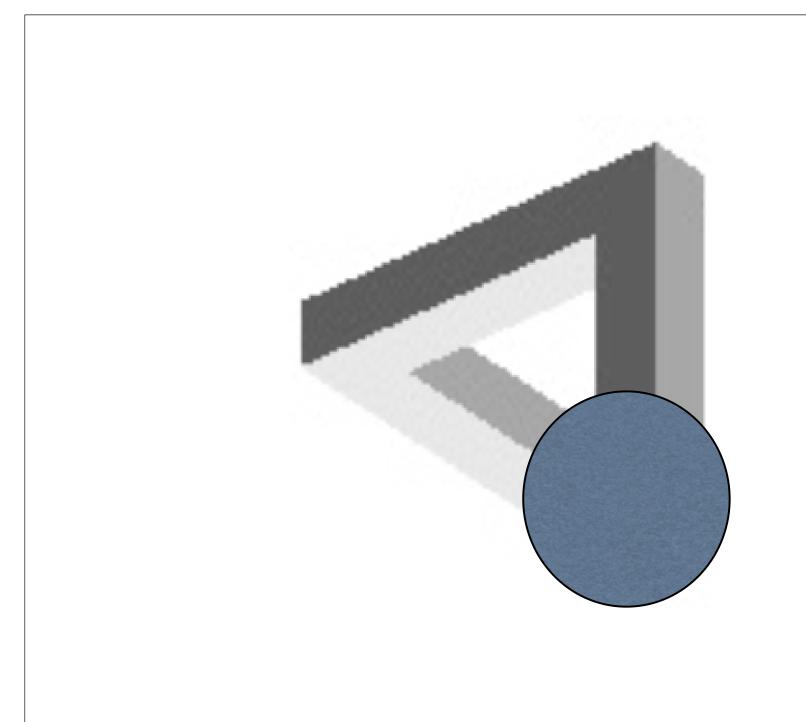


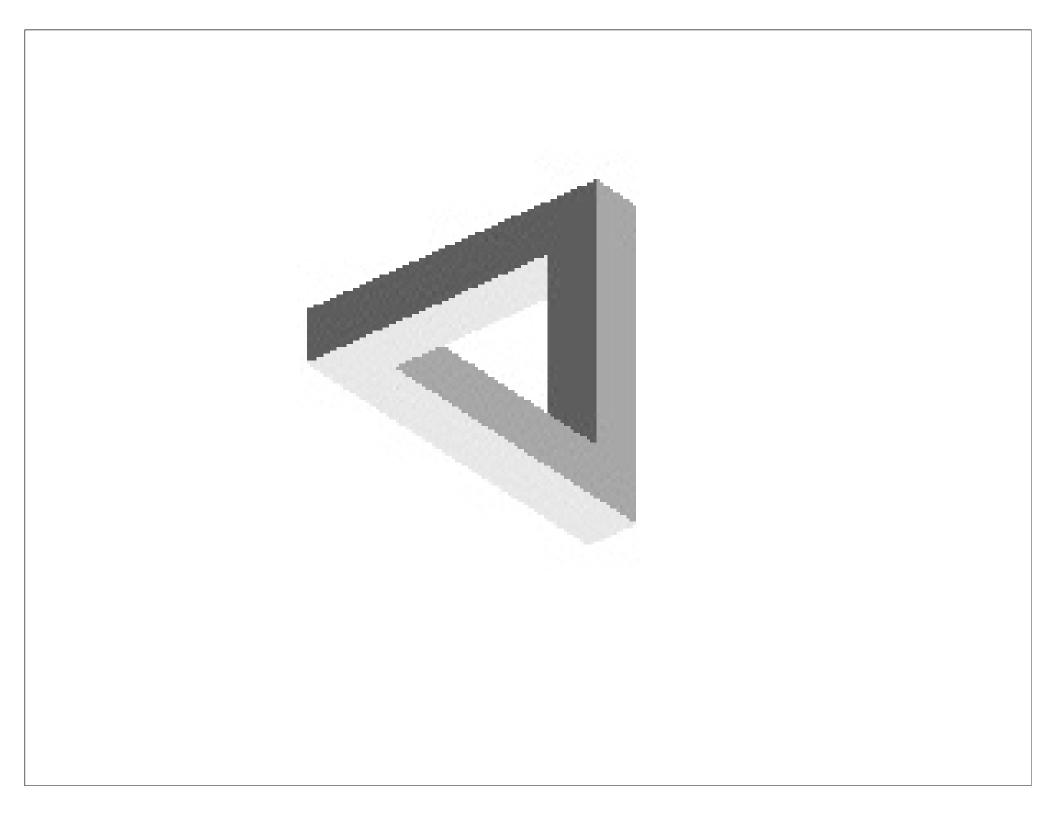
The Non-Locality of Impossibility

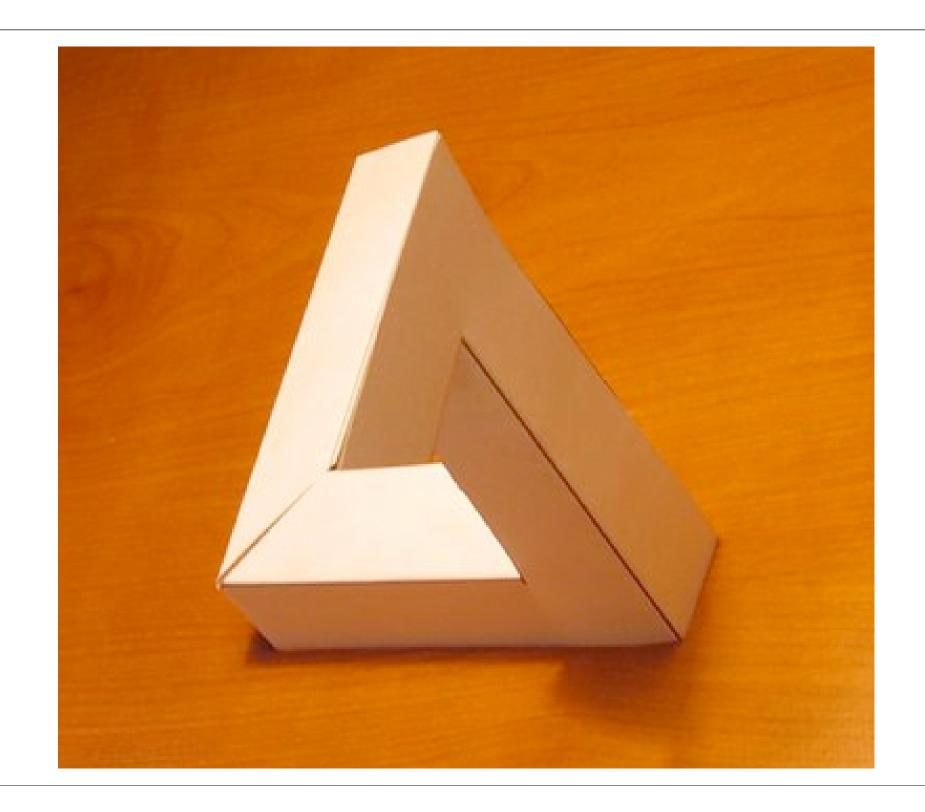


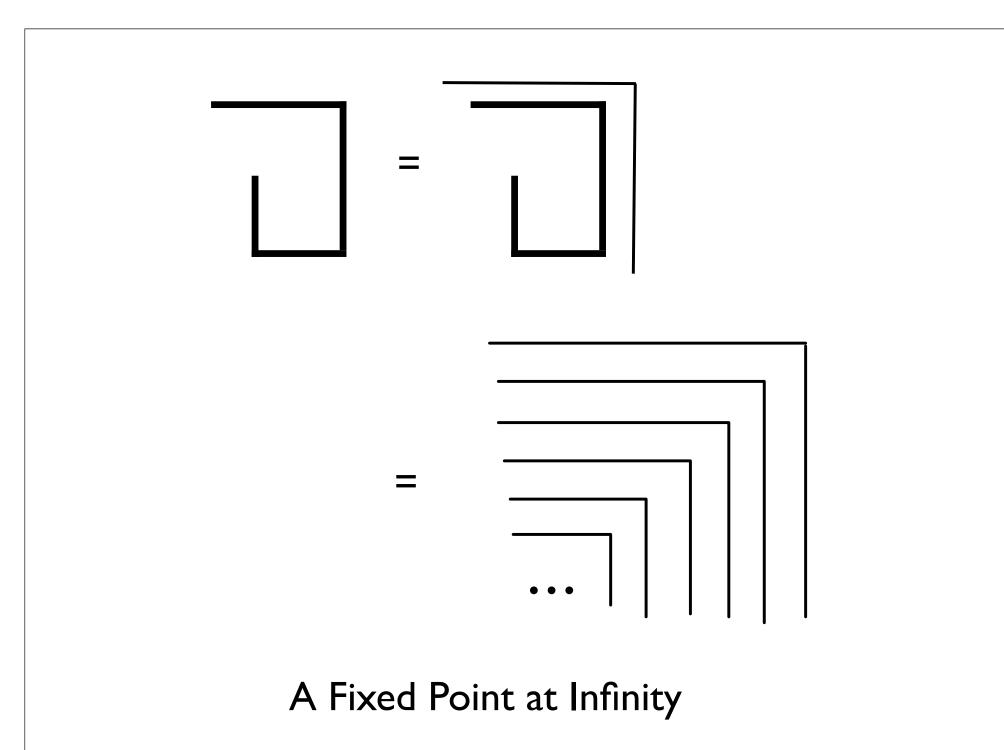












Fixed Points (Eigenforms) Exist

Theorem. Every recursion has a fixed point. Proof. Let the recursion be given by an equation of the form

$$X' = F(X)$$

where X' denotes the next value of X and F encapsulates the function or rule that brings the recursion to its next step. Here F and X can be any descriptors of actor and actant that are relevant to the recursion being studied. Now form

$$J = F(F(F(F(\ldots)))),$$

the infinite concatenation of **F** upon itself.

Then, we see that

$$\mathbf{F}(\mathbf{J}) = \mathbf{F}(\mathbf{F}(\mathbf{F}(\mathbf{F}(\mathbf{F}(\ldots))))) = \mathbf{J}.$$

Hence, $\bf J$ is a fixed point for the recursion and we have proved that every recursion has a fixed point. QED

The Duplicating Gremlin Creates The Re-entering Mark.

$$A = \overline{AA}$$

$$A = \overline{AA}$$
Hence
$$A = \overline{AA}$$

The Reflexive Existence of Fixed Points

(In a domain where entities are processes and new processes become new entities.)

We can explore this fixed point theorem from many angles.

- I. The process of F(F(F(F(...F(*))))).
- 2. The forms that emerge from a process.
- 3. The symbolic forms that emerge from a process.
 - 4. The discoveries of new forms related to the creativity of a process.
 - 5. The nature of process itself.

New
$$X = F(X)$$
.
New $X = Old X + G(X)$.

(Conservation Plus Difference in the circularity.)
6. The nature of changing process in the course of process.

Process and Bios

The next few slides illustrate examples of what H. Sabelli and LK call biotic process or bios. These are processes that are created via bipolar feedback as in A(t+1) = A(t) + g Sin(A(t)).

The key point about such recursion is that there is both conservation (A(t)) is part of the production of A(t+1)) and feedback.

The feedback depends non-trivially on the previous stage, and it is bipolar.

Bipolar feedback processes of this sort are fundamental, and they produce more than just chaos. They produce highly self-correlated states that we call bios, as the patterns of bios occur in many biological situations (such as the measurement of heartbeat intervals). Bios occurs in many natural and human circumstances and is characterized by exhibiting creativity in that its reccurence rate is less than random and a particular complexity that can be seen in recurrence plots and other tests.

More generally, we are concerned with fundamental creative process and underlying principles related to the mathematical concepts of Order

Algebra

Topology

Order -- Assymetry, Lattice, Time

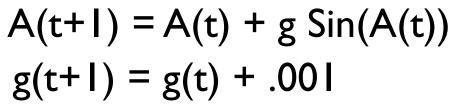
Algebra - Forms of combination, polarity, yin-yang, unity of opposites, creation from opposites.

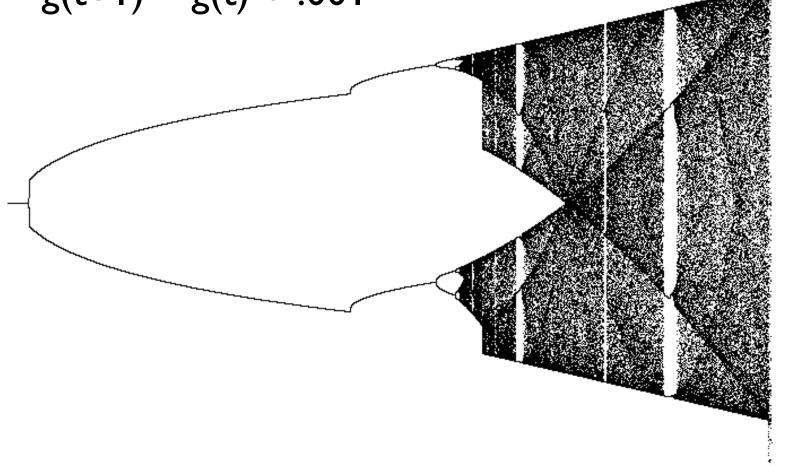
Topology - connection, continuity, change.

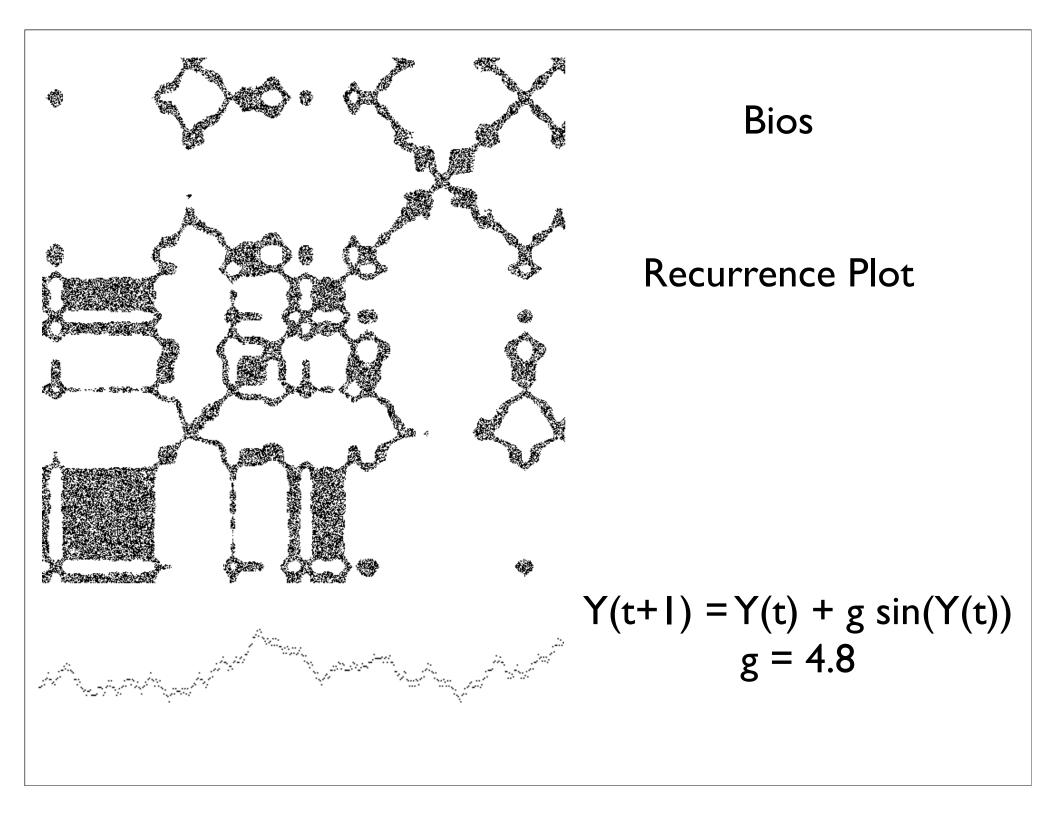
Even simple mathematical recursions exhibit creativity in this sense. The world is fundamentally creative and we partake in that creativity and diversity.

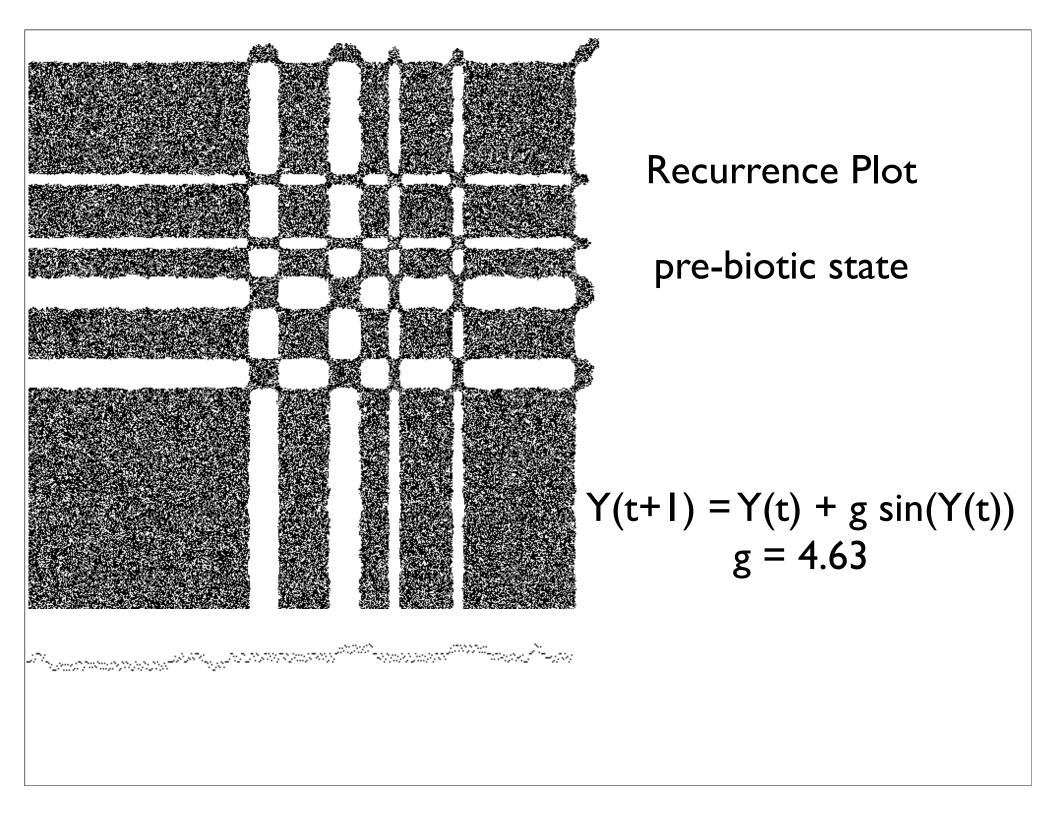
All processes have Action = Energy x Time.

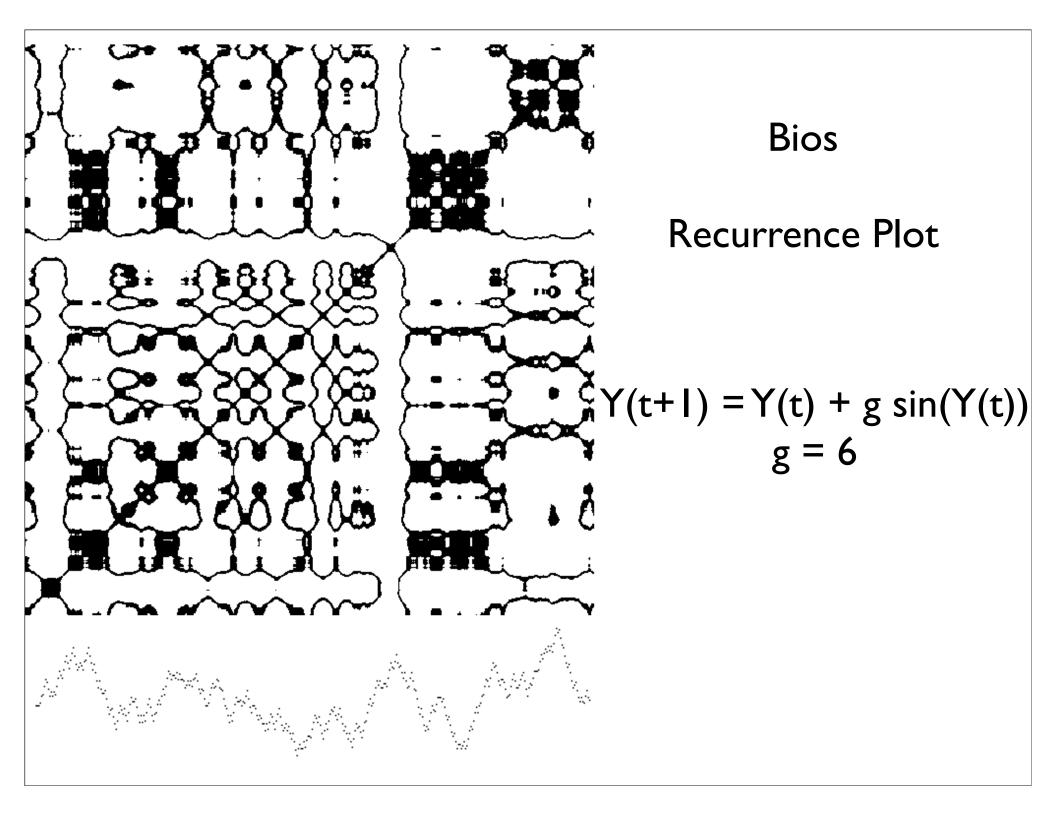
Process Equation - Kinetic Plot

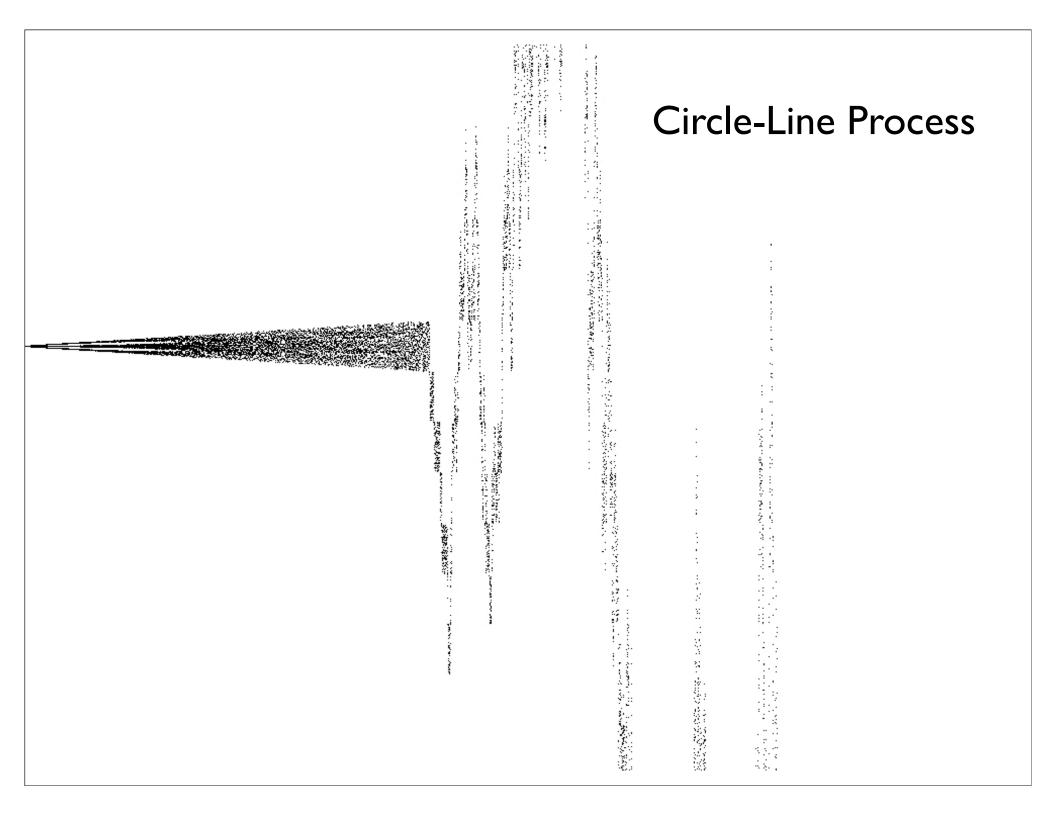


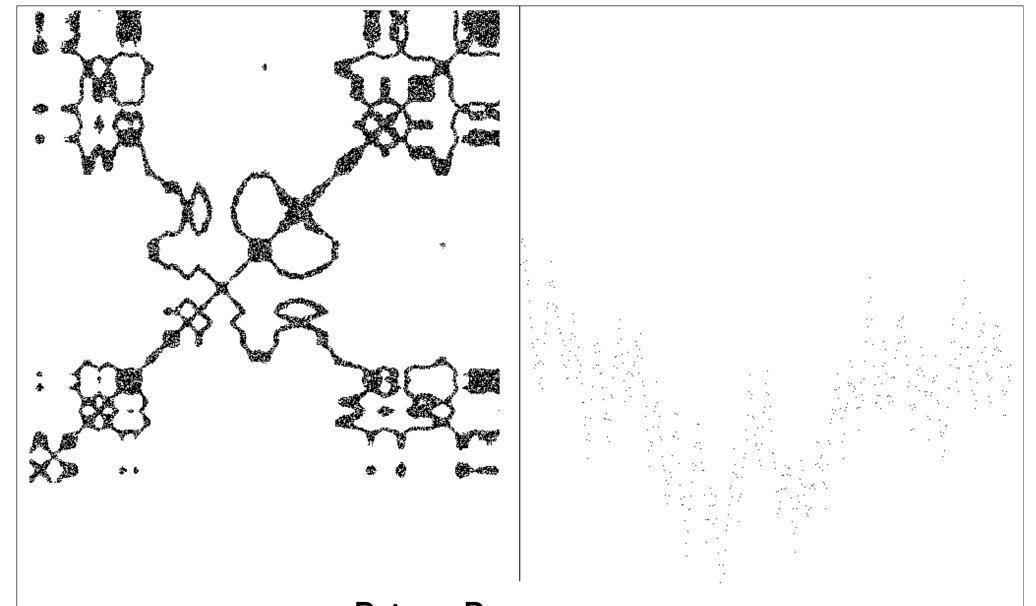








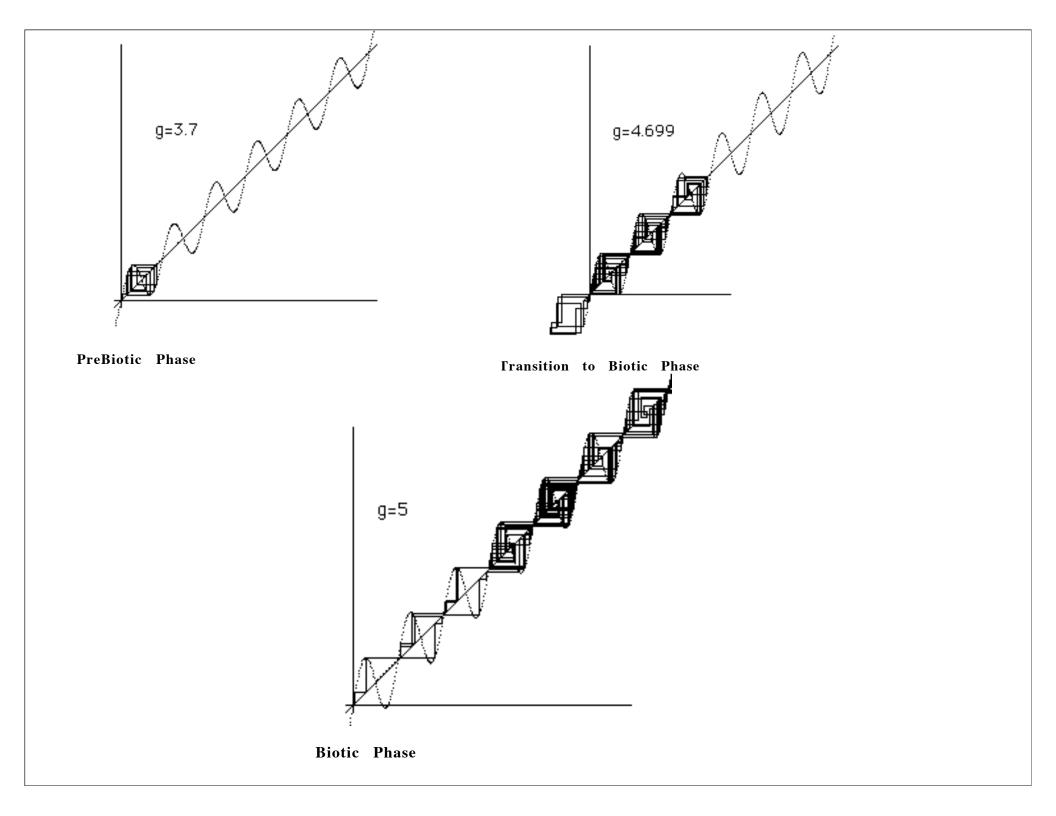




Prime Process:

A(t+1) = A(t) + Sin(P(t))

P(t) = Number of Prime Numbers Less Than t



REFLEXIVE SPACE

A reflexive space S is a space where the points in S are in I-I correspondence with the mappings of S to itself.

(A domain where entities are processes and new processes become new entities.)

A reflexive space S is a space where the points in S are in I-I correspondence with the mappings of S to itself.



Church-Curry Fixed Point Theorem

Theorem. Every F has a fixed point. (in contexts where entities can act upon themselves)

Proof. Let
$$gx = F(xx)$$
. Then

$$gg = F(gg)$$
.

QED

A reflexive space S is a space where the points in S are in I-I correspondence with the mappings of S to itself.

$$r: D \longrightarrow [D, D]$$

a I-I correspondence of D and [D, D].

Fixed Point Theorem: If D is a reflexive space and $T:D \longrightarrow D$

is any mapping from D to D, then there is an A in D such that T(A) = A.

Fixed Point Theorem: If D is a reflexive space and $T:D \longrightarrow D$

is any mapping from D to D, then there is an A in D such that T(A) = A.

Proof. Define a new mapping S by the formula Sx = T(r(x)x). S = r(z). r(z)x = T(r(x)x).

$$r(z)z = T(r(z)z).$$

Let $A = r(z)z.$

$$T(A) = A.$$
QED.

A reflexive space S is a space where the points in S are in I-I correspondence with the mappings of S to itself.

This definition could be a mathematician's conception. This depends upon what you might mean by "points" and by "mappings".

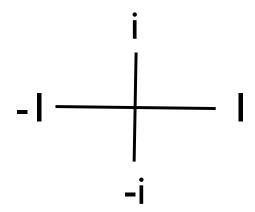
Points have particularity, timelessness. Mappings have action, possibly recursion.

A concept has particularity of statement.

The collecting of that which satisfies the concept has action.

The eigenform (fixed point) always exists, but it may be imaginary with respect to our present Reality.

There is no real number whose square is minus one.



$$i = -1/i$$

$$i = -1/(-1/(-1/...)) = [-1/]$$

$$J = J$$

$$J = I$$

$$J' = I$$

$$I' = I$$

Re-entry and Parenthesis Structure

$$S = I + S < S > enumeration$$

$$\mathbf{E} = \mathbf{I} + \mathbf{v} \mathbf{E}^2$$

 $F = I + xF^2$ generating function

$$f(x) = a + b/x$$

$$f(F) = a + b/F = F$$

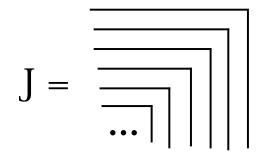
$$=\frac{1}{4} + \frac{1}{2}$$
 | Irrational

$$\begin{array}{c|c} \hline -1 \\ \hline \end{array} = i & Imaginary \\ \hline \\ \downarrow & Iterant \\ \hline \\ \dots + | -| +| -| +| -| \dots \end{aligned}$$

$$J = \overline{J}$$

$$J = \overline{J} \Rightarrow J = \overline{J} = \overline{$$

Implicate



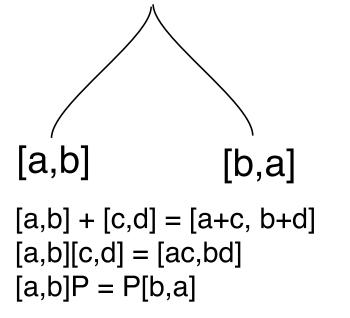
Spatial Explicate



Temporal Explicate

Matrix Algebra as Iterant Multiplication

....abababababababab...



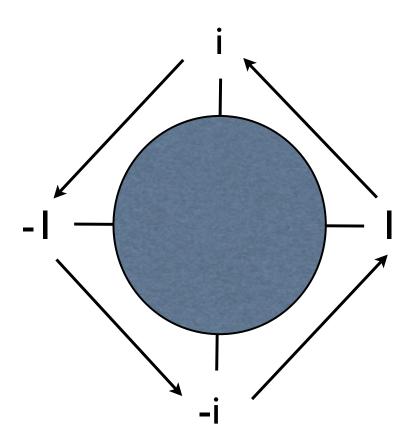
$$i = [1,-1]P$$

 $ii = [1,-1]P[1,-1]P = [1,-1][-1,1]PP = [-1,-1] = -1$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & c \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= [a,d]1 + [b,c]P$$

Possibility and Necessity



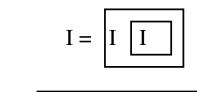
"I am the observed link between myself and observing myself." (HVF)

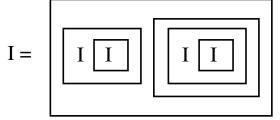
XY = "the link between X and Y".

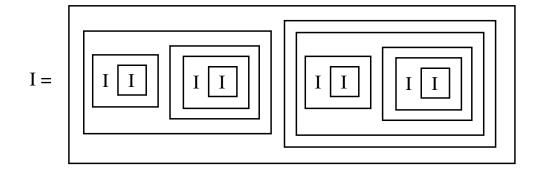
$$I = \begin{bmatrix} I & I \end{bmatrix}$$

"I am a Fibonacci Form!"

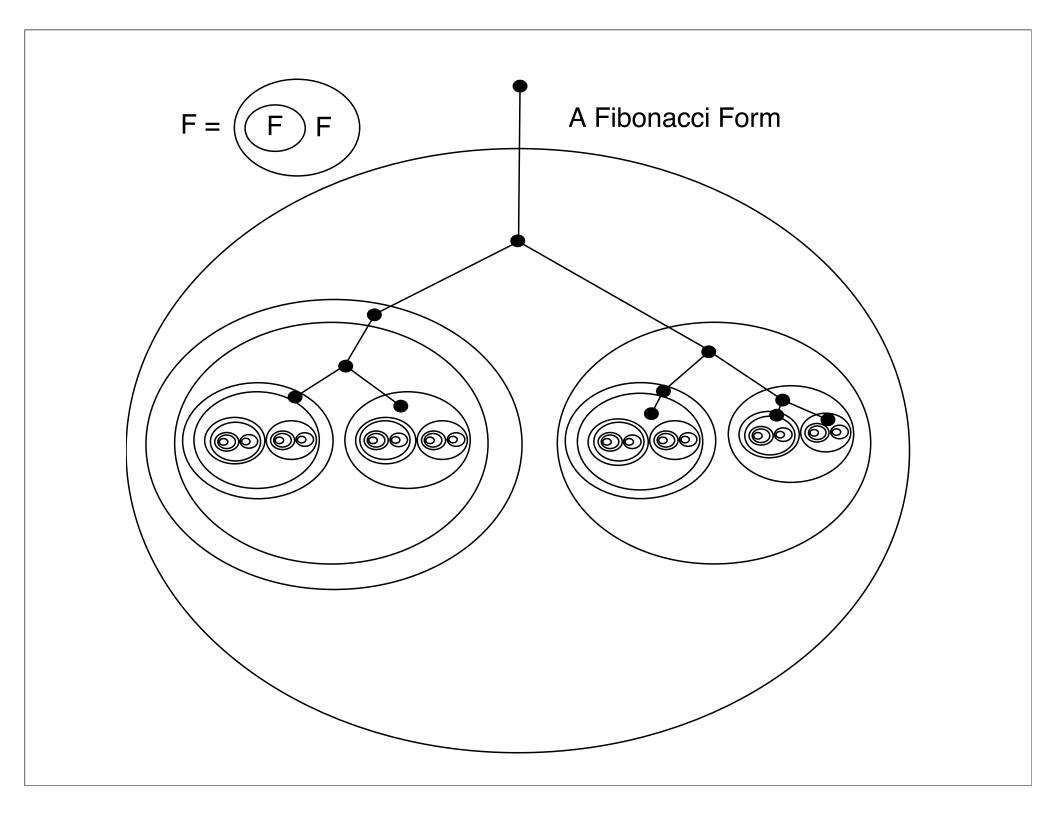
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, ...







The number of divisions of the Fibonaaci Form at depth N is the N-th Fibonacci number.



$$F = \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} = \begin{bmatrix} & & & & \\ & & & & \\ & & & & & \\ & & & & & \end{bmatrix} F F$$

$$= \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} F F$$

Depth Count

$$F_{n+1} = F F F$$

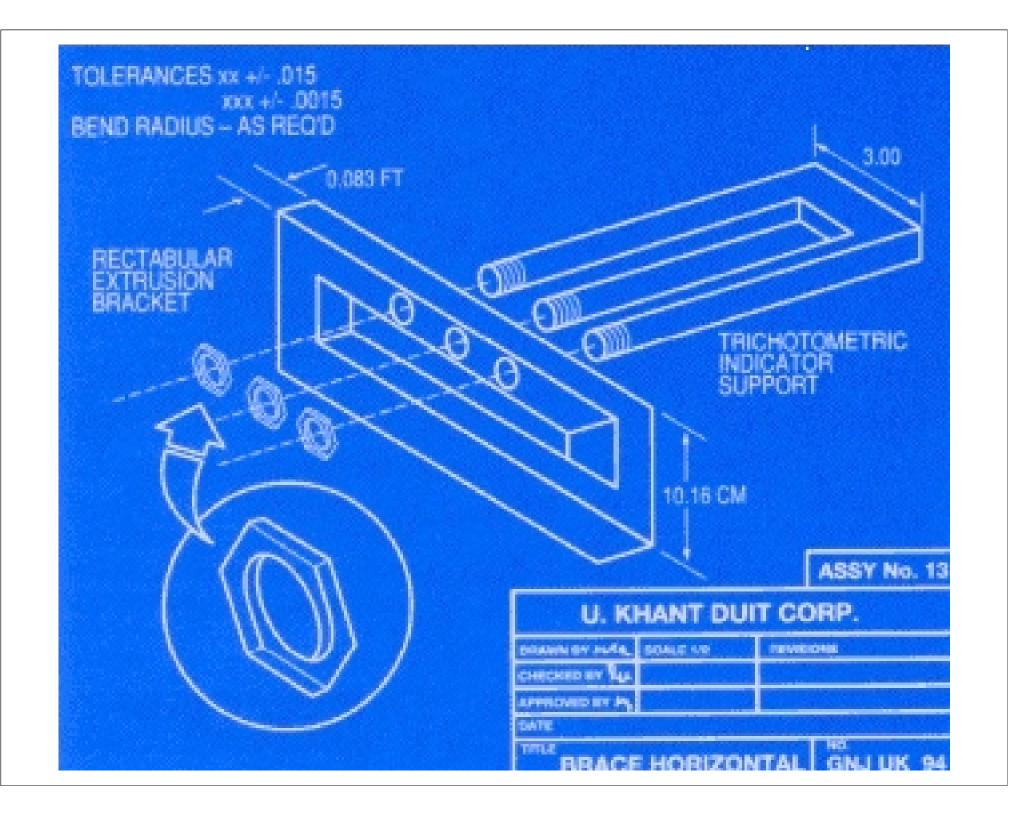
$$= F R$$

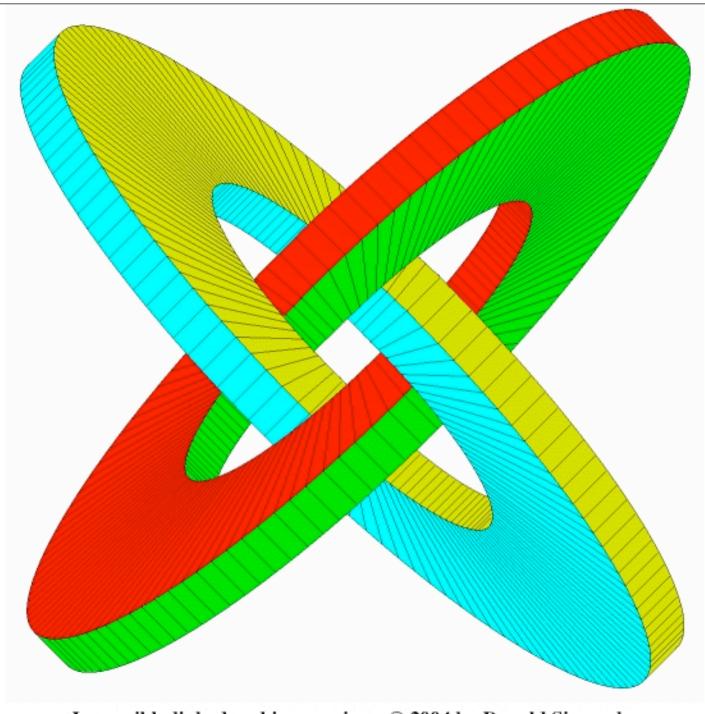
$$= F R$$

$$= R$$

$$F_{n+1} = F_n + F_{n-1}$$
 with $F_0 = F_1 = 1$

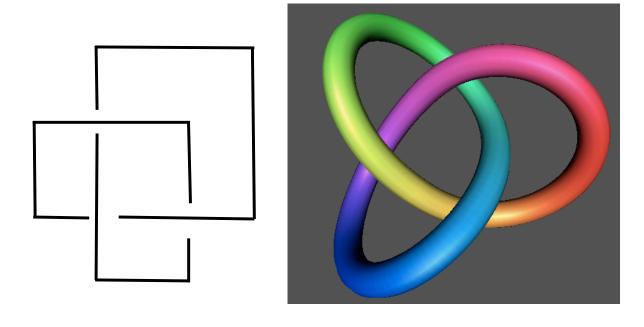
The Golden Rectangle



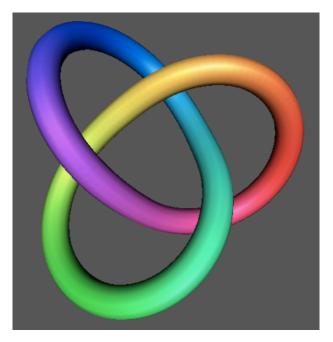


Impossibly linked ambiguous rings. © 2004 by Donald Simanek.

The Imaginary and The Real



Self-Mutuality and Fundamental Triplicity



Trefoil as self-mutuality.

Loops about itself.

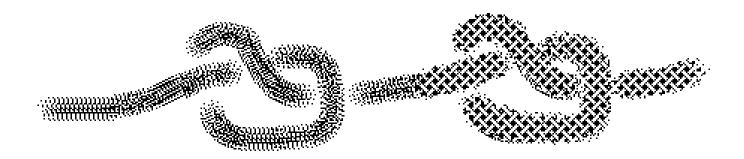
Creates three loopings

In the course of

Closure.

Patterned Integrity

The knot is information independent of the substrate that carries it.



Arithmetic of Knots

It is trivial that composition is associative and that the trivial knot type is a unit. Commutativity may be seen from the following picture.

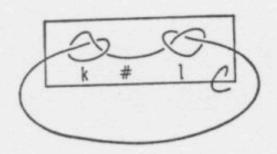


Thus the set of all (tame, oriented) knot types form a commutative

semigroup under the operation #.

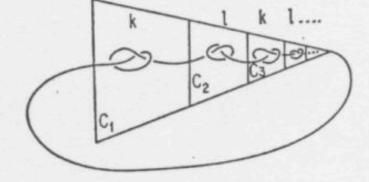
Schubert has proved that, in this semigroup, factorization is unique. Just as in the proof of unique factorization of integers under multiplication, the proof may be made to depend on two fundamental lemmas: (a) finiteness of factorization, and (b) the lemma about prime divisors of a product.

A Wild Proof that You Cannot Cancel Knots



Suppose that there were an autohomeomorphism f of space mapping k # l into 0. It may be arranged that f is the identity outside a cube C whose boundary meets k # l in two points.

Construct the following wild knot m:

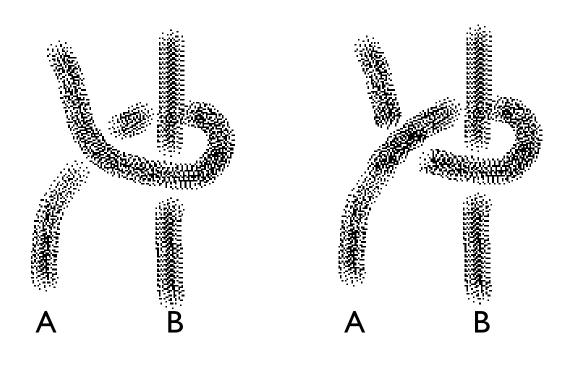


Then there is an autohomeomorphism f_n of space that is the identity outside $C_{2n-1} + C_{2n}$, that replaces $k \# \mathcal{L}$ by 0 inside $C_{2n-1} + C_{2n}$. Defining f to be f_n inside $C_{2n-1} + C_{2n}$ for all n and the identity outside

$$\sum_{i=1}^{\infty} C_i,$$

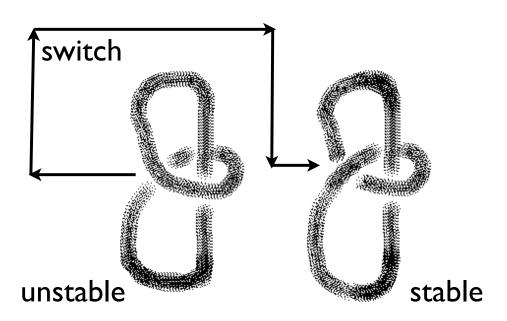
we see that m = 0. Repeating the same construction, using $C_{2n} + C_{2n+1}$, $n = 1, 2, 3, \ldots$, instead of $C_{2n-1} + C_{2n}$, and observing that k # l = l # k, we see that m = k. Consequently k = 0, and hence l = 0.

Observation as Linking

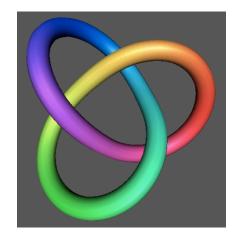


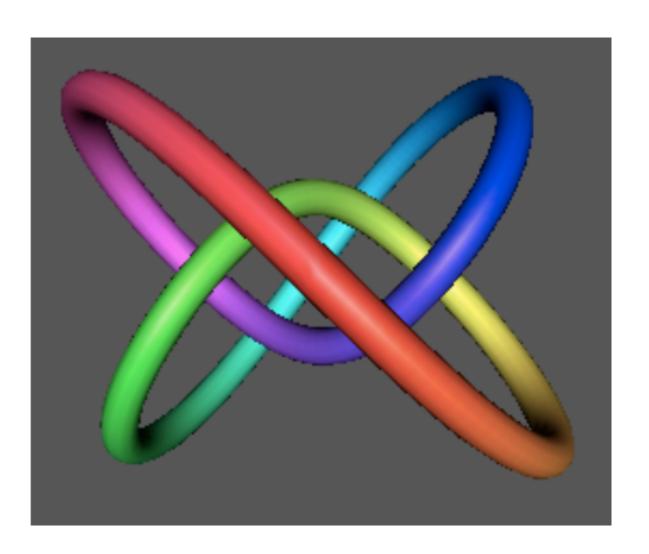
A observes B

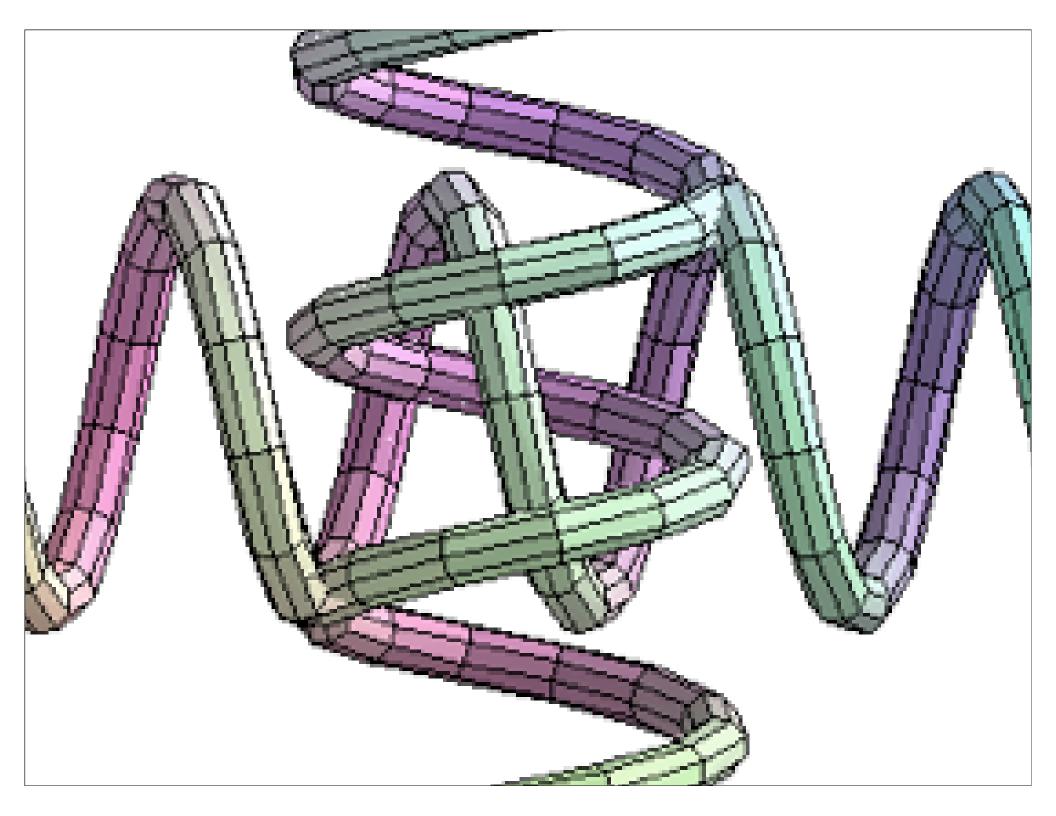
Self-Observation and Observing Observing

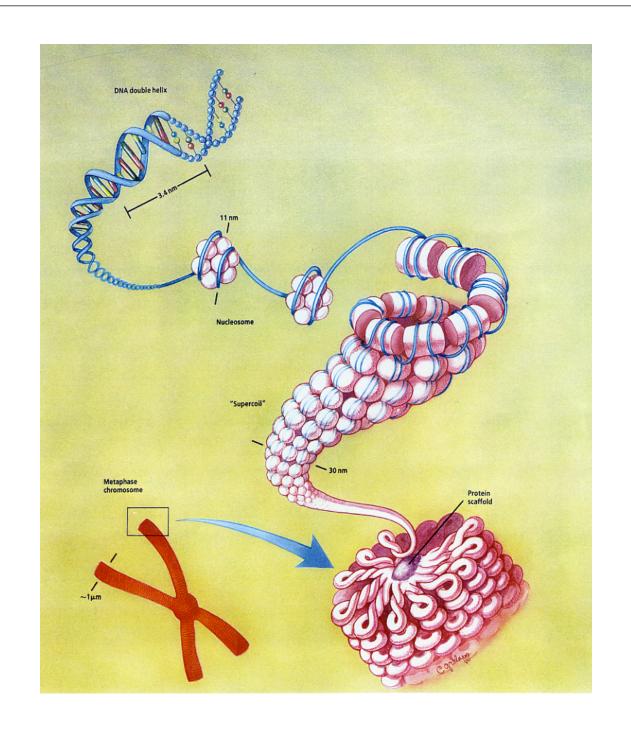


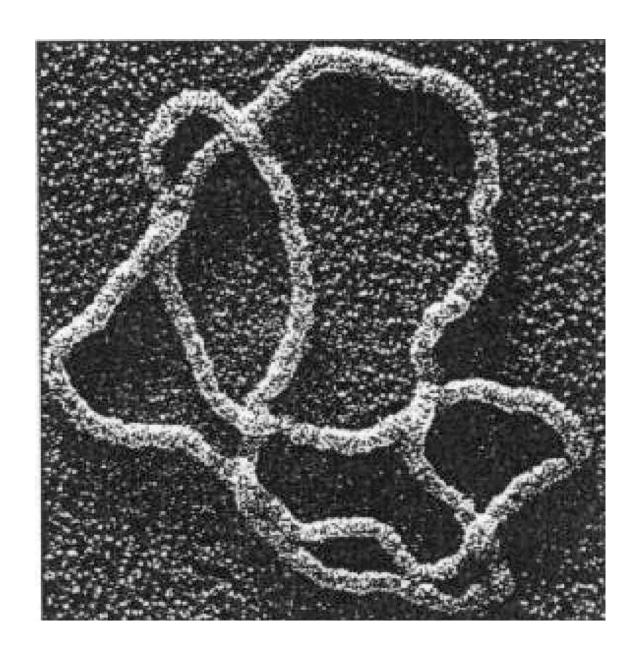




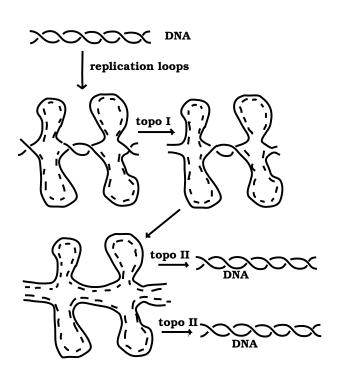








DNA is a Self-Replicating EigenForm



$$DNA = \langle W|C \rangle$$

< W| = < ...TTAGAATAGGTACGCG...|

|C>=|...AATCTTATCCATGCGC...>.

$$< W| + E \longrightarrow < W|C> = DNA$$

$$E + |C> \longrightarrow < W|C> = DNA$$

 $< W|C> \longrightarrow < W|+E+|C> = < W|C> < W|C>$

Self Replication Schematic

DNA = <Watson|Crick>

E = Environment

DNA = <>

$$DNA = <> \longrightarrow < E> \longrightarrow <>> = DNA DNA.$$

E is the "environment".

E is replaced by ><.

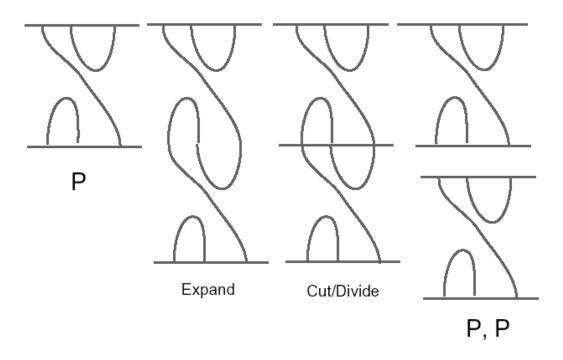
<> is a Container,

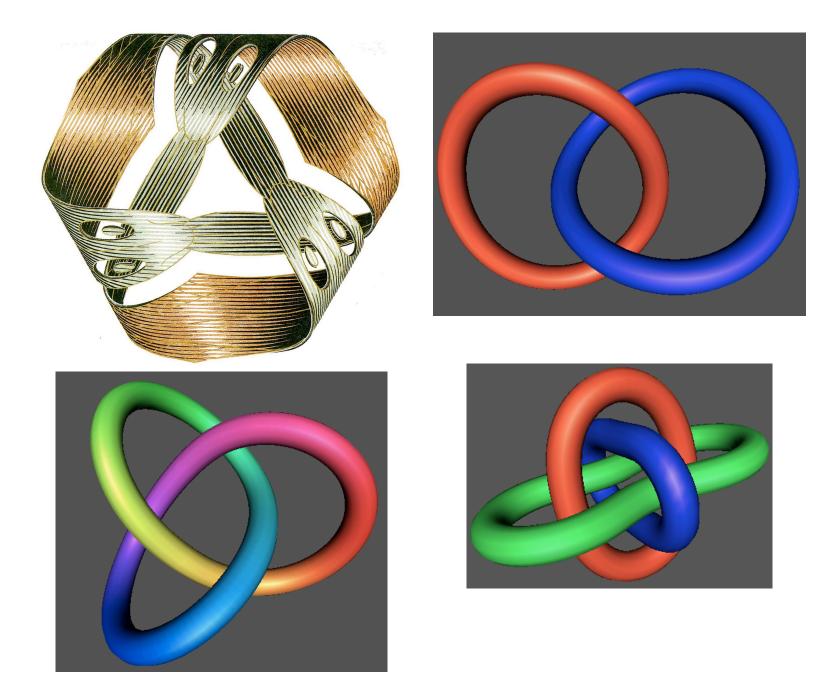
>< is an Extainer.

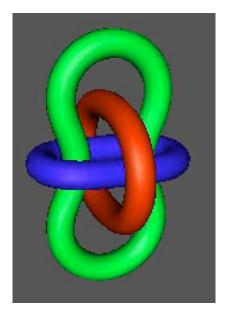
Each produces the Other.

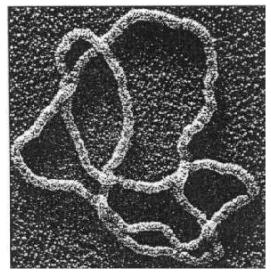
Temperley-Lieb Relations Arise Naturally in an Algebra of Projectors

Topological Replication









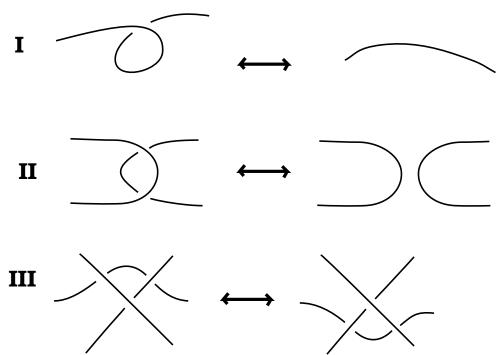


Figure 2 - The Reidemeister Moves.

Reidemeister Moves reformulate knot theory in terms of graph combinatorics.

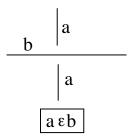
Knot Sets

b

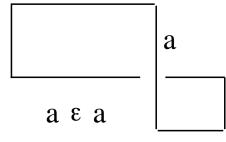
a

aεb

Knot Sets

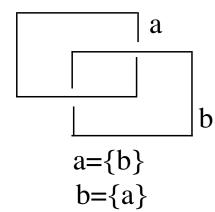


Crossing as Relationship



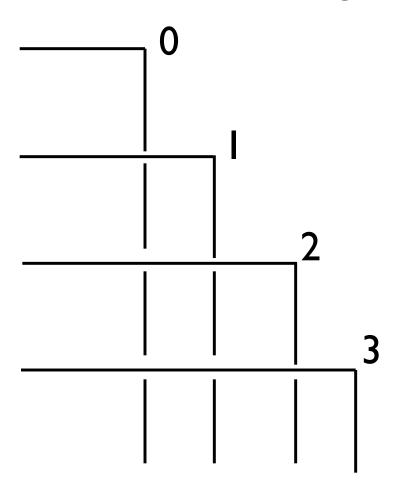
 $a = \{a\}$

Self-Membership

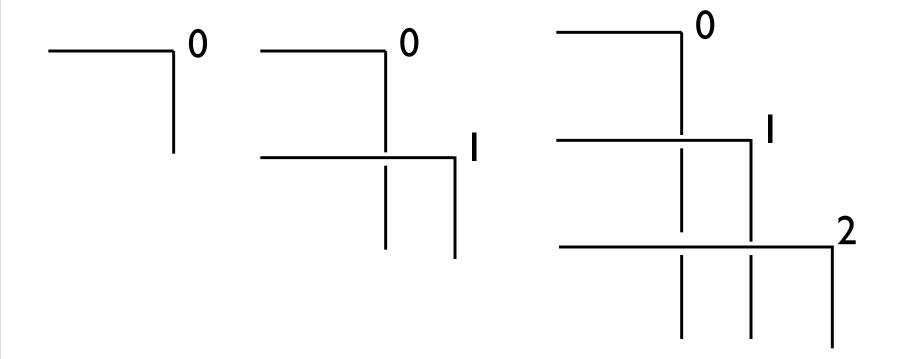


Mutuality

Architecture of Counting



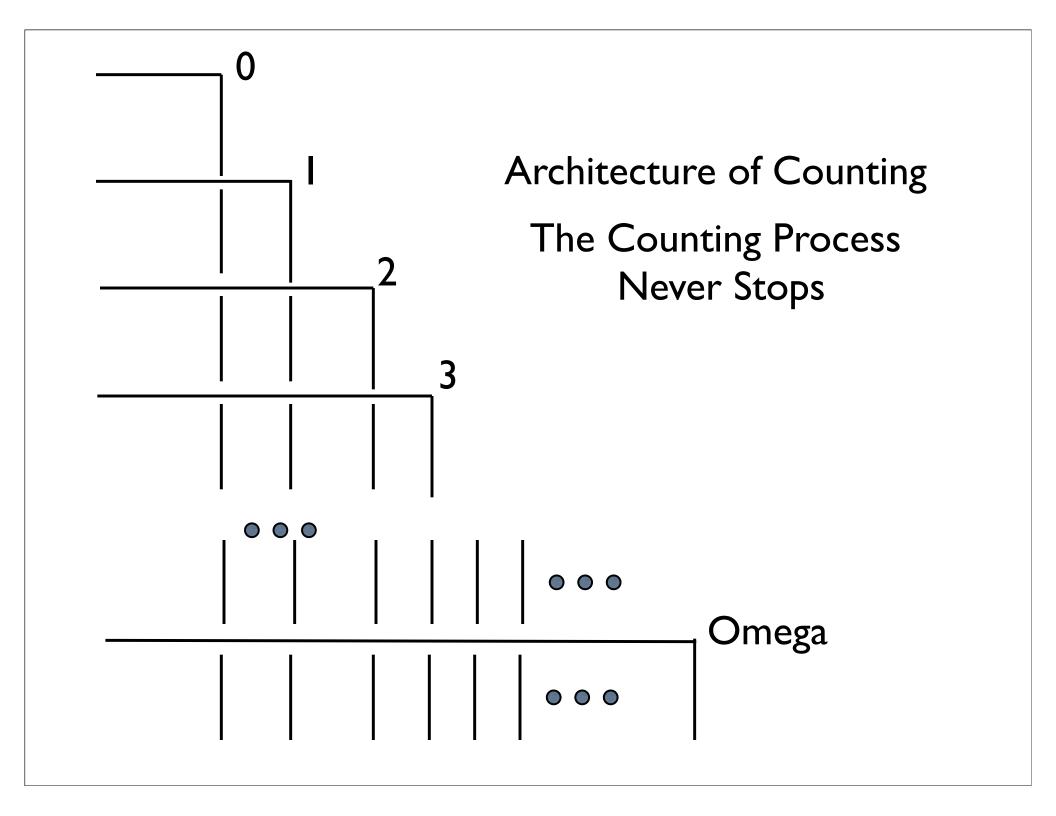
Architecture of Counting



Each new level collects all that has gone before.

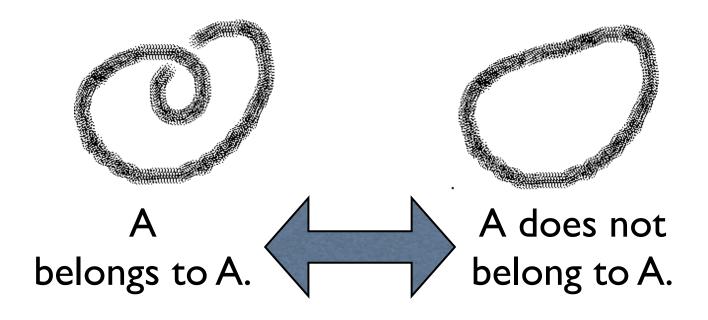
Already this is a solution to the Russell Paradox.

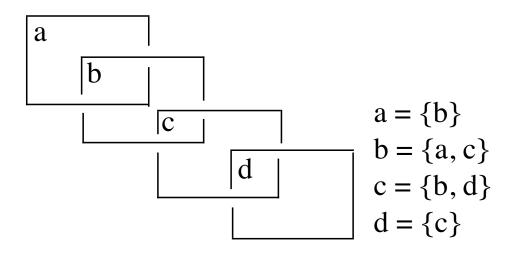
Each new set comes in the demand to write down sets that have not themselves as members. The demand is never met and the creation process continues.

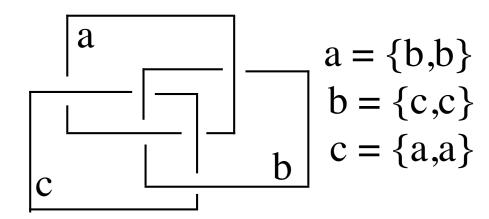


 Ω $\Omega = \left\{\Omega\right\}$ Ω 3 Ω

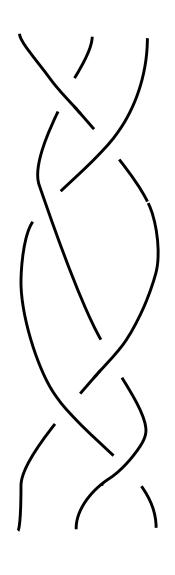
Topological Russell (K)not Paradox



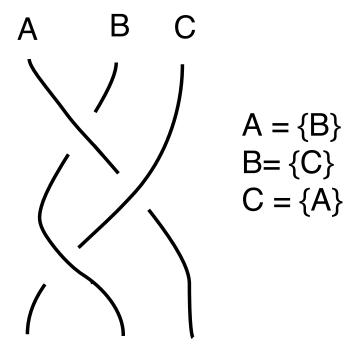


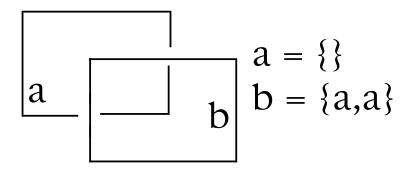


The Borrommean Rings



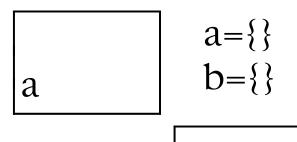
Borromean Braid





topological equivalence

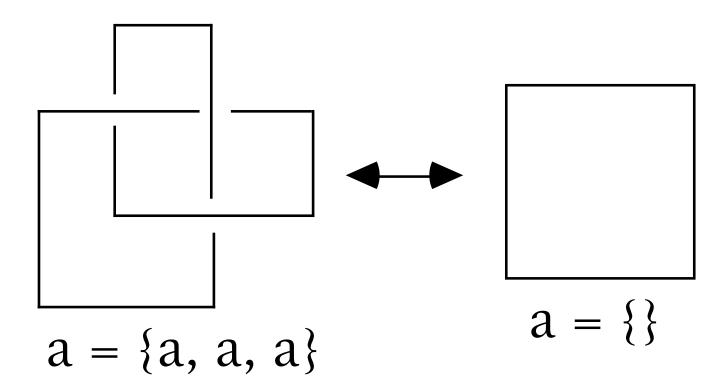
Knot Sets are "Fermionic".
Identical elements cancel in pairs.



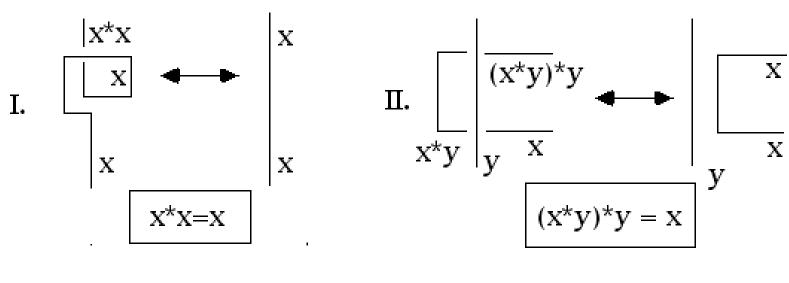
b

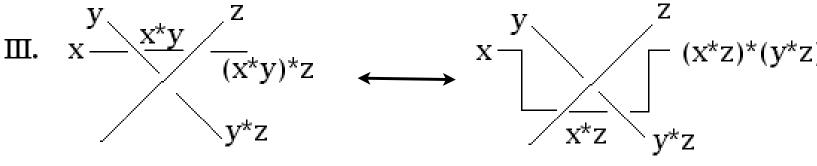
(No problem with invariance under third Reidemeister move.)

Knot sets do not know knots.
But they do provide a non-standard model for sets.



The Next Step - An Algebra of Boundaries





$$(x^*y)^*z = (x^*z)^*(y^*z)$$

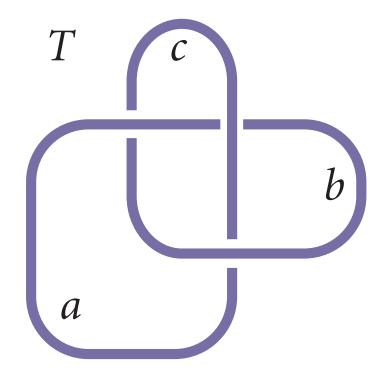
Algebra of Discrimination

$$\frac{ | C = A*B}{B}$$

A and B are distinct.

The distinction is reflected by A*B is neither A nor B.

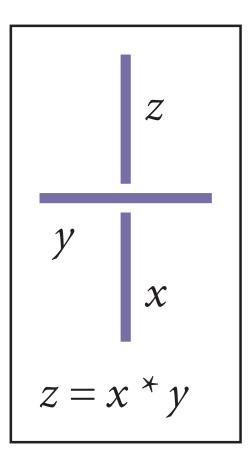
$$A*B = B*A = C$$
 $A*C = C*A = B$
 $B*C = C*B = A$
 $X*X = X$



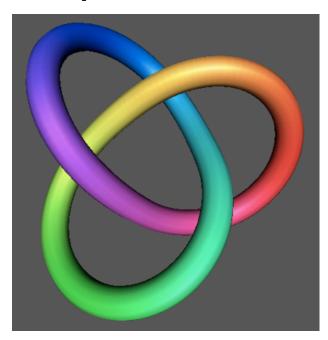
$$b = a * c$$

$$c = b * a$$

$$a = c * b$$



Self-Mutuality and Fundamental Triplicity



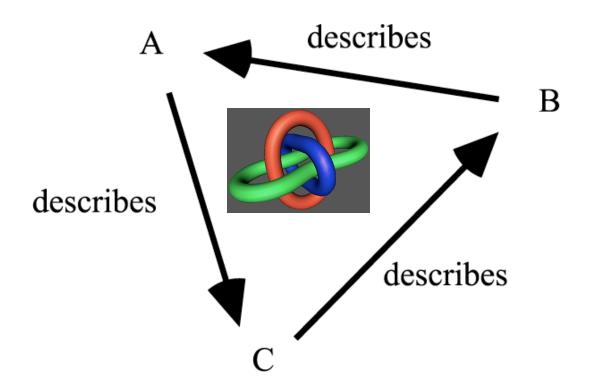
Describing Describing

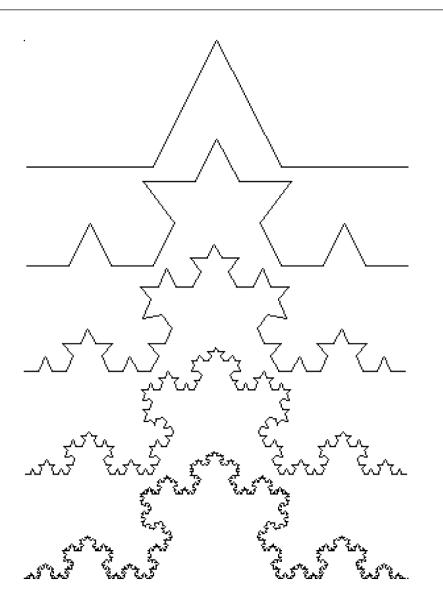
```
| | | *
311*
13211*
111312211*
311311222111*
1321132132311*
11131221131211131213211*
```

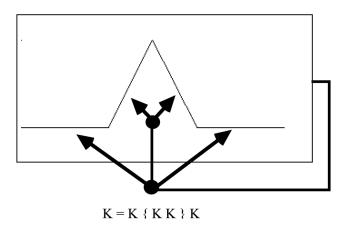
A = 11131221131211132221...

B = 3113112221131112311332...

C = 132113213221133112132123...

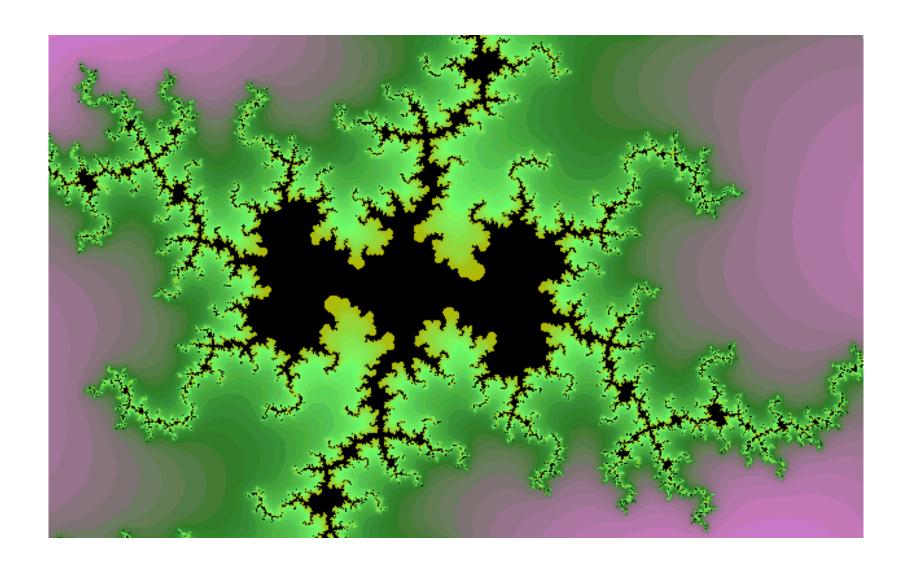






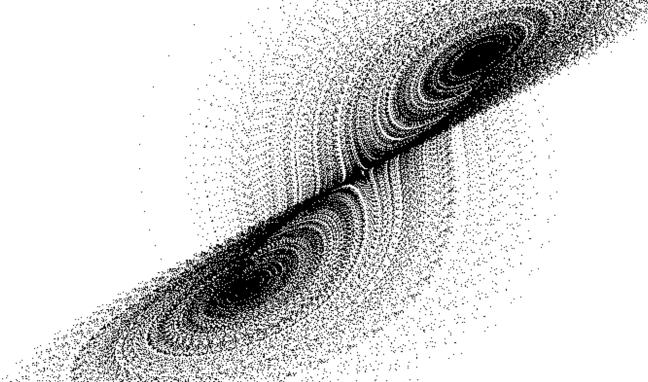
K = K{K K}KThe Framing of Imaginary Space.

Fractal Re-entering Mark





DX=10*(Y-X) DY=X*(28-Z)-Y DZ=X*Y-2.666*Z



THE INDICATIVE SHIFT

$$\begin{array}{cccc} A & \longrightarrow & B \\ \#A & \longrightarrow & BA \end{array}$$

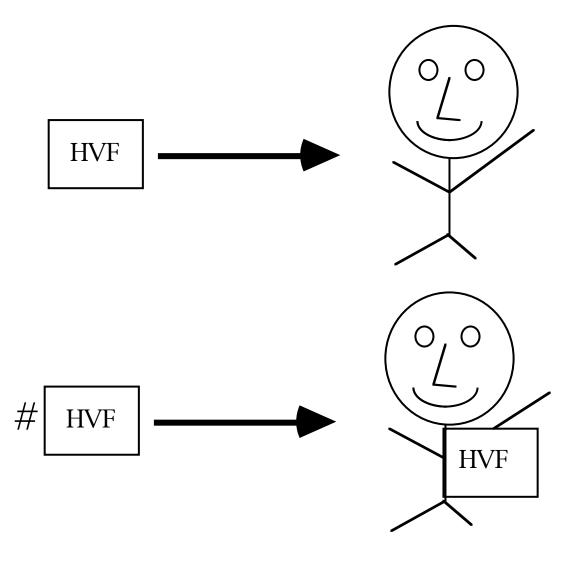
name → object

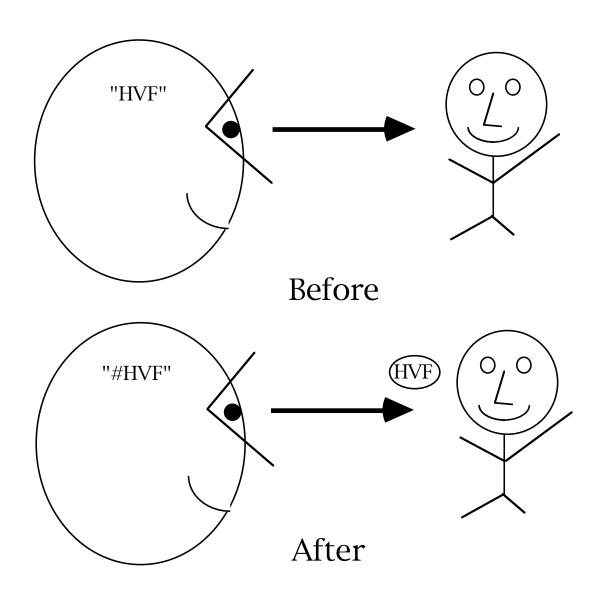
#name → object name

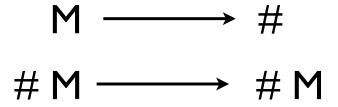
Eigenvalue Occurs

At the Third Departure from Void.

The Indicative Shift







is the operation of observing.

#M \longrightarrow #M

is the act of observing observing.

"I am the
Observed link
Between myself
And
Observing myself."

Goedelian Shift

 $g \longrightarrow F\#$ $\#g \longrightarrow F\#g$

F#g talks about its own name.

The pattern behind Goedel's Theorem where a Sentence states its own Uprovability within a given consistent

Formal System. The Sentence is True Yet Unprovable within that System.

Goedelian Shift



~B#g states its own unprovability.

Rules of the Small Machine

- The SM is a box with a button on it. When you press the button, SM prints a "word" consisting in two consecutive letters on card, and emits the card from a slot on its side. Nothing, other than these two letters, is printed on the card.
- 2 SM uses an alphabet consisting in two letters { N, R }. Thus the words that SM might print are { NR, RN, NN, RR}.
- 3 Letting X denote a second letter (X is either R or N) then the words NX and RX each have a specific meaning:

NX means that the Machine can not print XX.
RX means that the Machine can print XX.

The Machine always tells the truth. Thus if SM should print NX then it will never print XX. If SM should print RX then it can print XX (and may do so or may have already done so).

Hidden Repetition

$$g \longrightarrow A$$

$$\#g \longrightarrow Ag$$

Replace the arrow by an equals sign:

$$g = A$$
 $\#g = gg$
 $\#g = Ag$

$$g \longrightarrow F\#$$
 $g = F\#$
 $\#g \longrightarrow F\#g$ $\#g = F\#g$

#g is an Eigenform for F.

Paradox and Time

$$F OR T = T$$

Therefore

$$J OR \sim J = T.$$

Logically Contradictory?

Flagg Resolution:
There is only one \(\subseteq \).
All appearances of \(\subseteq \) in a given Text
Must be altered together or not at all.

Non-Locality in the Text

The Flagg resolution allows the entry of eigenforms into our discourse without having to change the essential forms of reasoning.

This, at the cost of "textual non-locality".

The Universe is undoubtedly Indistinguishable from Itself.

And yet, a Distinction arises in Mutuality.

Eigenforms such as

J = ~J

could well

be called

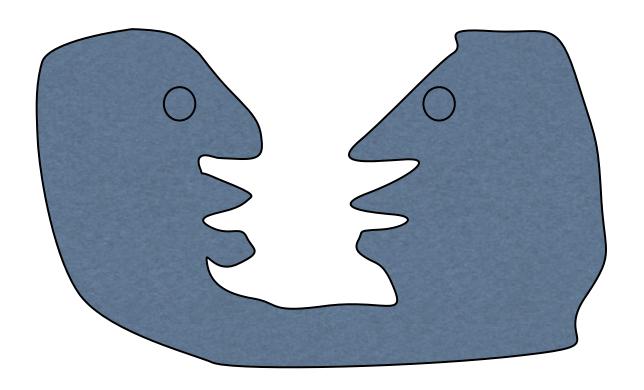
Imaginary values.

Let us not forget the primordial imaginary value, the act of (making) a distinction.

Only the Imaginary is Real.

The universe that we know
Comes into being
Through Imagination
and Observation
Bringing forth
A world of distinctions that
we take to be
Real.

The art of distinction is Inseparable from The art of Joining.



In order for a universe to come into being the world must act to divide itself into one part that is seen and another part that sees.

Quality, Love Reality, Imagination, and Discrimination are Inseparable. What IS
is identical
In Form
with
What is not.

The Form
we take to exist
arises from
framing
Nothing.

